classifying Simplex Tableaus

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Note: We assume that the tableau is set up so that the basic variable in the objective row is –Z rather than Z.

a. The number in the objective row of a ***nonbasic*** variable indicates the change in the objective function per unit increase in the variable.

- if maximizing, we call it ***relative profit***
- if minimizing, we call it ***reduced cost***

b. ***Any*** nonbasic variable with

- positive relative profit (if maximizing) or
- negative reduced cost (if minimizing)

can be chosen to enter the basis.
(That is, we need not choose the "most positive" relative profit or "most negative" reduced cost.)

Example: either X1 or X3 could be selected as the pivot column:

–Z (max)	X_1	X_2	X_3	X_4	X_5	RHS
1	1	0	3	0	0	–27
0	5	0	1	1	0	11
0	0	1	6	0	0	52
0	-2	0	2	0	1	33

For convenience, suppose we are minimizing— similar conclusions will follow the maximizing case:

c. If no nonbasic variable has a negative reduced cost, the basic solution is ***optimal***.

Example:

–Z (min)	X_1	X_2	X_3	X_4	X_5	RHS
1	1	0	3	0	0	–27
0	5	0	1	1	0	11
0	0	1	6	0	0	52
0	–2	0	2	0	1	33

d. If a nonbasic variable has zero reduced cost, there may be another basic solution with the ***same objective value***.

Example: a pivot in X_1 column does not change the objective value

–Z (min)	X_1	X_2	X_3	X_4	X_5	RHS
1	0	0	–3	0	0	–27
0	5	0	1	1	0	10
0	0	1	6	0	0	52
0	–2	0	2	0	1	32

(If reduced costs are ≥ 0, and one nonbasic reduced cost is zero, then there are ***multiple optimal solutions***.)

Example: A pivot in the X_1 column gives another basic solution with the same objective value:

–Z (min)	X_1	X_2	X_3	X_4	X_5	RHS
1	0	0	–3	0	0	–27
0	1	0	0.2	0.2	0	2
0	0	1	6	0	0	52
0	0	0	0.4	0.4	1	36

e. If a nonbasic variable with negative reduced cost has no positive substitution rate, i.e., no candidate for the pivot, the LP has an ***unbounded solution*** *(even if some other nonbasic variable does have an eligible pivot).*

Example: *Although X3 could enter the basis, the LP has an unbounded solution because X1 can increase without limit, sending the objective function to* $-\infty$:

$-Z$ (min)	X_1	X_2	X_3	X_4	X_5	RHS
1	–1	0	–3	0	0	–27
0	–5	0	1	1	0	11
0	0	1	6	0	0	52
0	–2	0	2	0	1	33

f. If a basic variable has the value zero, we call the basic solution ***degenerate***.

Example: the basic variable X_3 is zero:

$-Z$ (min)	X_1	X_2	X_3	X_4	X_5	RHS
1	–1	0	–3	0	0	–27
0	5	0	1	1	0	0
0	0	1	6	0	0	52
0	2	0	2	0	1	33

g. If the pivot column has a *positive substitution rate* in a row with zero on the right-hand-side (i.e., solution is *degenerate*), then

- the minimum ratio (RHS/positive substitution rate) is zero
- the simplex method is forced to pivot in that row
- the variable entering the basis will remain zero in the new tableau, that is, the new basic solution will also be degenerate
- there will be *no changes in the values of any variable* or the objective function

(Degenerate pivots will continue until a pivot column has only zero or negative substitution rates in rows with zero right-hand-sides.)

Example: Whether X_1 or X_3 is selected for the pivot row, X_4 will leave the basis but the entering variable will remain zero.

$-Z$ (min)	X_1	X_2	X_3	X_4	X_5	RHS
1	–1	0	–3	0	0	–27
0	5	0	1	1	0	0
0	0	1	6	0	0	52
0	2	0	2	0	1	33

h. *It is possible but very rare that the simplex method will get "stuck" in a* ***cycle*** *of degenerate basic solutions and unable to converge to the optimal solution.*
(Special rules for choosing the pivot column can guarantee this doesn't happen, but are seldom implemented in computer software.)