Introduction to QUEUEING: M/M/1/N/N

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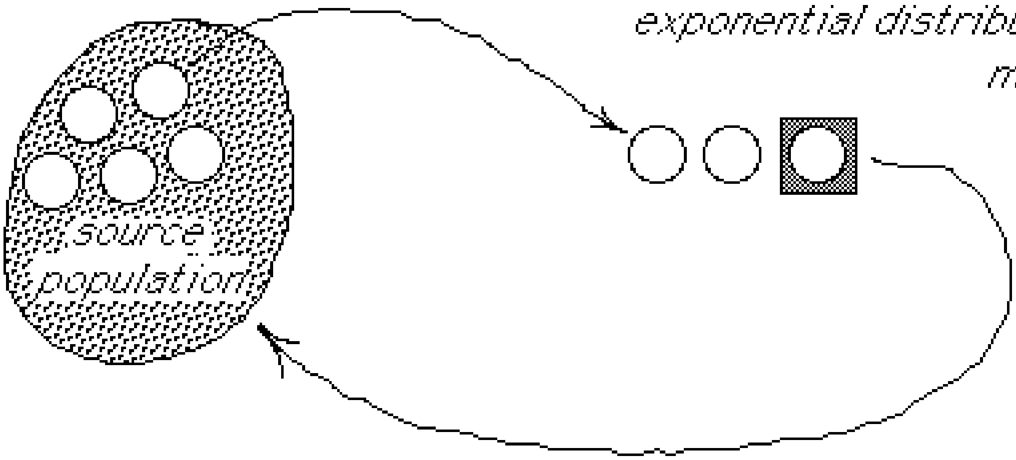
M/M/1/N/N

- *Single server*
- ***Finite Source Population** of size* N
- *Arrival & Service processes are **M**emoryless, i.e., service times have Exponential distribution with mean* $1/\mu$
- *A departing customer returns to the queue after a time having an Exponential distribution with mean* $1/\lambda$

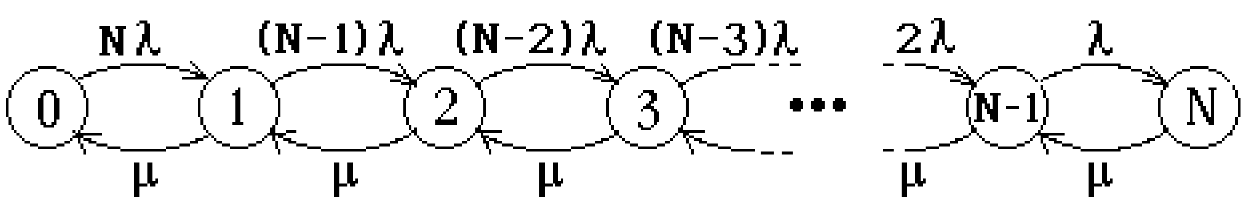
M/M/1/N/N

Each customer, after being served, returns to the source population for a length of time having exponential distribution with mean $1/\lambda$



M/M/1/N/N

Birth/Death Model



M/M/1/N/N

Steadystate Distribution

$$\pi_0 = \frac{1}{\sum_{j=0}^{N} \frac{N!}{(N-j)!} \rho^j}$$

$$\pi_j = \frac{N!}{(N-j)!} \rho^j \pi_0$$

First calculate the probability π_0 that the server is idle.

Other probabilities are then multiples of π_0

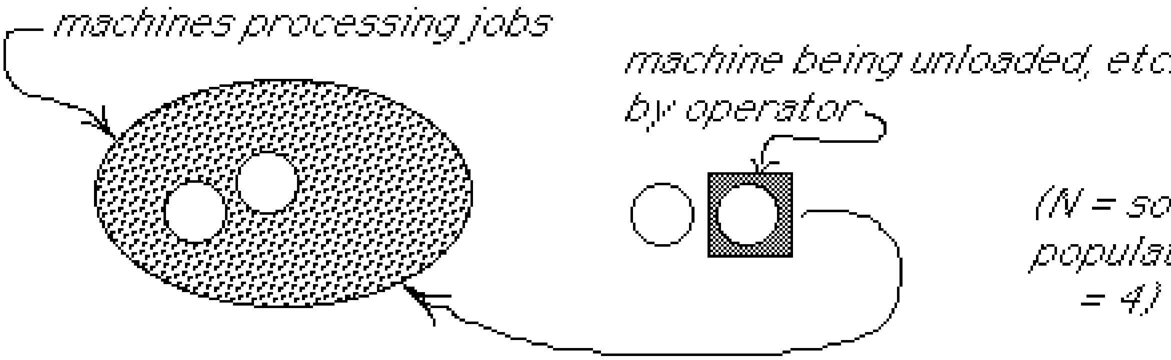
where $\rho = \frac{\lambda}{\mu}$

Example

An operator can be assigned to service (load, unload, adjust, etc.) several automatic machines in a factory

- Running time of each machine before it must be serviced has exponential distribution, with mean 120 minutes.
- Service time has an exponential distribution with mean 12 minutes.

To achieve a desired utilization of ≥ 87.5% for the machines, how many machines should be assigned to the operator?

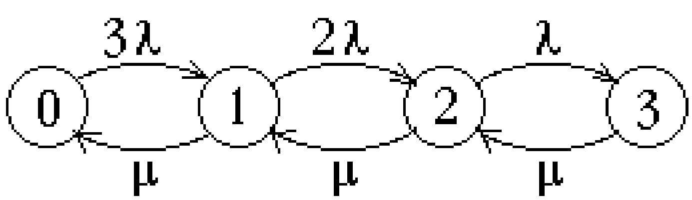


(N = source population = 4)

This can be modeled as a M/M/1 queueing system with finite source population.
Machine operator = server
Machines = customers
μ = 5/hour
λ = 0.5/hour

M/M/1/N/N

Birth/Death Model



$$\lambda = {}^1/_{2\text{ hrs}} = 0.5/\text{hr}$$
$$\mu = 5/\text{hr}$$
$$\rho = {}^\lambda/_\mu = 0.1$$

$$\frac{1}{\pi_0} = 1 + 3\rho + 3\times2\times\rho^2 + 3!\times\rho^3$$

Steadystate Distribution

$$\frac{1}{\pi_0} = \sum_{j=0}^{3} \frac{3!}{(3-j)!}(0.1)^j$$

$$= 1 + 0.3 + 0.06 + 0.006$$

$$= 1.366$$

$$\pi_0 = 1/1.366 = 0.732965$$

i.e., operator will be idle about 73% of the time!

$$\pi_1 = 0.3\,\pi_0 = 0.2196$$

$$\pi_2 = 0.06\,\pi_0 = 0.0439$$

$$\pi_3 = 0.006\,\pi_0 = 0.0044$$

$$\pi_0 = 0.732965$$
$$\pi_1 = 0.2196$$
$$\pi_2 = 0.0439$$
$$\pi_3 = 0.0044$$

If 0 machines are in system, then 3 are busy processing jobs;

if 1 machine is in system, then 2 are busy processing jobs, etc.

Average utilization of the machines will be

$$\frac{3\,\pi_0 + 2\,\pi_1 + 1\,\pi_2 + 0\,\pi_3}{3} = 89.3\%$$