Suppose that a new car costs $10,000, and that the annual operating cost & resale value are as follows:

<table>
<thead>
<tr>
<th>Age of car (yrs)</th>
<th>Resale Value</th>
<th>Operating cost in previous year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7000</td>
<td>$300</td>
</tr>
<tr>
<td>2</td>
<td>$6000</td>
<td>$500</td>
</tr>
<tr>
<td>3</td>
<td>$4000</td>
<td>$800</td>
</tr>
<tr>
<td>4</td>
<td>$3000</td>
<td>$1200</td>
</tr>
<tr>
<td>5</td>
<td>$2000</td>
<td>$1600</td>
</tr>
<tr>
<td>6</td>
<td>$1000</td>
<td>$2200</td>
</tr>
</tbody>
</table>

Starting with a new car, what is the replacement policy that minimizes the net cost of owning and operating a car for the next six years?

(Assuming that
1. Initial car has already been paid for
2. no car is needed at the end of the sixth year)

\[ G(t) = \text{minimum total cost incurred from time } t \text{ until the end of the planning horizon, if a new machine has just been purchased.} \]

\[ X^*(t) = \text{optimal replacement time for a machine which has been purchased at the beginning of period } t. \]

\[ G(t) = \min_{X \in S^T} \left\{ \sum_{i=1}^{X} C_i - S_{2t} + P_x G(x) \right\} \]

where

\[ P_x = \text{purchase price of a new machine at time } t \; (P_x = 0) \]

\[ C_i = \text{cost of operation & maintenance of a machine in its } i^{th} \text{ year} \]

\[ S_j = \text{salvage value of machine of age } j \]

Starting point:
New car at the beginning of the first year

What's the least-cost way to get from node 0 to node 6?

Termination:
No car at the end of the sixth year.
Computation of $G(5)$ - Minimum total cost until end of time period $5$, given a new machine at time $5$

\[
7000 + \frac{C}{6700} \cdot 6700 = 7000 + 6700 = 13700
\]

$X(5) = 6$ = optimal replacement time

$G(5) = 0$

Computation of $G(4)$ - Minimum total cost until end of time period $6$, given a new machine at time $4$

\[
\begin{array}{l}
G(4) = \text{minimum}(300 + 10000 - 7000 + G(5), \\
300 + 500 - 6000 + G(6)) \\
= \text{minimum}(-3400, -5200) = -5200
\end{array}
\]

$X(4) = 6$ = optimal replacement time

$G(4) = 5200$

Computation of $G(3)$ - Minimum total cost until end of time period $6$, given a new machine at time $3$

\[
\begin{array}{l}
G(3) = \text{minimum}(300 + 10000 - 7000 + G(4), \\
300 + 500 + 10000 - 6000 + G(5), \\
300 + 500 + 800 - 4000 + G(6)) \\
= \text{minimum}(-1900, -1900, -2400) = -2400
\end{array}
\]

$X(3) = 6$ = optimal replacement time

$G(3) = 2400$

Computation of $G(2)$ - Minimum total cost until end of time period $6$, given a new machine at time $2$

\[
\begin{array}{l}
G(2) = \text{minimum}(3300 + G(3), 4800 + G(4), \\
7600 + G(5), -200 + G(6)) \\
= \text{minimum}(900, -400, 900, -200) = -400
\end{array}
\]

$X(2) = 4$ = optimal replacement time

$G(2) = 400$
**Stage 1**

Computation of \( G(1) \) - Minimum total cost until end of time period 0, given a new machine at time 1

\[
\begin{array}{ccc}
\times & C & C+G \\
2 & 3000 & 3400 \\
3 & 4000 & 4400 \\
4 & 7600 & 2400 \\
5 & 9800 & 3160 \\
6 & 2400 & 2400 \\
\end{array}
\]

\( x^* (1) = 3 \) - optimal replacement time
\( G(1) = 2400 \)

Actually, one could replace at \( x = 3, 4, \) or 6!

**Stage 0**

Computation of \( G(0) \) - Minimum total cost until end of time period 0, given a new machine at time 0

\[
\begin{array}{ccc}
\times & C & C+G \\
1 & 3100 & 5700 \\
2 & 4800 & 4400 \\
3 & 7800 & 3700 \\
4 & 9800 & 4600 \\
5 & 12000 & 5700 \\
6 & 9500 & 5600 \\
\end{array}
\]

\( x^* (0) = 2 \) - optimal replacement time
\( G(0) = 4400 \)

**Summary**

Your expected total cost for 6 time periods will be 4400.00

The optimal plan is to replace the initial car after two years, i.e., \( x^* (1) = 2 \). Then, since \( x^* (2) = 4 \), you should replace at the end of the fourth year.

**Optimal replacement plan**

Starting point:
New car at the beginning of the first year

Termination:
No car at the end of the sixth year