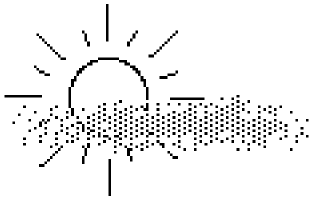
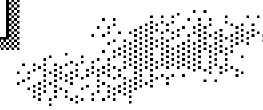


Summer Weather



Summer days are classified as either

sunny or *cloudy.*

Suppose that the weather on any summer day depends on the weather conditions for the previous **two** days.

To be exact, suppose that

- if it was **sunny** today and yesterday, then it will be **sunny** tomorrow with probability 80%
- if it was **sunny** today but **cloudy** yesterday, then it will be **sunny** tomorrow with probability of only 60%
- if it was **cloudy** today but **sunny** yesterday, then it will be **cloudy** tomorrow with probability 60%
- if it was **cloudy** for the last two days, then it will be **cloudy** tomorrow with probability 90%

If we define a stochastic process $\{X_n; n=1,2,\ldots\}$

$$\text{by} \quad X_n = \begin{cases} 0 & \text{if day } n \text{ is cloudy,} \\ 1 & \text{if day } n \text{ is sunny,} \end{cases}$$

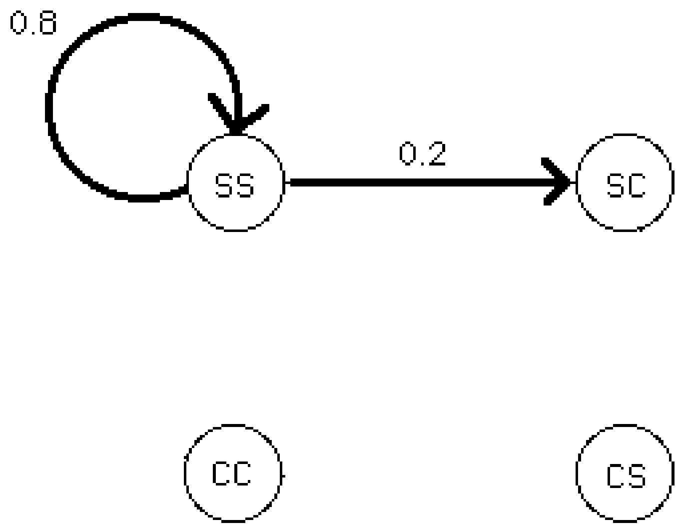
then the stochastic process is NOT a Markov chain, since it is not MEMORYLESS,

that is, the probability that (for example) tomorrow is SUNNY depends not only on today's state, but also yesterday's!

If we wish a Markov chain model, the state of the system must incorporate ALL relevant information required to determine the transition probabilities!

- There are four possible states:
  - i. (S,S): Sunny both yesterday & today
  - ii. (S,C): Sunny yesterday, cloudy today
  - iii. (C,S): Cloudy yesterday, sunny today
  - iv. (C,C): Cloudy both yesterday & today

From the state (S,S), there will be a transition to either (S,S) or (S,C); from (S,C), there will be a transition to either (C,S) or (C,C), etc. That is, tomorrow, today will be yesterday!



Suppose that we are in state SS.

If tomorrow is sunny, then we will again be in state SS

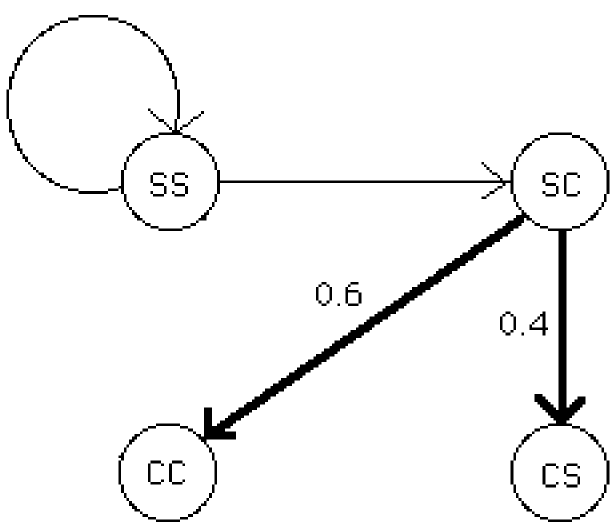
today's condition

tomorrow's condition

If tomorrow is cloudy, then we will be in the state SC

today's condition

tomorrow's condition



If we are currently in state SC, i.e., today is cloudy but yesterday was sunny,

then if tomorrow is sunny, we will be in state CS

today's condition

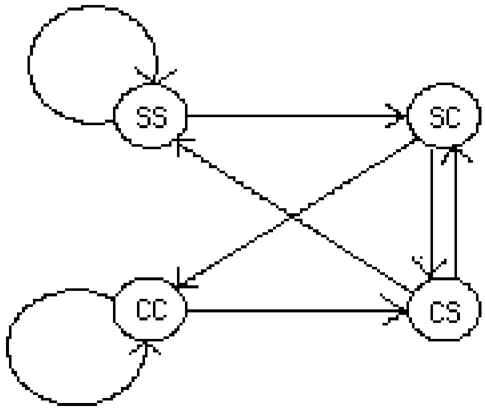
tomorrow's condition

if tomorrow is cloudy, we will be in the state CC

today's condition

tomorrow's condition

etc.



The transition probability matrix is

$$P = \begin{array}{c|cccc} & SS & SC & CS & CC \\ SS & 0.8 & 0.2 & 0 & 0 \\ SC & 0 & 0 & 0.4 & 0.6 \\ CS & 0.6 & 0.4 & 0 & 0 \\ CC & 0 & 0 & 0.1 & 0.9 \end{array}$$

Suppose that Sunday and Monday are both SUNNY. What is the probability that Friday will be sunny?

$$P^4 = \begin{bmatrix} .4936 & .1424 & .0844 & .2796 \\ .2532 & .0928 & .087 & .567 \\ .4272 & .1248 & .0928 & .3552 \\ .1398 & .0592 & .0945 & .7065 \end{bmatrix}$$

Beginning in state 1 (S,S), there is a probability of 0.4936+0.0844 = 0.578 that 4 days hence (***Friday***) the system will be in states 1 or 3, i.e. the day will be sunny.

*(Or, equivalently, one could find the probability that on **Saturday** the system will be in states 1 (S,S) or 2 (S,C), in which case **Friday** must have been sunny. Using the fifth power of P, we would find that this gives 0.44552+0.13248 = 0.578, the same value.)*

The steady-state distribution is

$$\pi_1 = 0.2727,\ \pi_2=0.0909,\ \pi_3=0.0909,\ \pi_4=0.5454,$$

so that in the long run, the probability that a day is sunny is the probability of being in states (S,S) or (C,S) is

$$\pi_1+\pi_3 = 0.\dot{3}636,$$

i.e. 36.36% of the days will be sunny.

Summary

When constructing a Markov chain model of a system, the state must be defined so as to incorporate ALL information necessary to determine the probability distribution of the transitions!

(That is, the probability distribution may depend ONLY on the current state, and not on the state of the system in any prior stages.)