LP

FURTHER SIMPLEX EXAMPLES

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Max $z = 2x_1 - x_2 + x_3$
subject to

$$\begin{cases} 3x_1 + x_2 + x_3 \le 60 \\ x_1 - x_2 + 2x_3 \le 10 \\ x_1 + x_2 - x_3 \le 20 \end{cases}$$

$$x_1, x_2, x_3 \ge 0$$

EXAMPLE ONE

Max $z = 2x_1 - x_2 + x_3$
subject to

$$\begin{cases} 3x_1 + x_2 + x_3 + x_4 = 60 \\ x_1 - x_2 + 2x_3 + x_5 = 10 \\ x_1 + x_2 - x_3 + x_6 = 20 \end{cases}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

Add slack variables to convert the inequalities to equations

	-z	X_1	X_2	X_3	X_4	X_5	X_6	
	1	2	3	4	5	6	7	B
MAX	1	2	-1	1	0	0	0	0
	0	3	1	1	1	0	0	60
	0	1	-1	2	0	1	0	10
	0	1	1	-1	0	0	1	20

EXAMPLE ONE

	1	2 ↓	3	4	5	6	7	B	
MAX	1	2	-1	1	0	0	0	0	
	0	3	1	1	1	0	0	60	
	0	(1)	-1	2	0	1	0	10	←
	0	1	1	-1	0	0	1	20	
	*				*	*	*		

⇓

	1	2	3	4	5	6	7	B
MAX	1	0	1	-3	0	-2	0	-20
	0	0	4	-5	1	-3	0	30
	0	1	-1	2	0	1	0	10
	0	0	2	-3	0	-1	1	10
	*	*			*		*	

EXAMPLE ONE

	1	2	3 ↓	4	5	6	7	B	
MAX	1	0	1	-3	0	-2	0	-20	
	0	0	4	-5	1	-3	0	30	
	0	1	-1	2	0	1	0	10	
	0	0	(2)	-3	0	-1	1	10	←
	*	*			*		*		

⇓

	1	2	3	4	5	6	7	B
MAX	1	0	0	-1.5	0	-1.5	-0.5	-25
	0	0	0	1	1	-1	-2	10
	0	1	0	0.5	0	0.5	0.5	15
	0	0	1	-1.5	0	-0.5	0.5	5
	*	*	*		*			

EXAMPLE ONE

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	
	1	2	3	4	5	6	7	B
MAX	1	0	0	-1.5	0	-1.5	-0.5	-25
	0	0	0	1	1	-1	-2	10
	0	1	0	0.5	0	0.5	0.5	15
	0	0	1	-1.5	0	-0.5	0.5	5
	*	*	*		*			

optimal tableau!

$-z = -25 \Rightarrow z = 25$

basic	*nonbasic*
$z = 25$	$x_3 = 0$
$x_1 = 15$	$x_5 = 0$
$x_2 = 5$	$x_6 = 0$
$x_4 = 10$	

EXAMPLE ONE

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$
subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 - 5x_3 = 20$$
$$x_1 + 2x_2 - x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \ge 0$$

-z and X_4 can serve as the basic variables in the top and bottom rows, respectively.

	-z	x_1	x_2	x_3	x_4	
	1	2	3	4	5	B
MAX	1	1	2	3	-1	0
	0	1	2	3	0	15
	0	2	1	-5	0	20
	0	1	2	-1	1	10
	*				*	

We need basic variables for rows 2 & 3 also!

EXAMPLE TWO

Minimize $w = x_5 + x_6$

$$-z + x_1 + 2x_2 + 3x_3 - x_4 = 0$$

subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 \quad + x_5 \quad &= 15 \\ 2x_1 + x_2 - 5x_3 \quad + x_6 &= 20 \\ x_1 + 2x_2 - \ x_3 + x_4 \quad &= 10 \\ x_1, x_2, x_3, x_4 \geq 0,\ x_5 \geq 0, x_6 &\geq 0 \end{aligned}$$

Artificial variables X_5 & X_6 are added to the first two constraints to serve as initial basic variables.

EXAMPLE TWO

Phase One

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	1	2	3	-1	0	0	0
	0	0	1	2	3	0	(1)	0	15
	0	0	2	1	-5	0	0	(1)	20
	0	0	1	2	-1	(1)	0	0	10
	*	*							

First, pivot so as to eliminate X_5 & X_6 from the top row and X_4 from the second row.

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	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	2	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	-5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

We now have a basic "pseudo-feasible" solution with which to begin the Simplex method.

EXAMPLE TWO

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	2	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	-5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

We are minimizing the Phase-One objective, and select a pivot column having a negative reduced cost in the objective row.

Two columns have a negative reduced cost.
Pivoting in either column should reduce the value of the objective.

EXAMPLE TWO

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	2	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	(2)	1	-5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

Arbitrarily choose X_1 rather than X_2.

Minimum ratio test:

$$\min\left\{\frac{15}{1}, \frac{20}{2}, \frac{10}{1}\right\} \quad \textit{tie!}$$

⇓

	1	2	3	4	5	6	7	8	B
	1	0	0	-1.5	-5.5	0	0	1.5	-5
	0	1	0	3	7	0	0	-1	-10
	0	0	0	1.5	5.5	0	1	-0.5	5
	0	0	1	0.5	-2.5	0	0	0.5	10
	0	0	0	1.5	1.5	1	0	-0.5	0
	*	*	*			*	*		

Arbitrarily we select row 4 for the pivot. This introduced a zero on the right-hand-side *(degeneracy!)*

EXAMPLE TWO

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-1.5	-5.5	0	0	1.5	-5
	0	1	0	3	7	0	0	-1	-10
	0	0	0	1.5	5.5	0	1	-0.5	5
	0	0	1	0.5	-2.5	0	0	0.5	10
	0	0	0	1.5	(1.5)	1	0	-0.5	0
	*	*	*			*	*		

Either columns 4 or 5 (X or X) could be selected as pivot column... let's choose column 5.

Minimum Ratio Test:

$$\min\left\{\frac{5}{5.5}, --, \frac{0}{1.5}\right\}$$

The minimum ratio is zero!

⇓

	1	2	3	4	5	6	7	8	B
	1	0	0	4	0	3.667	0	-0.3333	-5
	0	1	0	-4	0	-4.667	0	1.333	-10
	0	0	0	-4	0	-3.667	1	1.333	5
	0	0	1	3	0	1.667	0	-0.3333	10
	0	0	0	1	1	0.6667	0	-0.3333	0
	*	*	*		*		*		

EXAMPLE TWO

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	4	0	3.667	0	-0.3333	-5
	0	1	0	-4	0	-4.667	0	1.333	-10
	0	0	0	-4	0	-3.667	1	(1.333)	5
	0	0	1	3	0	1.667	0	-0.3333	10
	0	0	0	1	1	0.6667	0	-0.3333	0
	*	*	*		*		*		

Choose column #8 for the next pivot.

There is only one candidate row for the pivot.

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	1	2	3	4	5	6	7	8	B
	1	0	0	3	0	2.75	0.25	0	-3.75
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	-3	0	-2.75	0.75	1	3.75
	0	0	1	2	0	0.75	0.25	0	11.25
	0	0	0	0	1	-0.25	0.25	0	1.25
	*	*	*		*			*	

The resulting tableau is optimal (for Phase One), since no column has a negative reduced cost.

EXAMPLE TWO

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	3	0	2.75	0.25	0	-3.75
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	-3	0	-2.75	0.75	1	3.75
	0	0	1	2	0	0.75	0.25	0	11.25
	0	0	0	0	1	-0.25	0.25	0	1.25
	*	*	*		*			*	

In this tableau, one of the artificial variables remains basic (and positive).

This indicates that the original LP had no feasible solution, since a feasible solution (with artificial variables zero) would be optimal for Phase One, if such a solution exists!

EXAMPLE TWO

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 \quad &= 15 \\ 2x_1 + x_2 + 5x_3 \quad &= 20 \\ x_1 + 2x_2 - \ x_3 + x_4 &= 10 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

-z & X_4 can serve as basic variables in the first and last rows.

	1	2	3	4	5	B
MAX	1	1	2	3	-1	0
	0	1	2	3	0	15
	0	2	1	5	0	20
	0	1	2	-1	1	10
	*				*	

The second and third rows will require artificial variables to serve as basic variables.

EXAMPLE THREE

Minimize $w = x_5 + x_6$

$$-z + x_1 + 2x_2 + 3x_3 - x_4 = 0$$

subject to

$$x_1 + 2x_2 + 3x_3 + x_5 = 15$$
$$2x_1 + x_2 + 5x_3 + x_6 = 20$$
$$x_1 + 2x_2 - x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \ge 0, \quad x_5 \ge 0, x_6 \ge 0$$

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	1	2	3	-1	0	0	0
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*							

X_5 & X_6 are artificial variables, and the Phase-One objective is to minimize their sum.

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	1	2	3	-1	0	0	0
	0	0	1	2	3	0	(1)	0	15
	0	0	2	1	5	0	0	(1)	20
	0	0	1	2	-1	(1)	0	0	10
	*	*				*	*	*	

We must pivot to enter X_4, X_5, and X_6 into the basis (eliminating these variables from the two objective rows).

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	-8	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	-8	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

Any one of columns 3, 4, and 5 could be selected as the pivot column. Here, column 3 (X_1) is chosen.

Minimum ratio test:

$$\min\left\{\frac{15}{1}, \frac{20}{2}, \frac{10}{1}\right\} \textit{ tie!}$$

Arbitrarily break the tie

Row 4

Row 5

EXAMPLE THREE

If we decide to pivot in row #4:

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	-8	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	(2)	1	5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

Any one of columns 3, 4, and 5 could be selected as the pivot column. Here, column 3 (X_1) is chosen.

Minimum ratio test:

$$\min\left\{\frac{15}{1}, \frac{20}{2}, \frac{10}{1}\right\} \textit{ tie!}$$

	1	2	3	4	5	6	7	8	B
	1	0	0	-1.5	-0.5	0	0	1.5	-5
	0	1	0	3	-3	0	0	-1	-10
	0	0	0	1.5	0.5	0	1	-0.5	5
	0	0	1	0.5	2.5	0	0	0.5	10
	0	0	0	1.5	-3.5	1	0	-0.5	0
	*	*	*			*	*		

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-1.5	-0.5	0	0	1.5	-5
	0	1	0	3	-3	0	0	-1	-10
	0	0	0	1.5	0.5	0	1	-0.5	5
	0	0	1	0.5	2.5	0	0	0.5	10
	0	0	0	(1.5)	-3.5	1	0	-0.5	0
	*	*	*			*	*		

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	-4	1	0	1	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	4	-1	1	0	5
	0	0	1	0	3.667	-0.3333	0	0.6667	10
	0	0	0	1	-2.333	0.6667	0	-0.3333	0
	*	*	*	*			*		

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	-4	1	0	1	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	(4)	-1	1	0	5
	0	0	1	0	3.667	-0.3333	0	0.6667	10
	0	0	0	1	-2.333	0.6667	0	-0.3333	0
	*	*	*	*			*		

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	*	*	*	*	*				

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	*	*	*	*	*				

This tableau is optimal for the Phase-One objective, and provides us with a basic feasible solution with which to begin Phase Two.

	1	2	3	4	5	6	7	8	B
	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	*	*	*	*	*				

We can now delete the artificial variables (and w) and can delete the Phase-One objective row.

EXAMPLE THREE

MAX	1	0	0	0	-1	-15
	0	0	0	1	-0.25	1.25
	0	1	0	0	0.5833	5.417
	0	0	1	0	0.08333	2.917
	*	*	*	*		

Since the Phase Two objective is to be minimized, this tableau is optimal!

EXAMPLE THREE

	-z	X_1	X_2	X_3	X_4	B
MAX	1	0	0	0	-1	-15
	0	0	0	1	-0.25	1.25
	0	1	0	0	0.5833	5.417
	0	0	1	0	0.08333	2.917

* * * *

$-z = -15$, i.e., $z = 15$

Optimal solution

$X_3 = 1.25$
$X_1 = 5.417$
$X_2 = 2.917$
$X_4 = 0$

EXAMPLE THREE

If we pivot in row 5 rather than row 4:

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	-8	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	(1)	2	-1	1	0	0	10

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	1	2	3	4	5	6	7	8	B
MIN	1	0	0	3	-11	3	0	0	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	4	-1	1	0	5
	0	0	0	-3	7	-2	0	1	0
	0	0	1	2	-1	1	0	0	10

* * * * *

Here, the pivot is in the bottom row.

The pivot produces a zero on the right-hand-side *(degeneracy)*

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	3	-11	3	0	0	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	4	-1	1	0	5
	0	0	0	-3	(7)	-2	0	1	0
	0	0	1	2	-1	1	0	0	10

* * * * *

Column 5 is selected for the pivot.

Minimum Ratio Test:

$$\min\left\{\frac{5}{4}, \frac{0}{7}, --\right\} = 0$$

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	1	2	3	4	5	6	7	8	B
	1	0	0	-1.714	0	-0.1429	0	1.571	-5
	0	1	0	1.714	0	-0.8571	0	-0.5714	-10
	0	0	0	1.714	0	0.1429	1	-0.5714	5
	0	0	0	-0.4286	1	-0.2857	0	0.1429	0
	0	0	1	1.571	0	0.7143	0	0.1429	10

* * * * *

Pivot in row 4

No improvement in the objective!

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-1.714	0	-0.1429	0	1.571	-5
	0	1	0	1.714	0	-0.8571	0	-0.5714	-10
	0	0	0	(1.714)	0	0.1429	1	-0.5714	5
	0	0	0	-0.4286	1	-0.2857	0	0.1429	0
	0	0	1	1.571	0	0.7143	0	0.1429	10

* * * * *

$$\min\left\{\frac{5}{1.714}, --, \frac{10}{1.571}\right\}$$

⇓

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417

In this iteration, degeneracy didn't block the improvement!

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417

This tableau is optimal for Phase One.

Now we proceed to Phase Two, in which we optimize the original objective, starting from the basic feasible solution which we have just found in Phase One.

EXAMPLE THREE

	1	2	3	4	5	6	7	8	B
	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917

* * * * *

We can now delete the artificial variables (and w) and can delete the Phase-One objective row.

						B
MAX	1	0	0	0	-1	-15
	0	0	0	1	-0.25	1.25
	0	1	0	0	0.5833	5.417
	0	0	1	0	0.08333	2.917

* * * *

Since the Phase Two objective is to be minimized, this tableau is optimal!

EXAMPLE THREE

Minimize $z = 34x_1 + 5x_2 + 19x_3 + 9x_4$
subject to

$$2x_1 + x_2 + x_3 + x_4 \geq 9$$
$$4x_1 - 2x_2 + 5x_3 + x_4 \leq 8$$
$$4x_1 - x_2 + 3x_3 + x_4 \geq 5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Convert the inequalities to equations by adding slack & subtracting surplus variables

Minimize $z = 34x_1 + 5x_2 + 19x_3 + 9x_4$
subject to

$$2x_1 + x_2 + x_3 + x_4 - x_5 = 9$$
$$4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8$$
$$4x_1 - x_2 + 3x_3 + x_4 - x_7 = 5$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

EXAMPLE FOUR

Minimize $z = 34x_1 + 5x_2 + 19x_3 + 9x_4$
subject to

$$2x_1 + x_2 + x_3 + x_4 - x_5 = 9$$
$$4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8$$
$$4x_1 - x_2 + 3x_3 + x_4 - x_7 = 5$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

The first and last constraint require artificial variables:

Minimize $w = x_8 + x_9$

$$-z + 34x_1 + 5x_2 + 19x_3 + 9x_4 = 0$$

subject to

$$2x_1 + x_2 + x_3 + x_4 - x_5 + x_8 = 9$$
$$4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8$$
$$4x_1 - x_2 + 3x_3 + x_4 - x_7 + x_9 = 5$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0$$

EXAMPLE FOUR

Minimize $w = x_8 + x_9$

$$-z + 34x_1 + 5x_2 + 19x_3 + 9x_4 = 0$$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 + x_4 - x_5 + x_8 &= 9 \\ 4x_1 - 2x_2 + 5x_3 + x_4 + x_6 &= 8 \\ 4x_1 - x_2 + 3x_3 + x_4 - x_7 + x_9 &= 5 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 &\geq 0 \end{aligned}$$

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	0	0	0	0	0	0	1	1	0
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	-1	0	0	1	0	9
	0	0	4	-2	5	1	0	1	0	0	0	8
	0	0	4	-1	3	1	0	0	-1	0	1	5

EXAMPLE FOUR

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	0	0	0	0	0	0	1	1	0
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	-1	0	0	1	0	9
	0	0	4	-2	5	1	0	1	0	0	0	8
	0	0	4	-1	3	1	0	0	-1	0	1	5
	*	*						*		*	*	

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	-6	0	-4	-2	1	0	1	0	0	-14
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	-1	0	0	1	0	9
	0	0	4	-2	5	1	0	1	0	0	0	8
	0	0	4	-1	3	1	0	0	-1	0	1	5
	*	*						*		*	*	

Initial basic solution for Phase One

EXAMPLE FOUR

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	-6	0	-4	-2	1	0	1	0	0	-14
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	-1	0	0	1	0	9
	0	0	4	-2	5	1	0	1	0	0	0	8
	0	0	4	-1	3	1	0	0	-1	0	1	5
	*	*						*		*	*	

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	-1.5	0.5	-0.5	1	0	-0.5	0	1.5	-6.5
	0	1	0	13.5	-6.5	0.5	0	0	8.5	0	-8.5	-42.5
	0	0	0	1.5	-0.5	0.5	-1	0	0.5	1	-0.5	6.5
	0	0	0	-1	2	0	0	1	1	0	-1	3
	0	0	1	-0.25	0.75	0.25	0	0	-0.25	0	0.25	1.25
	*	*	*					*		*		

EXAMPLE FOUR

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	-1.5	0.5	-0.5	1	0	-0.5	0	1.5	-6.5
	0	1	0	13.5	-6.5	0.5	0	0	8.5	0	-8.5	-42.5
	0	0	0	1.5	-0.5	0.5	-1	0	0.5	1	-0.5	6.5
	0	0	0	-1	2	0	0	1	1	0	-1	3
	0	0	1	-0.25	0.75	0.25	0	0	-0.25	0	0.25	1.25
	*	*	*					*		*		

1	2	3	4	5	6	7	8	9	10	11	B
1	0	0	0	0	0	0	0	0	1	1	0
0	1	0	0	-2	-4	9	0	4	-9	-4	-101
0	0	0	1	-0.333	0.333	-0.666	0	0.333	0.6667	-0.333	4.333
0	0	0	0	1.667	0.333	-0.666	1	1.333	0.6667	-1.333	7.333
0	0	1	0	0.666	0.333	-0.166	0	-0.166	0.1667	0.166	2.333
*	*	*	*				*				

EXAMPLE FOUR

1	2	3	4	5	6	7	8	9	10	11	B
1	0	0	0	0	0	0	0	0	1	1	0
0	1	0	0	-2	-4	9	0	4	-9	-4	-101
0	0	0	1	-0.333	0.333	-0.666	0	0.333	0.6667	-0.333	4.333
0	0	0	0	1.667	0.333	-0.666	1	1.333	0.6667	-1.333	7.333
0	0	1	0	0.666	0.333	-0.166	0	-0.166	0.1667	0.166	2.333
*	*	*	*				*				

This tableau is optimal for Phase One.

We may now delete the artificial variables and the Phase One objective row to obtain a basic feasible solution with which to begin Phase Two.

EXAMPLE FOUR

1	2	3	4	5	6	7	8	9	10	11	B
~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~	~~0~~
0	1	0	0	-2	-4	9	0	4	-9	-4	-101
0	0	0	1	-0.333	0.333	-0.666	0	0.333	0.6667	-0.333	4.333
0	0	0	0	1.667	0.333	-0.666	1	1.333	0.6667	-1.333	7.333
0	0	1	0	0.666	0.333	-0.166	0	-0.166	0.1667	0.166	2.333
*	*	*	*				*				

Phase Two:

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-2	-4	9	0	4	-101
	0	0	1	-0.333	0.333	-0.666	0	0.333	4.333
	0	0	0	1.667	0.333	-0.666	1	1.333	7.333
	0	1	0	0.666	0.333	-0.166	0	-0.166	2.333
	*	*	*				*		

EXAMPLE FOUR

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-2	-4	9	0	4	-101
	0	0	1	-0.333	0.333	-0.666	0	0.333	4.333
	0	0	0	1.667	0.333	-0.666	1	1.333	7.333
	0	1	0	0.666	0.333	-0.166	0	-0.166	2.333
	*	*	*				*		

	1	2	3	4	5	6	7	8	B
MIN	1	12	0	6	0	7	0	2	-73
	0	-1	1	-1	0	-0.5	0	0.5	2
	0	-1	0	1	0	-0.5	1	1.5	5
	0	3	0	2	1	-0.5	0	-0.5	7
	*		*		*		*		

Tableau is now optimal for Phase Two!

$-z = -73$, i.e., $z = 73$

$X_2 = 2$

$X_6 = 5$

$X_4 = 7$

EXAMPLE FOUR