**Example**

Set Coverage Matrix (A)

<table>
<thead>
<tr>
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<td>1</td>
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</tbody>
</table>

\[ C: \begin{bmatrix} 2 & 3 & 3 & 4 & 4 & 4 & 5 & 5 & 5 & 5 & 5 & 5 & 6 & 7 & 7 & 7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 4 & 4 & 6 & 6 & 7 & 2 \end{bmatrix} \]

Density = 34.00%

---

**Initial values of Lagrangian multipliers**

For point \( i \), choose the value:

- zero
- smallest of costs of sets covering point \( i \)
- average of costs of sets covering point \( i \)

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Iteration # 1

Current multipliers:

\[
\begin{array}{c|cccccccc}
  i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  \mu[i] & 3 & 2 & 3 & 3 & 5 & 4 & 3 & 2 \\
\end{array}
\]

Solving Lagrangian relaxation:

Set(s) 23 24 25 removed from problem
(P= 16 17 22 > 14 = incumbent!)
# sets remaining is 22

*** Dual value is -20 ***
Sets in cover: # 1 2 3 4 7 8 9 10 11 13 14 16 19 22
Points not covered: (none)

Heuristic Adjustment

n_take= 1, c_type= 1 row selection: smallest
Try removing sets from cover:
set     1 2 3 4 7 8 9 10 11 13 14 16 19 22
Set Cost  2 3 3 4 5 5 5 5 7 7 11 12 14
Reduced cost -2 -1 -2 -6 -11 -2 -3 -2 0 -2 -1 -2 -1 0
Set 22 can be removed!
Set 19 can be removed!
Set 16 can be removed!
Set 14 can be removed!
Set 13 can be removed!
Set 11 can be removed!
Set 10 can be removed!
Set 9 can be removed!
Set 8 can be removed!
Set 3 can be removed!
Set 1 can be removed!
Covering sets are 2 4 7
Heuristic solution cost is 12
*** New incumbent! *** Cover is 2 4 7
with cost 12

Subgradient of Dual Objective:
+1 for points:
≤ -2 for points: 1 2 3 4 5 6 7 8
Norm squared is 156
Stepsize is 0.205128

Current multipliers:

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>wi</td>
<td>i</td>
<td>2.179</td>
<td>1.589</td>
<td>1.769</td>
<td>2.384</td>
<td>4.179</td>
<td>2.974</td>
<td>1.974</td>
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</tbody>
</table>

Solving Lagrangian relaxation:

Set(s) 22 removed from problem
(P= 14 > 12 = incumbent!)
# sets remaining is 21

*** Dual value is -0.666667 ***
Sets in cover: # 1 2 3 4 7 8 9 10
Points not covered: (none)
Computation of $P_{22}$

Since $X_{22}=0$ in the solution of the Lagrangian relaxation, $P_{22} = \Phi(\lambda) + \overline{C}_{22}$, where $\overline{C}_{22}$ is the reduced cost of $X_{22}$, namely

$$\overline{C}_{22} = C_{22} - [\lambda_3 + \lambda_5 + \lambda_6 + \lambda_8] = 14 - [1.769 + 4.179 + 2.974 + 0.9743] = 14 - 9.896 = 4.1037$$

$$\Phi(\lambda) = -0.66667$$

Heuristic Adjustment

n_take= 1, c_type= 1 row selection: smallest

Try removing sets from cover:

set 1 2 3 4 7
Set Cost 2 3 3 4 5
Reduced cost -0.564103 -6.89744 -0.358974 -2.71795 -6.28205

8 9 10
5 5 5
-0.153846 -1.5641 -0.153846

Set 10 can be removed!
Set 9 can be removed!
Set 8 can be removed!
Set 3 can be removed!
Set 1 can be removed!
Covering sets are 2 4 7
Heuristic solution cost is 12
Subgradient of Dual Objective:
*1 for points:
-1 for points: 1 6
≤ -2 for points: 2 3 4 5 7 8
Norm squared is 31
Stepsize is 0.408602

Current multipliers:

<table>
<thead>
<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>w[i]</td>
<td>1.770</td>
<td>0.7725</td>
<td>0.5434</td>
<td>1.567</td>
<td>3.362</td>
<td>2.565</td>
<td>1.157</td>
<td>0.1571</td>
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</tbody>
</table>

Solving Lagrangian relaxation:
Set(s) 17 18 19 20 21 removed from problem

# sets remaining is 16
*** Dual value is 7.03143 ***  (Improvement: 7.6981)

Sets in cover: # 2 4 7
Points not covered: (none)
Heuristic Adjustment

n_take = 1, c_type = 1 row selection: smallest
  Try removing sets from cover:
    set  2  4  7
    Set Cost  3  4  5
    Reduced cost -2.81141 -0.266336 -1.78743
    Covering sets are 2 4 7
    Heuristic solution cost is 12

Subgradient of Dual Objective:
  +1 for points:
  -1 for points: 4
  ≤ -2 for points: 3 7
  Norm squared is 9
  Step size is 0.552063

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Iteration # 4

Current multipliers:

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<tbody>
<tr>
<td>wi</td>
<td>1.770</td>
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<td>0</td>
<td>1.015</td>
<td>3.362</td>
<td>2.565</td>
<td>0.05302</td>
<td>0.1571</td>
</tr>
</tbody>
</table>

Solving Lagrangian relaxation:

Set(s) 5 6 12 13 14 15 16 removed from problem
 > 12 = incumbent!)
# sets remaining is 9

*** Dual value is 9.08519 *** (Improvement: 2.05376)

Sets in cover: # 2
Points not covered: 5 6 8

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Heuristic Adjustment

n_take = 1, c_type = 1,
row selection rule: row with maximum multiplier

Add set 7 to cover row 5
Add set 4 to cover row 6
Try removing sets from cover:

<table>
<thead>
<tr>
<th>Set</th>
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<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Reduced cost = 0.6118 1.381 0.4122
Covering sets are 2 4 7
Heuristic solution cost is 12

Subgradient of Dual Objective:
+1 for points: 5 6 8
Norm squared is 3
Stepsize is 0.9716

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Iteration # 5

Current multipliers:

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</tr>
</thead>
<tbody>
<tr>
<td>wi</td>
<td>1.770</td>
<td>0.7725</td>
<td>0</td>
<td>1.015</td>
<td>4.333</td>
<td>3.537</td>
<td>0.05302</td>
<td>1.128</td>
</tr>
</tbody>
</table>

Solving Lagrangian relaxation:

*** Dual value is 9.34886 *** (Improvement: 0.263671)

Sets in cover: # 2 7 8 9 10
Points not covered: (none)

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Heuristic Adjustment

n_take = 1, c_type = 1 row selection: smallest
Try removing sets from cover:

<table>
<thead>
<tr>
<th>set</th>
<th>2</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Reduced cost</td>
<td>-0.6118</td>
<td>-1.53102</td>
<td>-0.308244</td>
<td>-0.349233</td>
<td>-0.462641</td>
</tr>
</tbody>
</table>
Set 10 can be removed!
Set 9 can be removed!
Covering sets are 2 7 8
Heuristic solution cost is 13

Subgradient of Dual Objective:
+1 for points:
-1 for points: 1 7 8
≤ -2 for points: 4 5
Norm squared is 11
Stepsize is 0.241012

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Iteration # 6

Current multipliers:

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<tbody>
<tr>
<td>w[i]</td>
<td>1.53</td>
<td>0.7725</td>
<td>0</td>
<td>0.5333</td>
<td>3.852</td>
<td>3.537</td>
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<td>0.8877</td>
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Solving Lagrangian relaxation:

Set(s) 3 removed from problem
(P = 13 > 12 = incumbent!)
# sets remaining is 8
*** Dual value is 10.77 *** (Improvement: 1.424)

Sets in cover: # 7 8
Points not covered: 2

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Heuristic Adjustment

n_take = 1, c_type = 1,
row selection rule: row with maximum multiplier

Add set 1 to cover row 2
Try removing sets from cover:

<table>
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<tr>
<th>set</th>
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<tr>
<td>Set Cost</td>
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<td>5</td>
</tr>
<tr>
<td>Reduced cost</td>
<td>0.3397</td>
<td>-0.2729</td>
<td>-0.06723</td>
</tr>
</tbody>
</table>

Covering sets are 1 7 8
Heuristic solution cost is 12

Subgradient of Dual Objective:
*1 for points: 2
Norm squared is 1
Stepsize is 1.227

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Iteration #10

Current multipliers:

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<td>W[j]</td>
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Solving Lagrangian relaxation:

Set(s) 11 removed from problem
(P = 12.09 > 12 = incumbent!)
# sets remaining is 7
*** Dual value is 11.14 ***
(Improvement: 0.5077)

Sets in cover: # 1 2
Points not covered: 5 6

Converged!
Solution is sets # 2 4 7
with value 12

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