Exchange Algorithm for the Traveling Salesman Problem

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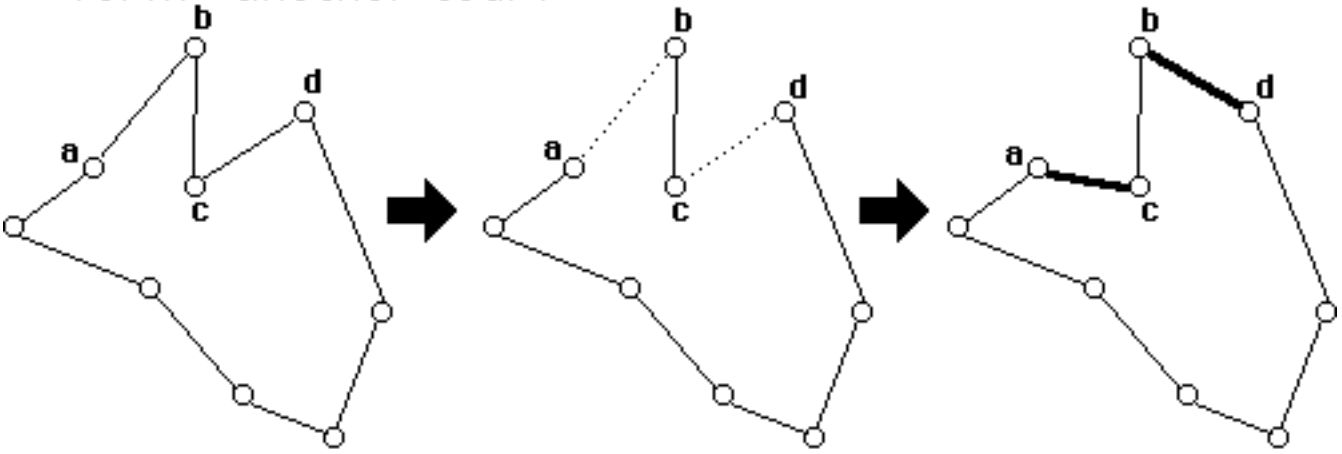
Exchange heuristics, given an initial tour, try to replace *k* edges of the tour with k edges not on the tour in order to find a shorter tour.

A ***k-exchange*** is performed by deleting k edges of a tour, and reconnecting the segments so as to form another tour.



Example: a 2-exchange

Edges **ab** and **cd** are replaced by **ac** and **bd**

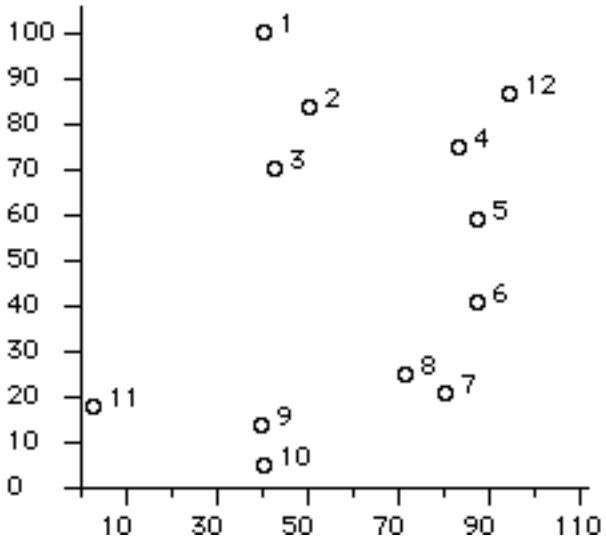
For a specified integer ***k***, a ***k-neighborhood*** of a tour is one which might be obtained by a k-exchange.

If no shorter tour exists in a k-neighborhood of a tour, that tour is said to be ***k-optimal***.

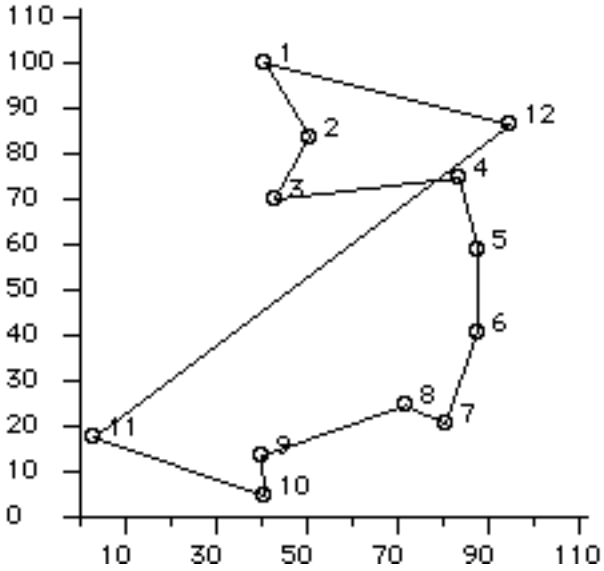
(Only if a tour is k-optimal for every $k \leq N/2$ can we be certain that the tour is truly optimal!)

Example

Random Symmetric TSP
(seed= 133398)

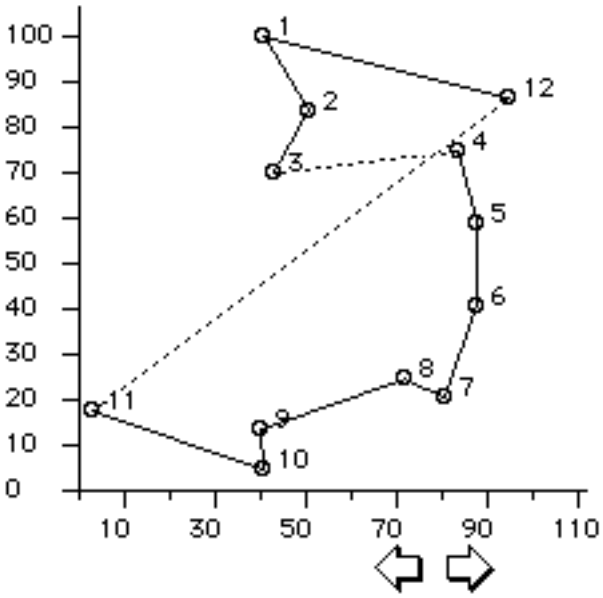


Lin's 2-exchange heuristic



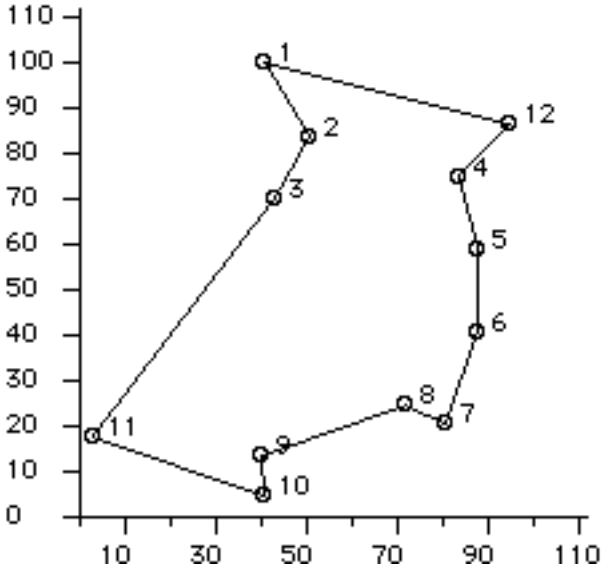
Initial tour (found by nearest neighbor heuristic)

Tour # 1 is 1 2 3 / 11 10 9 8 7 6 5 4 / 12 1
Length: 321 Improvement: 74



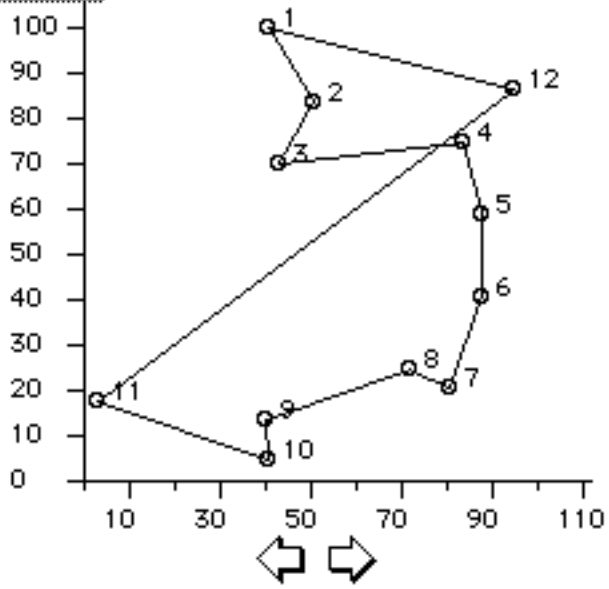
Edges (3,4) & (11,12) are removed, breaking tour into 2 paths. These can then be reconnected in only one other way.

2-Optimal Tour: 1 2 3 11 10 9 8 7 6 5 4 12 1,
with length 321



No 2-neighbor tour gives any improvement

3-exchange heuristic



Initial tour (found by nearest neighbor heuristic)

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Exchange type: 1
   Replace edges: 1 3 11, i.e., (1 2), (3 4), & (11 12)
                  having length 175
      with edges (1 2), (3 11), & (4 12) having length 101
Tour # 1 is 1 2 3 11 10 9 8 7 6 5 4 12 1 with length: 321
       Improvement: 74
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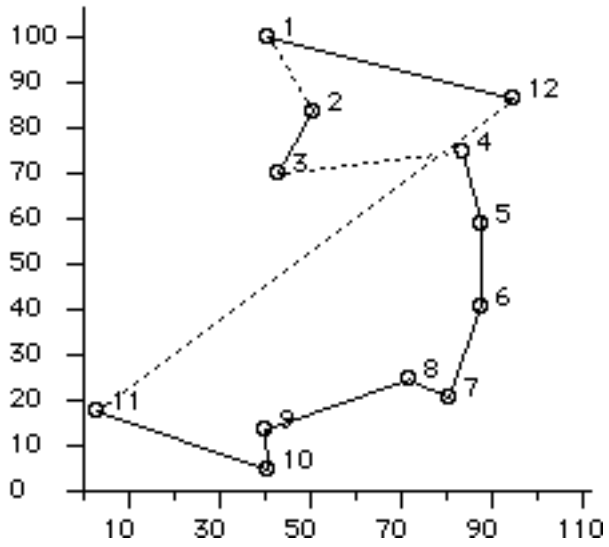
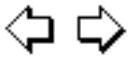
Because edge (1,2) was re-inserted, this is actually a 2-neighboring tour!

3-exchange heuristic





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3-Optimal Tour: 1 2 3 11 10 9 8 7 6 5 4 12 1,
     with length 321
```

No further improvement was found.