LP models for Curve-Fitting

Dennis Bricker
University of Iowa

Problem Formulation. In constructing a hydrological model, it is required to obtain the expected runoff, denoted by R_i during the i^{th} period, as a linear function of the observed precipitation. From hydrological considerations the expected runoff depends on the precipitation during period i (p_i) and the previous two periods. So the model for expected runoff is

$$R_i = b_o p_i + b_1 p_{i-1} + b_2 p_{i-2}$$

where p_i is the precipitation during the i^{th} period; b_0, b_1, and b_2 are the coefficients that are to be estimated. These coefficients have to satisfy the following constraints from hydrological considerations:

$$b_0 \geq b_1 \geq b_2 \geq 0.$$

The following data is available:

Period	1	2	3	4	5	6	7	8	9	10	11	12
Precipitation (inch-hours)	3.8	4.4	5.7	5.2	7.7	6.0	5.4	5.7	5.5	2.5	0.8	0.4
Runoff (acre-feet)	0.05	0.35	1.0	2.1	3.7	4.2	4.3	4.4	4.3	4.2	3.6	2.7

Thus we require that R_3 (i.e., 1.0) be *approximately* $5.7b_0 + 4.4b_1 + 3.8b_2$.

a. Obtain the best estimates for b0, b1, and b2 if the objective is to minimize the *sum of absolute deviations*:

$$\sum_{i=3}^{12} \left| R_i - \left(b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2} \right) \right|.$$

b. Obtain the best estimates for b_0, b_1, and b_2 if the objective is to minimize the *maximum absolute deviation*:

$$\underset{i}{Max}\left|R_i - \left(b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2}\right)\right|.$$

c. *(Optional)* Use the standard linear regression model which minimizes the sum of the squares of the deviations, but without any restrictions on the coefficients b_0, b_1, and b_2:

$$\sum_{i=3}^{12}\left[R_i - \left(b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2}\right)\right]^2$$

Do the regression model coefficients satisfy $b_0 \geq b_1 \geq b_2 \geq 0$?