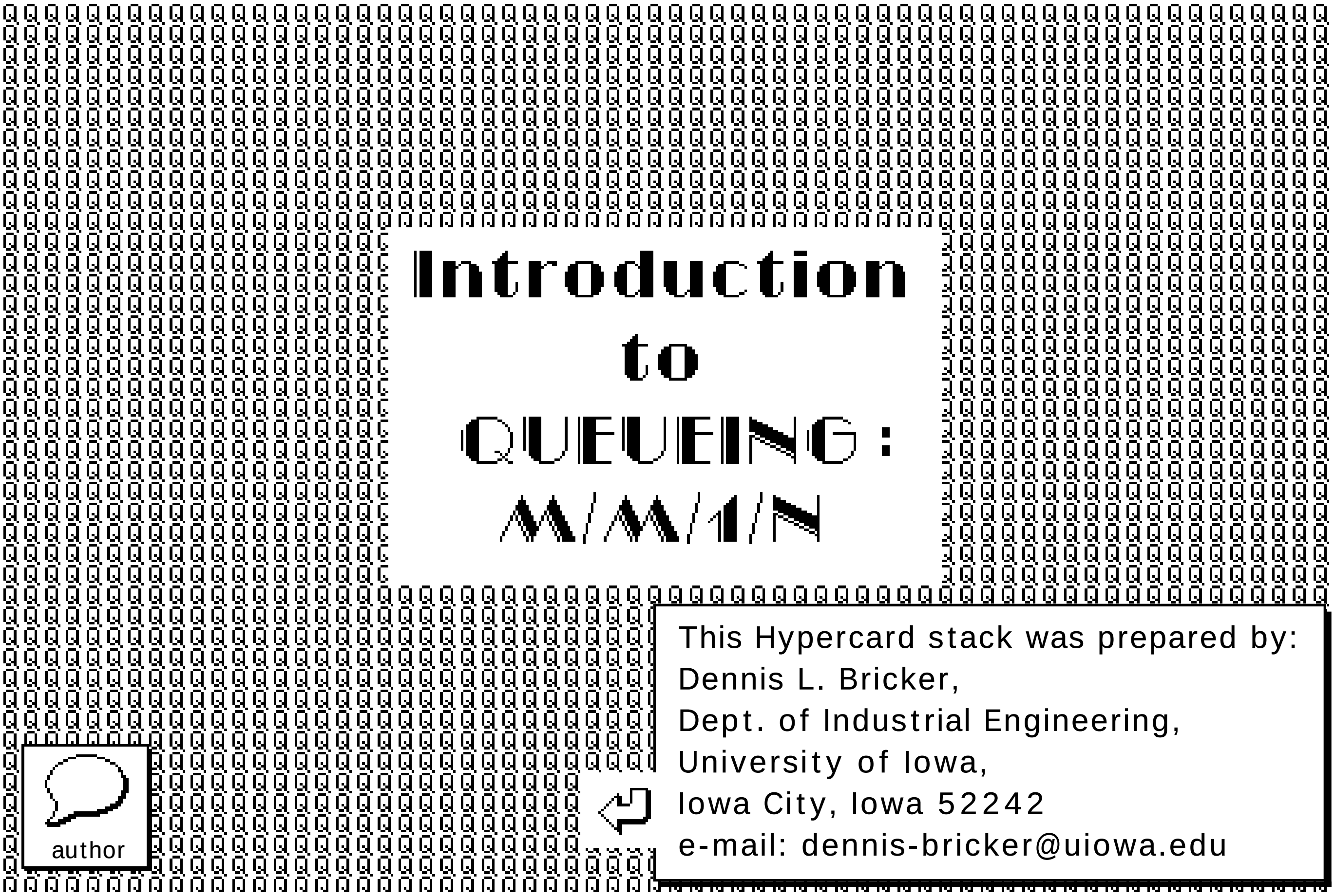
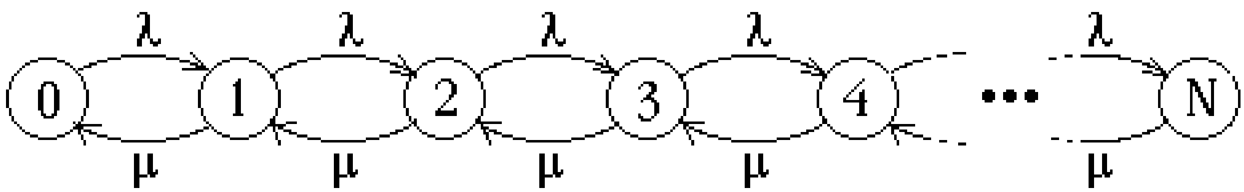


M/M/1/N

- *Arrival & Service processes are **M**emoryless, i.e.,*
  interarrival times have Exponential distribution with mean **1/λ**
  service times have Exponential distribution with mean **1/μ**
- *Single server*
- ***Capacity** of queueing system is **finite**:* N
  (including customer currently being served)
- *Arriving customers **balk** when queue is full.*

M/M/1/N

Birth/Death Model



M/M/1/N

Steadystate Distribution

$$\pi_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$$

$$\pi_j = \rho^j \pi_0 = \rho^j \left(\frac{1 - \rho}{1 - \rho^{N+1}}\right)$$

where $\rho = \frac{\lambda}{\mu} \neq 1$

Note that ρ is not restricted to be less than 1 for steady state to exist!

Average Number of Customers in System

$$L = \sum_{j=0}^{N} j\,\pi_j$$

$$L = \frac{\rho\left[1 - (N+1)\rho^{N} + N\rho^{N+1}\right]}{(1-\rho^{N+1})\,(1-\rho)}$$

where $\rho = \lambda/\mu \neq 1$

M/M/1/N

Special Case: $\lambda = \mu$, i.e., $\rho = \lambda/\mu = 1$

Arrival rate = Service rate

$$\pi_j = 1/N+1$$

$$L = N/2$$

All states are equally likely!

System is, on average, half-full!

Average Time in System per Customer

Little's Formula: $\mathbf{L} = \underline{\boldsymbol{\lambda}}\,\mathbf{W}$

↑ *average arrival rate*

$$\underline{\lambda} = \sum_{j=0}^{N-1} \lambda\, \pi_j = \lambda \sum_{j=0}^{N-1} \pi_j = \lambda\,(1 - \pi_N)$$

since arrival rate is zero when there are N in system

$$W = \frac{L}{\underline{\lambda}} = \frac{L}{\lambda\,(1 - \pi_N)}$$