Introduction to QUEUEING: M/G/1

- Arrival process is Memoryless, i.e., interarrival times have Exponential distribution with mean $1/\lambda$.
- Single server
- Service times are independent, identically-distributed, but not necessarily exponential. Mean service time is $1/\mu$ with variance $\sigma^2$.
- Queue capacity is infinite

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A steady state distribution exists if \( \rho = \frac{\lambda}{\mu} < 1 \) i.e., if service rate exceeds the arrival rate.

\[
\begin{align*}
\pi_0 &= 1 - \rho \quad \text{probability that server is idle} \\
1 - \pi_0 &= \rho \quad \text{probability that server is busy} \\
\text{i.e., utilization of server}
\end{align*}
\]

There is no convenient formula for the probability of \( j \) customers in the system when \( j > 0 \).

\[L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2 (1 - \rho)}\] average number of customers waiting

After calculating \( L_q \), Little's Formula allows us to compute:

\[W_q = \frac{L_q}{\lambda}, \quad W = W_q + \frac{1}{\mu}, \quad \lambda W = L_q + \rho\]

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For the M/M/1 queue, the standard deviation equals the mean service time, i.e., $\sigma = \frac{1}{\mu}$.

Using these formulae for the M/G/1 queueing system with $\sigma^2 = \frac{1}{\mu^2}$ will give results consistent with the formulae for M/M/1.