**Problem Formulation.** In constructing a hydrological model, it is required to obtain the expected runoff, denoted by $R_i$ during the $i$th period, as a linear function of the observed precipitation. From hydrological considerations the expected runoff depends on the precipitation during period $i$ ($p_i$) and the previous two periods. So the model for expected runoff is

$$R_i = b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2}$$

where $p_i$ is the precipitation during the $i$th period; $b_0$, $b_1$, and $b_2$ are the coefficients that are to be estimated. These coefficients have to satisfy the following constraints from hydrological considerations:

$$b_0 \geq b_1 \geq b_2 \geq 0.$$

The following data is available:

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precipitation (inch-hours)</td>
<td>3.8</td>
<td>4.4</td>
<td>5.7</td>
<td>5.2</td>
<td>7.7</td>
<td>6.0</td>
<td>5.4</td>
<td>5.7</td>
<td>5.5</td>
<td>2.5</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Runoff (acre-feet)</td>
<td>0.05</td>
<td>0.35</td>
<td>1.0</td>
<td>2.1</td>
<td>3.7</td>
<td>4.2</td>
<td>4.3</td>
<td>4.4</td>
<td>4.3</td>
<td>4.2</td>
<td>3.6</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Thus we require that $R_3$ (i.e., 1.0) be approximately $5.7b_0 + 4.4b_1 + 3.8b_2$.

a. Obtain the best estimates for $b_0$, $b_1$, and $b_2$ if the objective is to minimize the sum of *absolute deviations*:

$$\sum_{i=3}^{12} \left| R_i - (b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2}) \right|.$$
b. Obtain the best estimates for $b_0$, $b_1$, and $b_2$ if the objective is to minimize the maximum absolute deviation:

$$\max_{i} \left| R_i - \left( b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2} \right) \right|.$$ 

c. (Optional) Use the standard linear regression model which minimizes the sum of the squares of the deviations, but without any restrictions on the coefficients $b_0$, $b_1$, and $b_2$:

$$\sum_{i=3}^{\text{last}} \left[ R_i - \left( b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2} \right) \right]^2$$

Do the regression model coefficients satisfy $b_0 \geq b_1 \geq b_2 \geq 0$?