Chicken Feed

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A firm manufactures **chicken feed** by mixing three different ingredients.

Each ingredient contains **four key nutrients**:
protein, fat, vitamin A, and vitamin B.
The amount of each nutrient contained in 1 kilogram of the **three basic ingredients** is:

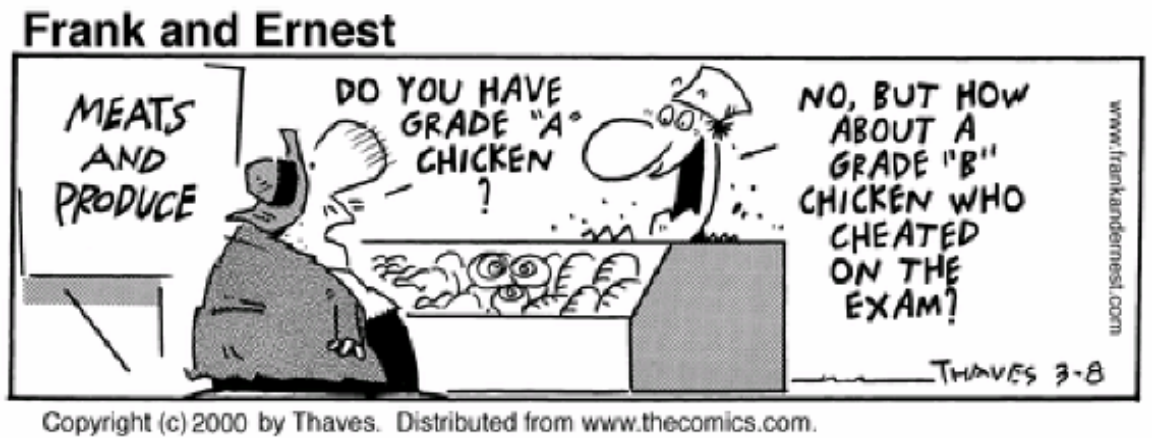
Ingredient	Protein (grams)	Fat (grams)	Vitamin A (units)	Vitamin B (units)
1	25	11	235	12
2	45	10	160	6
3	32	7	190	10

The costs per kg of Ingredients 1, 2, and 3 are
$0.55, $0.42, and $0.38, respectively.

Each kilogram of the feed must contain

- *at least* 35 grams of protein,
- a *minimum* of 8 grams (and a *maximum* of 10 grams) of fat,
- *at least* 180 units of vitamin A and
- *at least* 9 units of vitamin B.

Formulate an LP model for finding the feed mix that has the minimum cost per kg.



Decision variables

X1 = kg. of Ingredient 1 included in mixture
X2 = kg. of Ingredient 2 included in mixture
X3 = kg. of Ingredient 3 included in mixture

Complete LP Formulation (**LINDO**)

```
MIN     0.55 X1 + 0.42 X2 + 0.38 X3
st
       25 X1 +  45 X2 +  32 X3 >=    35  ! Protein constraint
       11 X1 +  10 X2 +   7 X3 >=     8  ! Min Fat constraint
       11 X1 +  10 X2 +   7 X3 <=    10  ! Max Fat constraint
      235 X1 + 160 X2 + 190 X3 >=   180  ! Vitamin A constraint
       12 X1 +   6 X2 +  10 X3 >=     9  ! Vitamin B constraint
          X1 +     X2 +     X3  =     1  ! total wt of mixture
END
```

```
       OBJECTIVE FUNCTION VALUE
       1)     0.3986364

  VARIABLE         VALUE          REDUCED COST
       X1          0.045455          0.000000
       X2          0.272727          0.000000
       X3          0.681818          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          0.227273          0.000000
       3)          0.000000         -0.034545
       4)          2.000000          0.000000
       5)          3.863636          0.000000
       6)          0.000000         -0.015909
       7)          0.000000          0.020909
```

The minimum cost mixture costs $0.398/kg and consists of

0.045 kg of Ingredient 1,
0.273 kg of Ingredient 2 and
0.682 kg of Ingredient 3

Which constraints are "tight" or "binding"?
Which are "loose"?

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	0.550000	INFINITY	0.116667
X2	0.420000	0.087500	0.380000
X3	0.380000	0.126667	0.350000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	35.000000	0.227273	INFINITY
3	8.000000	2.000000	0.250000
4	10.000000	INFINITY	2.000000
5	180.000000	3.863636	INFINITY
6	9.000000	0.068493	0.333333
7	1.000000	0.017241	0.003453

The optimal tableau:

ROW	(BASIS)	X1	X2	X3	SLK 2	SLK 3	SLK 4
1	ART	0.000	0.000	0.000	0.000	0.035	0.000
2	SLK 5	0.000	0.000	0.000	0.000	-5.455	0.000
3	X1	1.000	0.000	0.000	0.000	-0.182	0.000
4	SLK 4	0.000	0.000	0.000	0.000	1.000	1.000
5	X2	0.000	1.000	0.000	0.000	-0.091	0.000
6	X3	0.000	0.000	1.000	0.000	0.273	0.000
7	SLK 2	0.000	0.000	0.000	1.000	0.091	0.000

ROW	SLK 5	SLK 6	
1	0.000	0.016	-0.399
2	1.000	-11.591	3.864
3	0.000	-0.136	0.045
4	0.000	0.000	2.000
5	0.000	0.182	0.273
6	0.000	-0.045	0.682
7	0.000	3.318	0.227

Sensitivity Analysis

- Suppose that the cost of ingredient #2 varies considerably from day to day. If all other ingredient costs remain the same, by how much can the cost of ingredient increase or decrease without changing the optimal formula for chicken feed? _____

- If experts change their recommendation of vitamin A content from 180 units/kg to 181 units/kg,
  will this change the optimal cost? _____
  will it change the optimal formula? _____ *(If so, how?)*

- If experts change their recommendation of vitamin B content from 9 units/kg to 10 units/kg,
  will this change the optimal cost? _____
  will it change the optimal formula? _____ *(If so, how?)*

LINGO *model:*

```
SETS:
    INGREDIENT /1..3/: PROTEIN, FAT, VITA, VITB, COST, X;
ENDSETS

DATA:
    PROTEIN = 25 45 32;
    FAT = 11 10 7;
    VITA = 235 160 190;
    VITB = 12 6 10;
    COST = 0.55 0.42 0.38;
ENDDATA

MIN = @SUM(INGREDIENT: COST*X);
    @SUM(INGREDIENT: PROTEIN * X) >= 35; ! minimum protein ;
    @SUM(INGREDIENT: FAT * X) >= 8;      ! minimum fat ;
    @SUM(INGREDIENT: FAT * X) <= 10;     ! maximum fat ;
    @SUM(INGREDIENT: VITA * X) >= 180;   ! minimum vitamin A ;
    @SUM(INGREDIENT: VITB * X) >= 9;     ! minimum vitamin B ;
    @SUM(INGREDIENT: X) = 1 ;            ! total weight = 1 kg;
END
```

Suppose that a new ingredient has become available with the following characteristics:

Protein	2.2%
Fat	0.9%
Vitamin A (units/kg)	200
Vitamin B (units/kg)	5
Cost ($/kg)	$0.36

- Modify the LINGO model in order to consider this ingredient.
- Is the solution changed?