

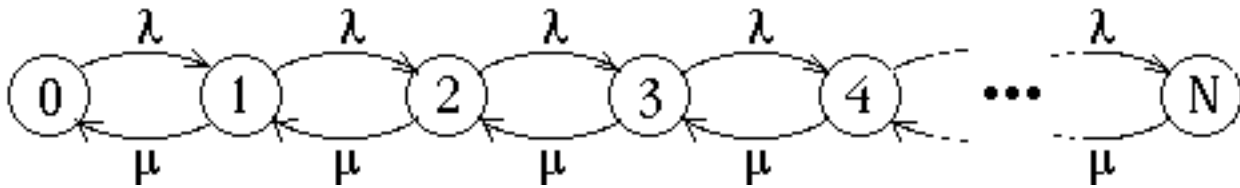
Introduction to QUEUEING: M/M/1/N

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M/M/1/N

- *Arrival & Service processes are **Memoryless**, i.e., interarrival times have Exponential distribution with mean $1/\lambda$, service times have Exponential distribution with mean $1/\mu$*
- *Single server*
- ***Capacity** of queueing system is **finite**: N (including customer currently being served)*
- *Arriving customers **balk** when queue is full.*

M/M/1/N**Birth/Death Model**

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M/M/1/N***Steadystate Distribution***

$$\pi_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$$

$$\pi_j = \rho^j \pi_0 = \rho^j \left(\frac{1 - \rho}{1 - \rho^{N+1}} \right)$$

where $\rho = \frac{\lambda}{\mu} \neq 1$

Note that ρ is not restricted to be less than 1 for steady state to exist!

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Average Number of Customers in System

$$L = \sum_{j=0}^N j \pi_j$$

$$L = \frac{\rho [1 - (N+1)\rho^N + N\rho^{N+1}]}{(1 - \rho^{N+1})(1 - \rho)}$$

where $\rho = \frac{\lambda}{\mu} \neq 1$

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M/M/1/N

Special Case: $\lambda = \mu$, i.e., $\rho = \frac{\lambda}{\mu} = 1$

Arrival rate = Service rate

$$\pi_j = \frac{1}{N+1}$$

$$L = \frac{N}{2}$$

All states are equally likely!

System is, on average, half-full!

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Average Time in System per Customer

Little's Formula: $L = \underline{\lambda} W$

↑
average arrival rate

$$\underline{\lambda} = \sum_{j=0}^{N-1} \lambda \pi_j = \lambda \sum_{j=0}^{N-1} \pi_j = \lambda (1 - \pi_N) \quad \text{since arrival rate is zero when there are } N \text{ in system}$$

$$W = \frac{L}{\underline{\lambda}} = \frac{L}{\lambda(1 - \pi_N)}$$

