

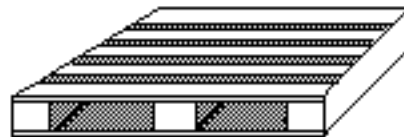
Labatt Brewery



*based upon "Labatt's Breweries",
in the book
Cases in Operations Research
by C.H. von Lanzener,
published by Holden-Day, 1975.*

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An analyst for Labatt's Ontario Breweries was faced with a question posed by the warehouse manager concerning the repair and replacement policy for pallets used in the shipment of beer.



Should the pallets be repaired when damaged, and if so, how often? This question always arose in early summer when the supply of pallets became tighter as a result of increased summer shipments.

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Labatt's belongs to a pallet pool with the other breweries in Ontario and Quebec and their common retail outlets.

The pallets were used for shipments to the retail outlets from the breweries and for the return of empties. Each brewery shipped its products to many locations throughout both provinces. The use of a common bottle and pallet by breweries in Canada allowed the return of empties to any brewery.

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New bottles, when shipped to the breweries from the glass manufacturers, were shipped on new pallets. The bill for the new pallets was included with the bill for the new bottles.

From time to time, additional pallets could be ordered by the individual breweries, if required. The number of new pallets purchased by each brewery was recorded, and at the end of each fiscal year, breweries that had purchased more than their share were compensated by those breweries that had purchased less than their share. (A brewery's share of new pallets to be purchased was determined by its share of the market.)

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Damaged pallets were repaired by individual breweries if feasible. If a center block had been damaged, the pallet was not repaired.

When questioned, the foreman at Labatt's in charge of pallet repair said: "Pallets are damaged through abuse by the handlers." Often, in repairing a pallet, the new nails would split a center block which would necessitate scrapping of the pallet.

Approximately 10 percent of the damaged pallets were unrepairable.

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Average cost of a new pallet is \$4.47

Average cost of repairing a pallet was \$2.07

The foreman in charge of repairs considered those over 2 years old not worth repairing, and they were scrapped.

Labatt's current repair policy is to repair a damaged pallet only if the age is no more than two years.

\$0.75 was the most ever received for damaged pallets, but the warehouse manager thought that as much as \$1.50 might be obtained if the scrapped pallets were in a better condition.

©Dennis Bricker, U. of Iowa, 1997 **Average Damage Rates**

<i>description</i>	<i>% damaged in year #</i>			
	<i>one</i>	<i>two</i>	<i>three</i>	<i>four</i>
New pallets	22	45	33	--
Pallets repaired in year #1	--	47	48	5
Pallets repaired in year #2	--	--	83	17

Using the damage rates, we will compute the transition probabilities

Note that each row sums to 100%!

Current Policy

pallets over two years old are scrapped when damaged

Consider the pallet to age 1 year on the last day of the year. Assume that the state of the system is observed on the last day of the year.

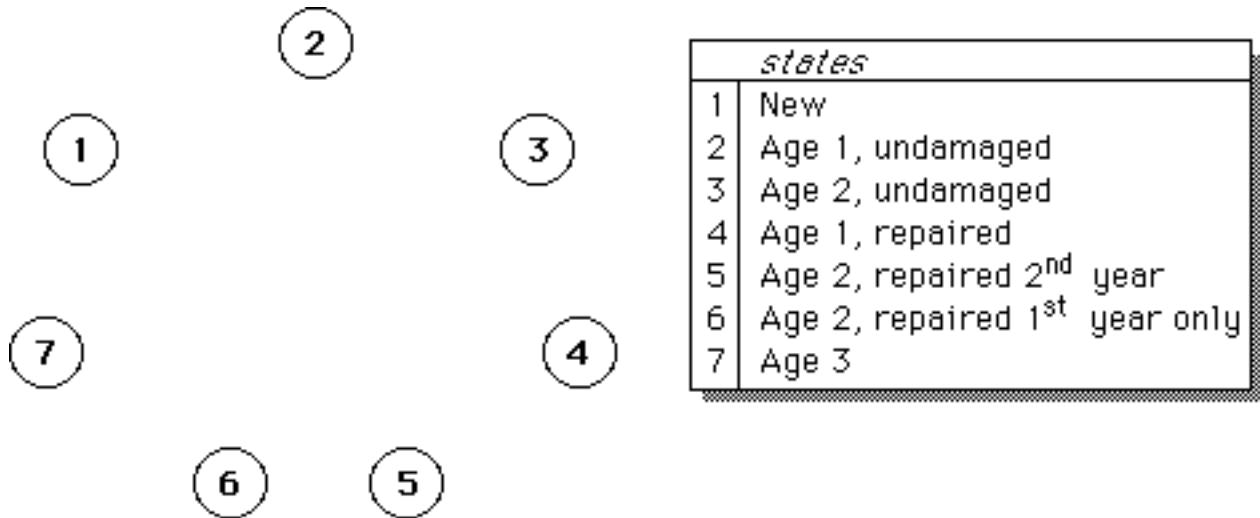
Possible states:

- 1 New (less than one year old)
- 2 Age 1, never damaged
- 3 Age 2, never damaged
- 4 Age 1, repaired
- 5 Age 2, repaired second year (& possibly first)
- 6 Age 2, repaired first year only
- 7 Age 3

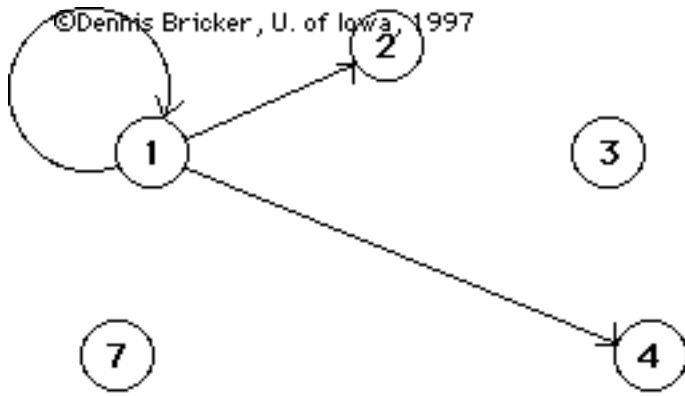
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Rather than model a "typical" pallet, which has a limited lifetime (no more than 4 years), we will model a pallet AND its future replacements, so that when a pallet is scrapped, the state of the pallet returns to "New", i.e., the state of the replacement pallet.

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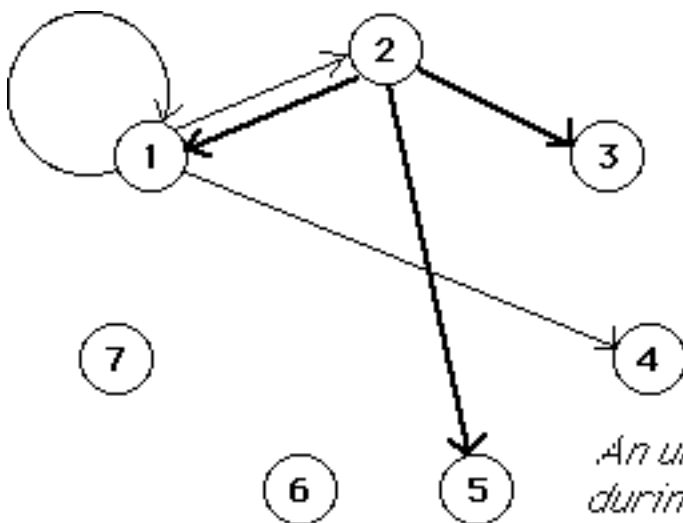
What transitions are possible?



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

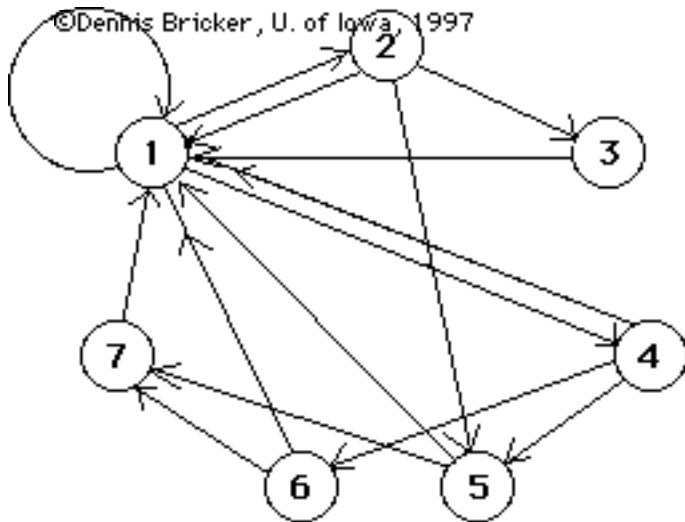
- 6 5 *A pallet which is new may or may not be damaged during the next year;*
- *if damaged & repairable, it will next be in state 4*
 - *if damaged & not repairable, it will be replaced by a new pallet*
 - *if not damaged, it will next be in state 2*

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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

- 6 5 *An undamaged one-year old pallet, during the next year:*
- *if damaged & repairable, it will next be in state 5*
 - *if damaged & not repairable, it will be replaced by a new pallet*
 - *if not damaged, it will next be in state 3*



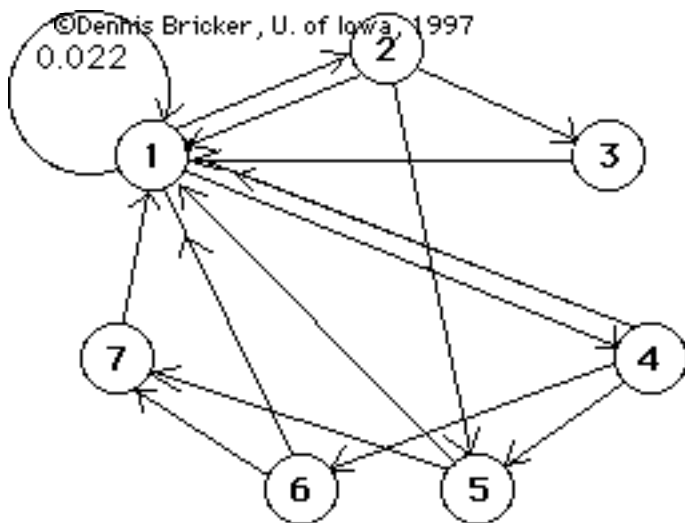
<i>states</i>	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

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Average Damage Rates

<i>description</i>	<i>% damaged in year #</i>			
	<i>one</i>	<i>two</i>	<i>three</i>	<i>four</i>
New pallets	22	45	33	--
Pallets repaired in year #1	--	47	48	5
Pallets repaired in year #2	--	--	83	17

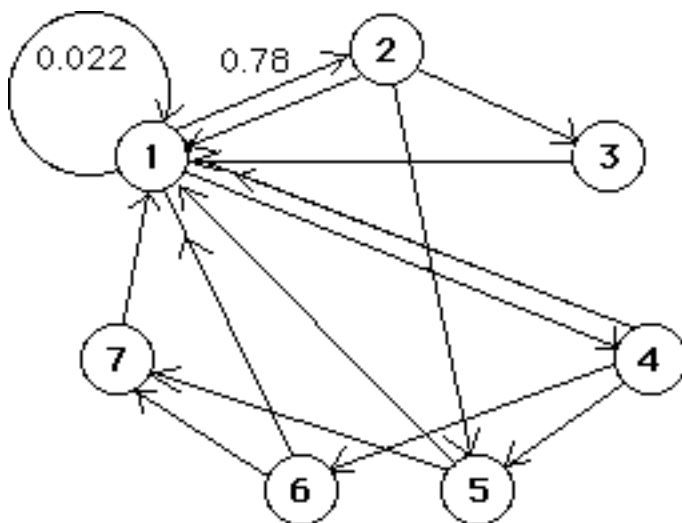
Using the damage rates, we will compute the transition probabilities



<i>states</i>	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

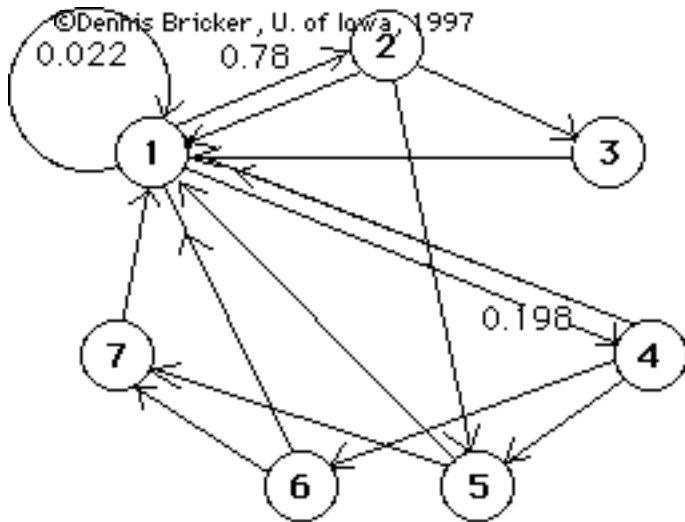
p_{11} = probability that new pallet is damaged in first year, but is not repairable
 = $0.22 \times 0.10 = 0.022$

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<i>states</i>	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

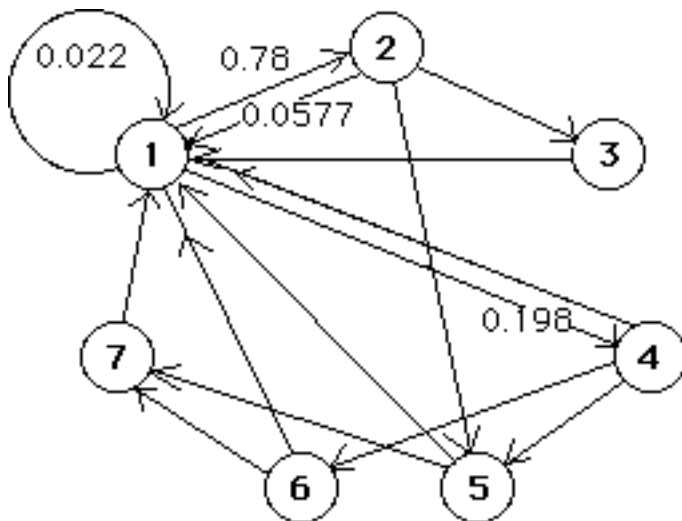
p_{12} = probability that new pallet is not damaged in its first year
 = $1 - 0.22 = 0.78$



<i>states</i>	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

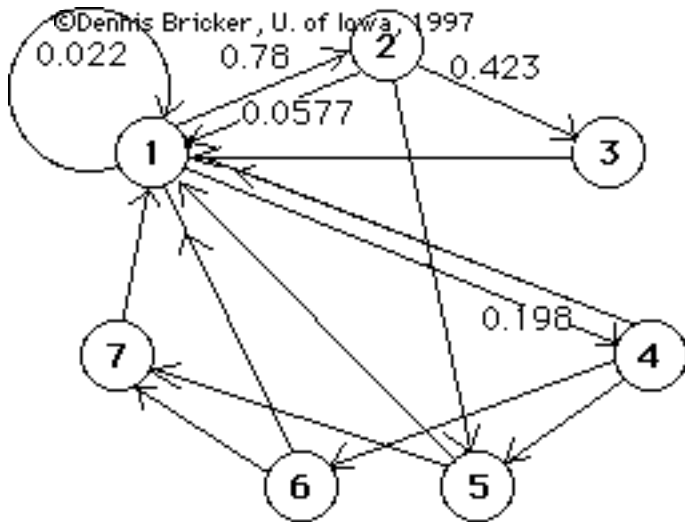
p_{14} = probability that new pallet is damaged in its first year, and is repairable
 = $0.22 \times 0.90 = 0.198$

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<i>states</i>	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

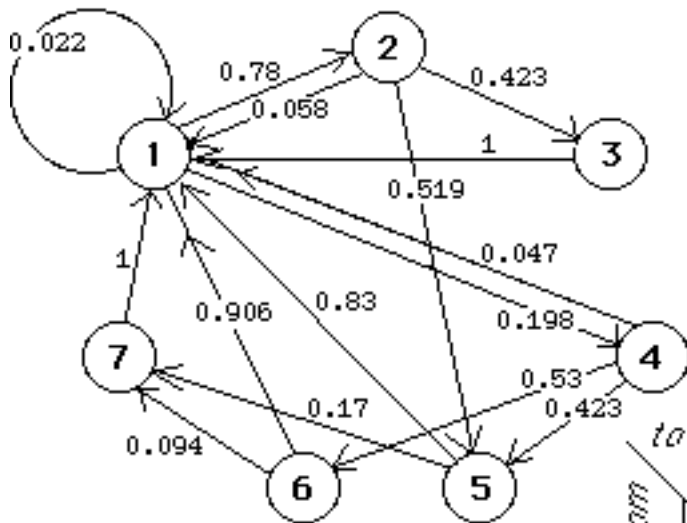
p_{21} = probability that undamaged 1-yr-old pallet is damaged next year & is unreparable
 = $\frac{0.45}{1-0.22} \times 0.10 = 0.0577$



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

p_{23} = probability that undamaged 1-yr-old pallet is not damaged in its second year
 $= \frac{0.33}{0.33+0.45} = 0.423$

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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transition Probability Matrix

		to						
		1	2	3	4	5	6	7
from	1	0.022	0.78	0	0.198	0	0	0
	2	0.058	0	0.423	0	0.519	0	0
	3	1	0	0	0	0	0	0
	4	0.047	0	0	0	0.423	0.53	0
	5	0.83	0	0	0	0	0	0.17
	6	0.906	0	0	0	0	0	0.094
	7	1	0	0	0	0	0	0

etc.

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Steady State Distribution

Current Policy

i		P<i>
1	New	0.3339594
2	Age 1, undamaged	0.26048833
3	Age 2, undamaged	0.11018657
4	Age 1, repaired	0.066123962
5	Age 2, Repair yr 2	0.16316388
6	Age 2, repair yr 1	0.0350457
7	Age 3	0.031032156

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System costs/revenues by state

Current Policy

i		--COST--
1	New	3.72
2	Age 1, undamaged	0
3	Age 2, undamaged	0
4	Age 1, repaired	2.07
5	Age 2, Repair yr 2	2.07
6	Age 2, repair yr 1	0
7	Age 3	0

assuming 75¢ scrap value

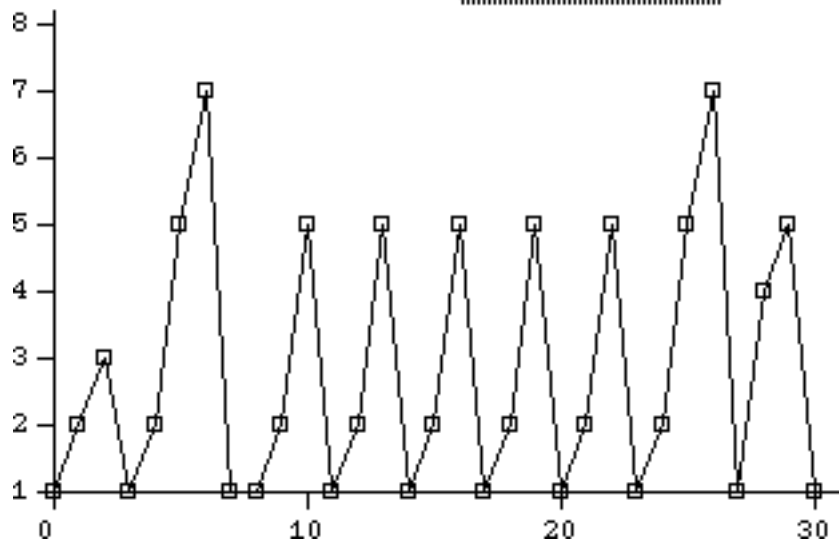
Current Policy

i	state	Pi	C	Pi×C
1	New	0.3339594	3.72	1.242329
2	Age 1, undamaged	0.26048833	0	0
3	Age 2, undamaged	0.11018657	0	0
4	Age 1, repaired	0.066123962	2.07	0.1368766
5	Age 2, Repair yr 2	0.16316388	2.07	0.33774923
6	Age 2, repair yr 1	0.0350457	0	0
7	Age 3	0.031032156	0	0

The average cost/period in steady state is 1.7169548

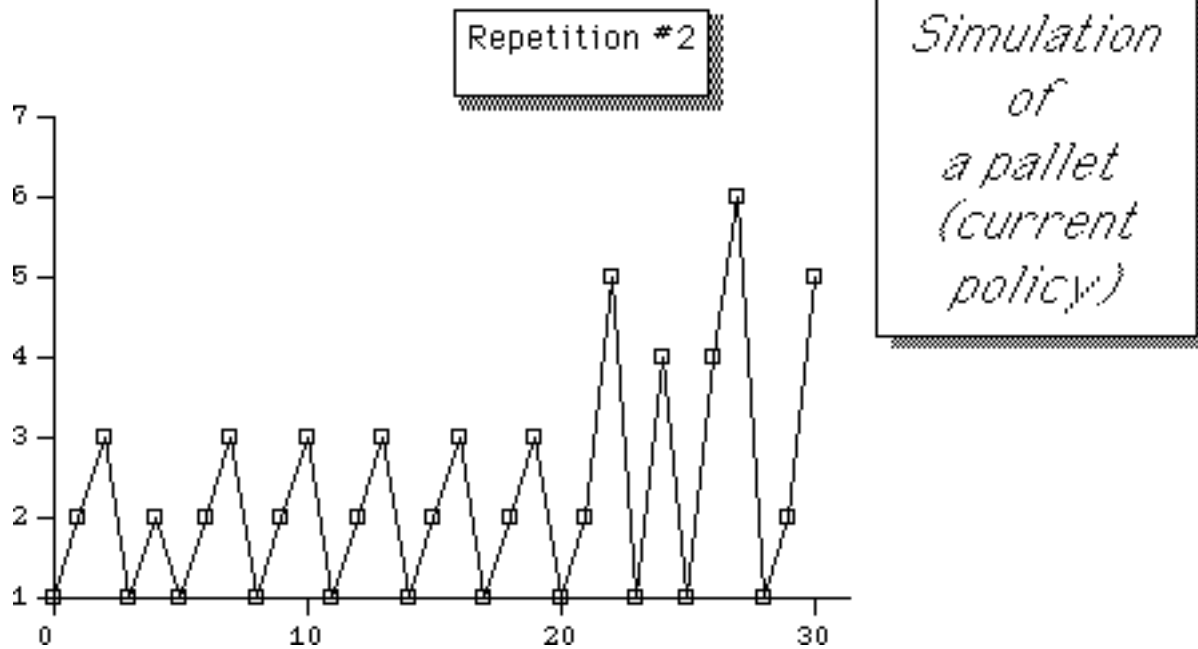
Annual cost of a pallet, under the current policy, is \$1.7169548

Repetition #1

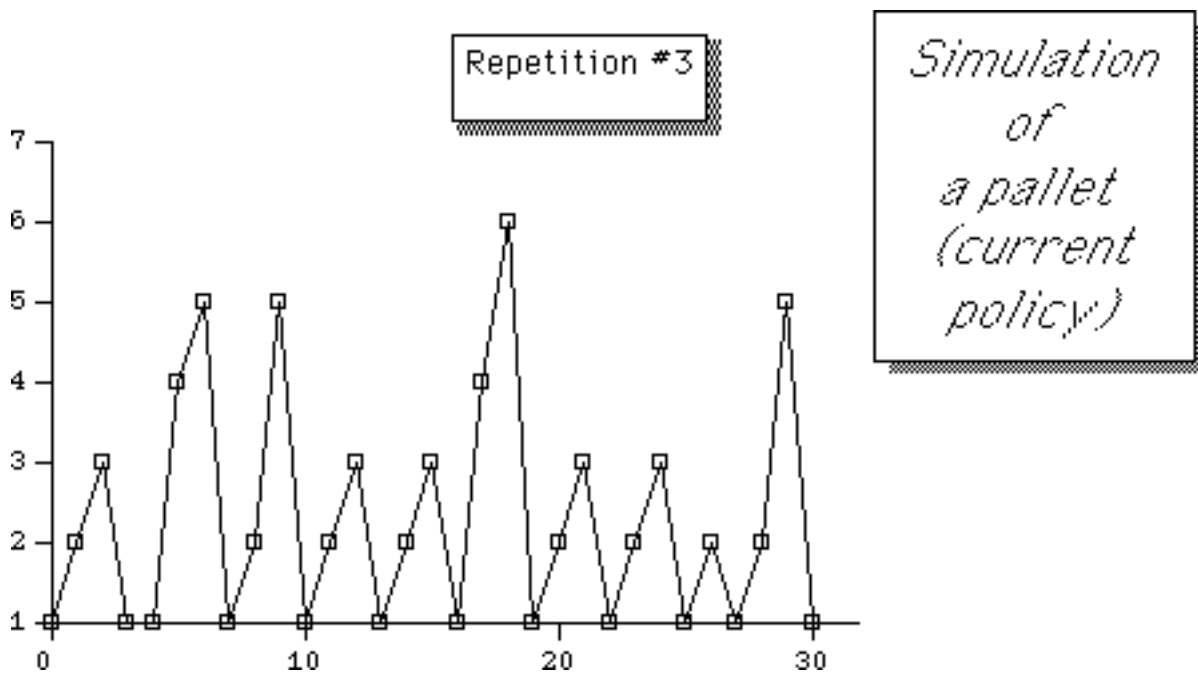


Simulation of a pallet (current policy)

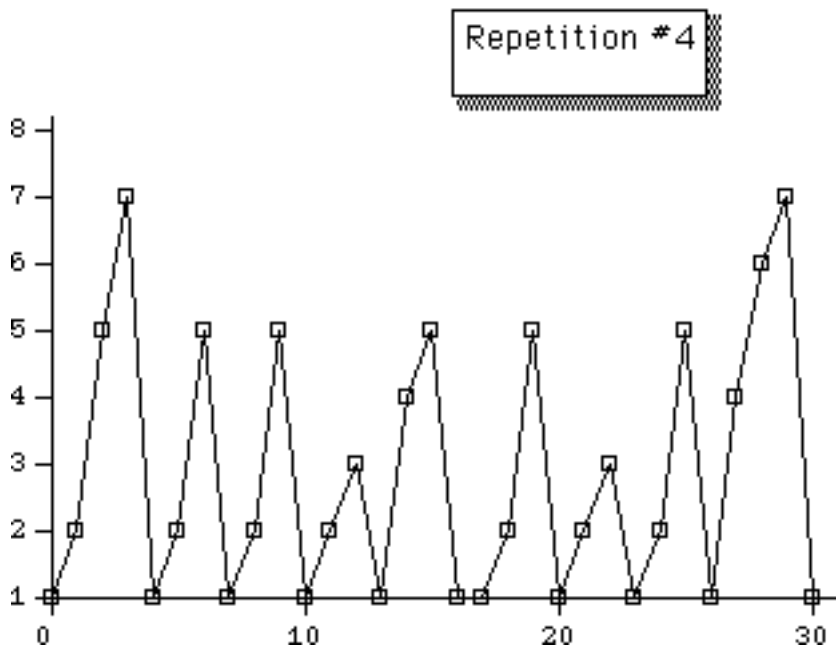
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*Simulation
of
a pallet
(current
policy)*

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Simulation results:

										1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3
0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0
1	2	3	1	2	5	7	1	1	2	5	1	2	5	1	2	5	1	2	5	1	2	5	1	2	5	1	2	5	7	1
1	2	3	1	2	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	5	1	4	1	4	6	1	2	5
1	2	3	1	1	4	5	1	2	5	1	2	3	1	2	3	1	4	6	1	2	3	1	2	3	1	2	1	2	5	
1	2	5	7	1	2	5	1	2	5	1	2	3	1	4	5	1	1	2	5	1	2	3	1	2	5	1	4	6	7	

Current
Policy

The array RUN has now been globally defined in the workspace.
 Each row of the array represents a repetition of the simulation.
 Note: Column 1 represents stage 0, i.e. the initial state.

```

+/COST[RUN]
59.55 49.2 54.99 57.48

(+/COST[RUN])÷31
1.9209677 1.5870968 1.773871 1.8541935  annual costs
    
```

*Simulation
of
a pallet
(current
policy)*

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Present Values by Period

Current Policy

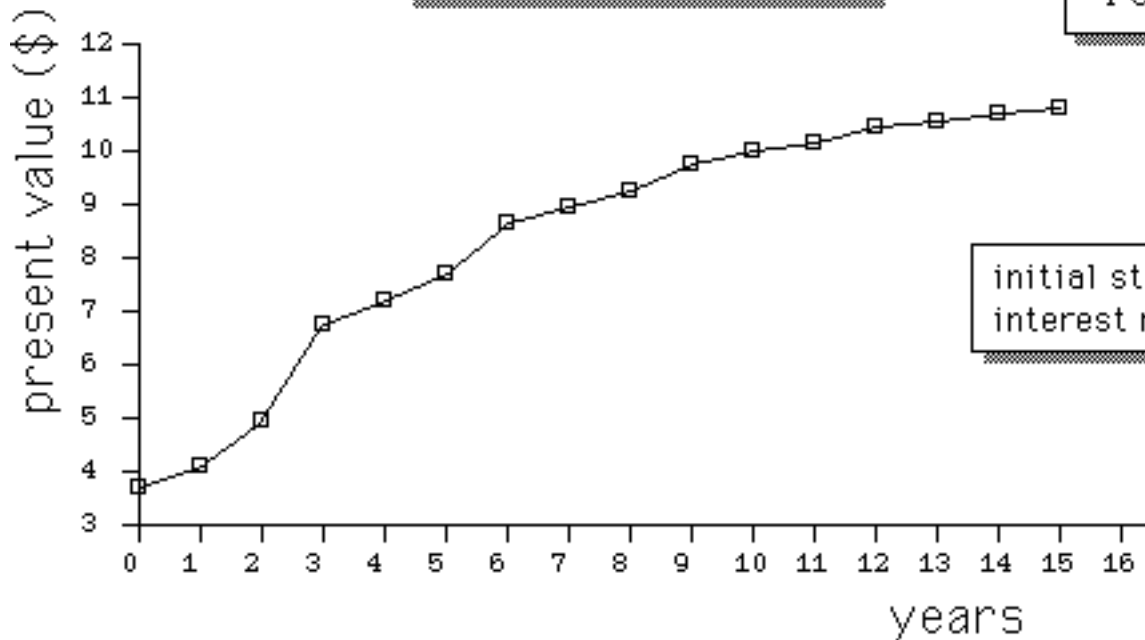
year n	cost C _n	present value PVC _n	cumulative present value V _n
0	3.72	3.72	3.72
1	0.4917	0.40975	4.12975
2	1.2250767	0.85074771	4.9804977
3	3.1433512	1.819069	6.7995667
4	0.89001899	0.42921441	7.2287811
5	1.2541928	0.50403198	7.7328131
6	2.8006297	0.93792521	8.6707383
7	1.1613007	0.3240977	8.994836
8	1.3026614	0.30295741	9.2977934
9	2.5345576	0.49121425	9.7890077
10	1.3515526	0.21828329	10.007291
11	1.3577965	0.1827431	10.190034
12	2.3296705	0.26128805	10.451322
13	1.4833389	0.13863861	10.589961
14	1.4129901	0.11005295	10.700014
15	2.1730311	0.14104161	10.841055

initial state = 1
interest rate = 20%

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Cumulative present value of annual costs

Current Policy



initial state = 1
interest rate = 20%

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Present value of all future costs, for each initial state:
 (using interest rate 20%, i.e. discount factor 0.83333333)

Current
Policy

i	initial state	PV
1	New	11.387469
2	Age 1, undamaged	8.7786854
3	Age 2, undamaged	9.4895575
4	Age 1, repaired	10.621535
5	Age 2, Repair yr 2	11.290687
6	Age 2, repair yr 1	9.3408878
7	Age 3	9.4895575

assuming
20% interest
rate

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What are some alternative repair policies?

Policy:
Repair only once

If a pallet has been repaired previously and is again damaged, replace it.

Policy:
Repair in first year only

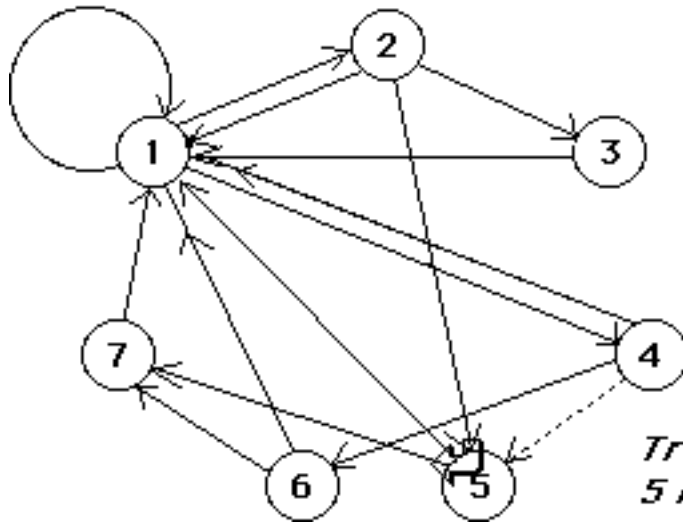
If a pallet is age 1 or greater, and is damaged, replace it

Policy:
No repairs

Always replace a damaged pallet (thereby eliminating the need for a repair shop.)

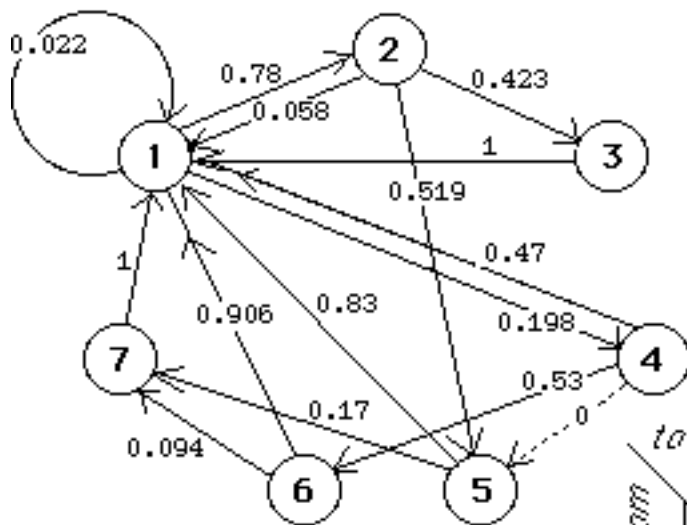
**Policy:
Repair only once**

If a pallet has been repaired previously and is again damaged, replace it.



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transitions from state 4 to state 5 NEVER occur, under this policy!



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transition Probability Matrix

		to						
		1	2	3	4	5	6	7
from	1	0.022	0.78	0	0.198	0	0	0
	2	0.058	0	0.423	0	0.519	0	0
	3	1	0	0	0	0	0	0
	4	0.47	0	0	0	0	0.53	0
	5	0.83	0	0	0	0	0	0.17
	6	0.906	0	0	0	0	0	0.094
	7	1	0	0	0	0	0	0

**Policy:
Repair only once**

**Policy:
Repair only once**

Steady State Distribution

i	P<i>
1 New	0.34525812
2 Age 1, undamaged	0.26930133
3 Age 2, undamaged	0.11391446
4 Age 1, repaired	0.068361107
5 Age 2, Repair yr 2	0.13976739
6 Age 2, repair yr 1	0.036231387
7 Age 3	0.027166207

Assuming 75¢ scrap value:

i	state	Pi	C	Pi×C
1	New	0.34525812	3.72	1.2843602
2	Age 1, undamaged	0.26930133	0	0
3	Age 2, undamaged	0.11391446	0	0
4	Age 1, repaired	0.068361107	2.07	0.14150749
5	Age 2, Repair yr 2	0.13976739	2.07	0.2893185
6	Age 2, repair yr 1	0.036231387	0	0
7	Age 3	0.027166207	0	0

The average cost/period in steady state is 1.7151862

Annual cost per pallet, under this policy, is

\$ 1.7151862

which is slightly less than that of the current policy,

\$ 1.7169548

**Policy:
Repair only once**

Assuming 75¢ scrap value:

Present value of all future costs, for each initial state:

(using interest rate 20%, i.e. discount factor 0.83333333)

i	PV
1	11.40129
2	8.7882537
3	9.5010753
4	10.666072
5	11.301878
6	9.3522251
7	9.5010753

Present Values by Period

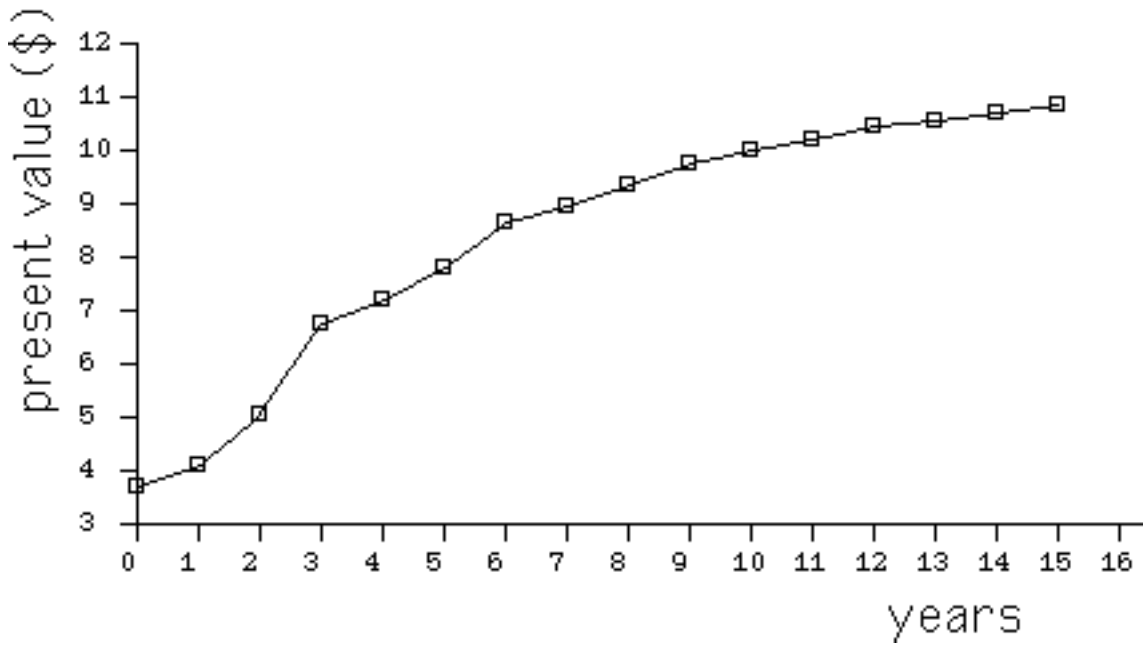
**Policy:
Repair only once**

n	Cn	PVCn	Vn
0	3.72	3.72	3.72
1	0.4917	0.40975	4.12975
2	1.3632708	0.94671583	5.0764658
3	2.9289745	1.6950084	6.7714742
4	0.91987306	0.44361162	7.2150858
5	1.5014744	0.60340889	7.8184947
6	2.4965153	0.83607792	8.6545726
7	1.1930783	0.33296626	8.9875389
8	1.5865413	0.36897879	9.3565177
9	2.217937	0.42985104	9.7863687
10	1.3726005	0.22168264	10.008051
11	1.6381974	0.22048169	10.228533
12	2.0385921	0.22864167	10.457175
13	1.4904986	0.13930778	10.596483
14	1.6694245	0.13002574	10.726508
15	1.9231645	0.1248239	10.851332

r=20%
initial state: 1

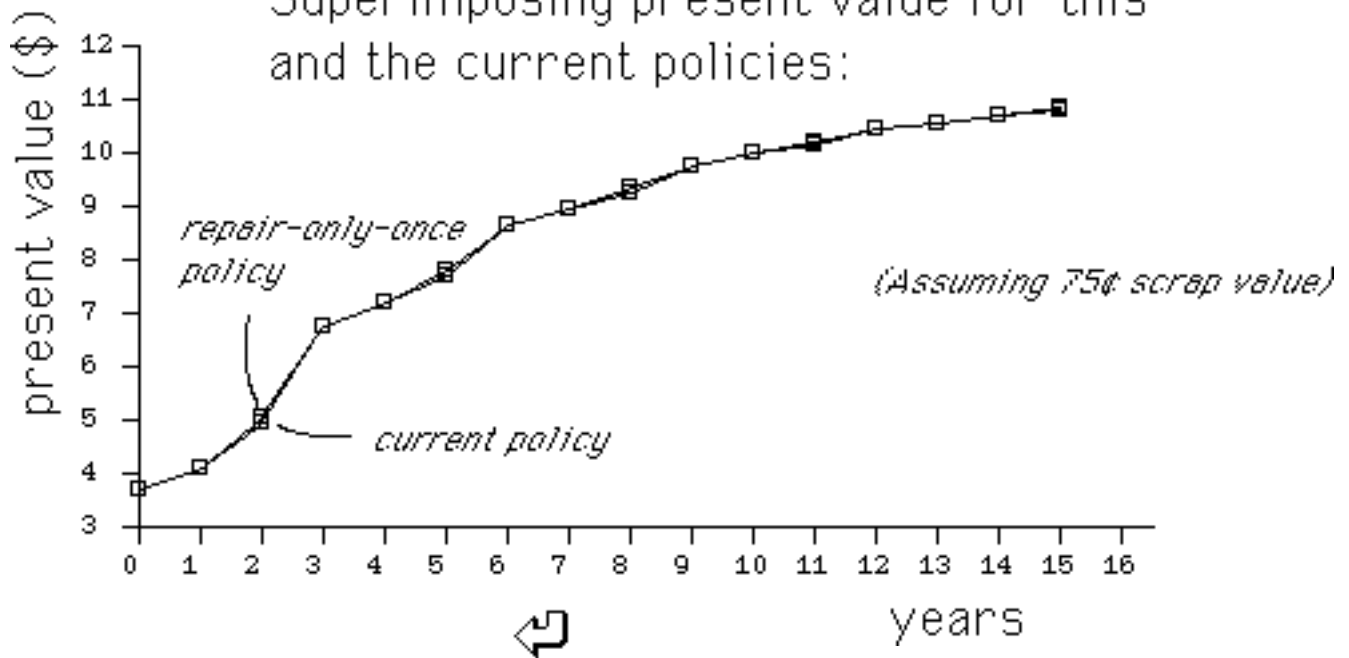
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Cumulative present value of costs or revenues vs period



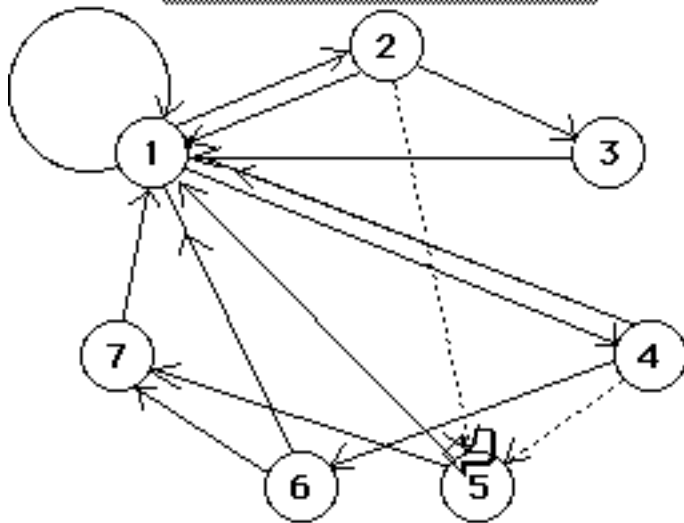
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Superimposing present value for this and the current policies:



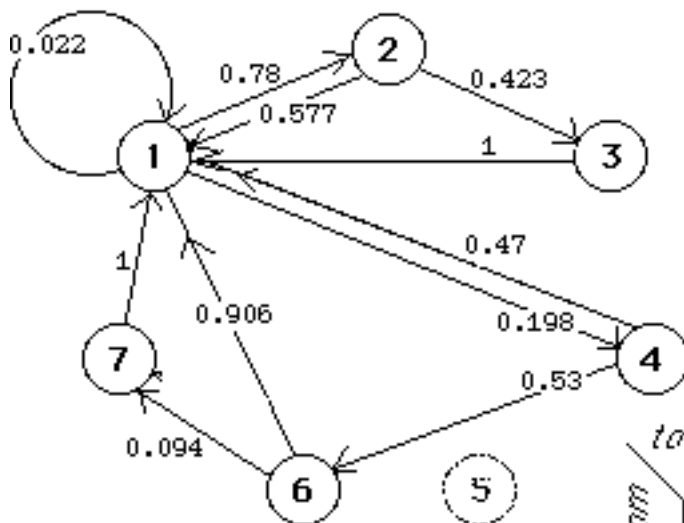
Policy:
 Repair in first
 year only

*If a pallet is age 1 or greater,
 and is damaged, replace it*



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

*Transitions into state 5
 will never occur!*



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transition Probability Matrix

		to						
		1	2	3	4	5	6	7
from	1	0.022	0.78	0	0.198	0	0	0
	2	0.577	0	0.423	0	0	0	0
	3	1	0	0	0	0	0	0
	4	0.47	0	0	0	0	0.53	0
	5	0	0	0	0	1	0	0
	6	0.906	0	0	0	0	0	0.094
	7	1	0	0	0	0	0	0

Policy:
 Repair in first
 year only

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Transition Probability Matrix

f						
r						
o	1	2	3	4	5	6
m	-----	-----	-----	-----	-----	-----
1	0.022	0.78	0	0.198	0	0
2	0.577	0	0.423	0	0	0
3	1	0	0	0	0	0
4	0.47	0	0	0	0.53	0
5	0.906	0	0	0	0	0.094
6	1	0	0	0	0	0

- i
- 1 New
- 2 Age 1, undamaged
- 3 Age 2, undamaged
- 4 Age 1, repaired
- 5 Age 2, repaired
- 6 Age 3

**Policy:
Repair in first
year only**

*Let's eliminate the state
Age 2, repaired 2nd year
and renumber the states*

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Assuming 75¢ scrap value:

i	State	Pi	C	Pi×C
1	New	0.41275506	3.72	1.5354488
2	Age 1, undamaged	0.32194895	0	0
3	Age 2, undamaged	0.13618441	0	0
4	Age 1, repaired	0.081725502	2.07	0.16917179
5	Age 2, repaired	0.043314516	0	0
6	Age 3	0.0040715645	0	0

**Policy:
Repair in first
year only**

The average cost/period in steady state is 1.7046206

The annual cost per pallet will be \$1.7046206

**Policy:
Repair in first
year only**

Present value of all future costs, for each initial state:

(using interest rate 20%, i.e. discount factor 0.83333333)

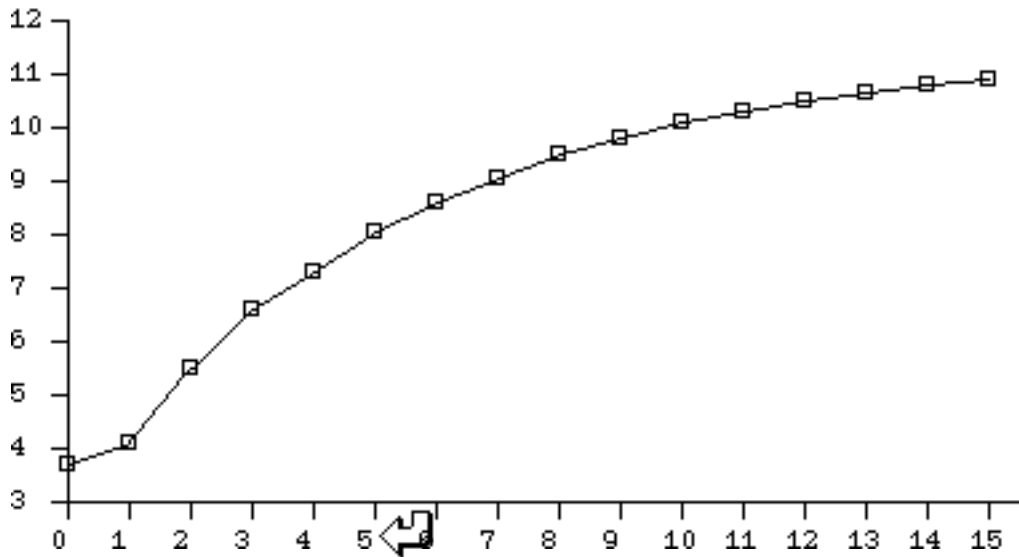
i	State	PV
1	New	11.480285
2	Age 1, undamaged	8.8924378
3	Age 2, undamaged	9.5669045
4	Age 1, repaired	10.72563
5	Age 2, repaired	9.417023
6	Age 3	9.5669045

Present Values by Period

n	Cn	PVCn	Vn
0	3.72	3.72	3.72
1	0.4917	0.40975	4.12975
2	2.0312238	1.4105721	5.5403221
3	1.8927972	1.0953688	6.6356908
4	1.3905154	0.67058035	7.3062712
5	1.9267595	0.77432145	8.0805926
6	1.6221106	0.54324155	8.6238342
7	1.6918101	0.47215315	9.0959873
8	1.75084	0.40718943	9.5031768
9	1.665803	0.32284378	9.8260206
10	1.7226107	0.27821125	10.104232
11	1.7034514	0.22926409	10.333496
12	1.6983235	0.19047828	10.523974
13	1.7111102	0.159927	10.683901
14	1.7010238	0.1324869	10.816388
15	1.7053782	0.11068838	10.927076

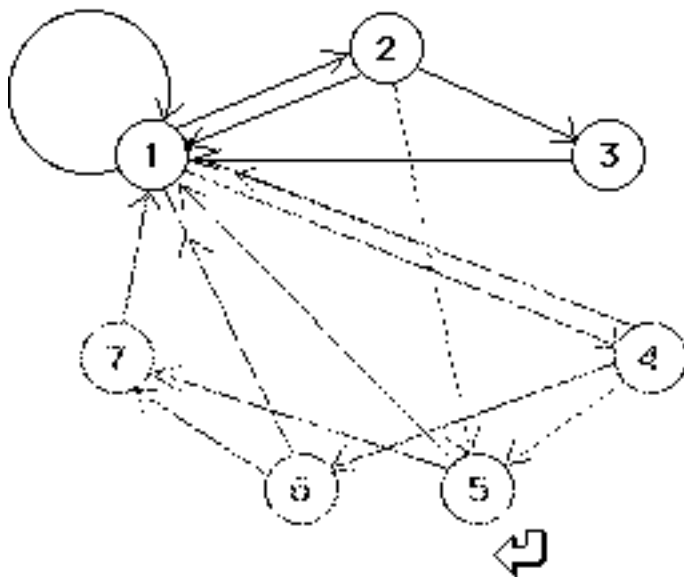
**Policy:
Repair in first
year only**

Cumulative present value of costs or revenues vs period



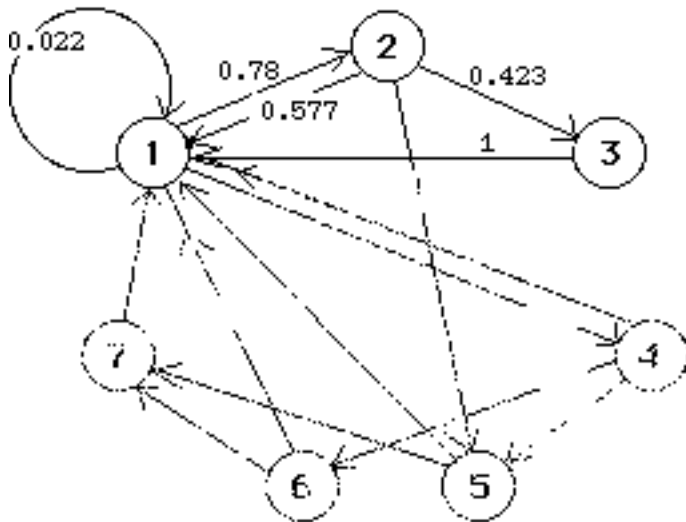
**Policy:
No repairs**

*Always replace a damaged pallet
(thereby eliminating the need for
a repair shop.)*



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

*Only states 1, 2, & 3
are possible!*



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged

Transition Probability Matrix

		to		
		1	2	3
from	1	0.22	0.78	0
	2	0.577	0	0.423
	3	1	0	0

**Policy:
No repairs**

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Assuming a 75¢ scrap value:

**Policy:
No repairs**

i	Pi	C	Pi×C
1	0.47394713	3.72	1.7630833
2	0.36967876	0	0
3	0.15637411	0	0

The average cost/period in steady state is 1.7630833

The annual cost per pallet will be \$1.7630833

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Assuming a 75¢ scrap value:

**Policy:
No repairs**

Present value of all future costs, for each initial state:

(using interest rate 20%, i.e. discount factor 0.83333333)

i	State	PV
1	New	11.877869
2	Age 1, undamaged	9.2003991
3	Age 2, undamaged	9.8982239

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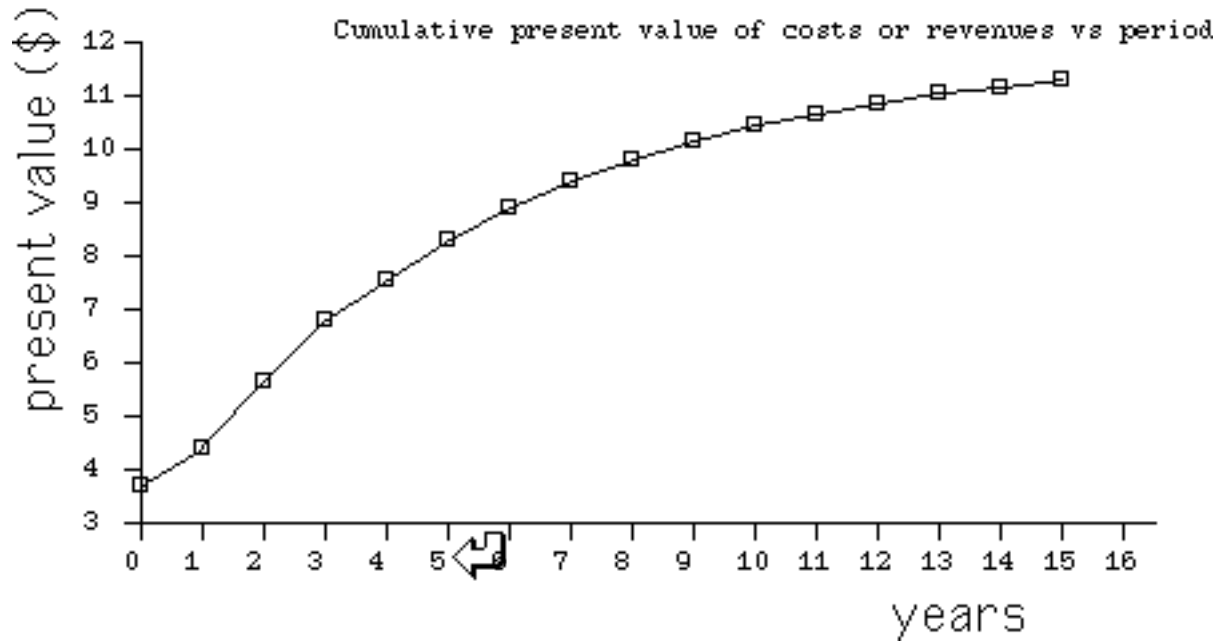
Initial state: 1, interest rate= 20%

Present Values by Period

n	Cn	PVCn	Vn
0	3.72	3.72	3.72
1	0.8184	0.682	4.402
2	1.8542712	1.2876883	5.6896883
3	2.0036456	1.1595171	6.8492054
4	1.5453582	0.74525377	7.5944592
5	1.8535378	0.74489526	8.3393545
6	1.764365	0.59088228	8.9302368
7	1.732239	0.48343612	9.4136729
8	1.786719	0.41553373	9.8292066
9	1.7548243	0.3400967	10.169303
10	1.761727	0.28452875	10.453832
11	1.7668662	0.23779896	10.691631
12	1.7605801	0.19746078	10.889092
13	1.7637876	0.16485044	11.053942
14	1.7633598	0.13734204	11.191284
15	1.7626352	0.11440467	11.305689

**Policy:
No repairs**

Assuming a 75¢ scrap value:



Summary

Policy	Annual Cost per Pallet	
	75¢ scrap value	\$1.50 scrap value
Current policy	\$1.7169548	
Repair only once	\$1.7151862	
Repair First Year Only	\$1.7046206	
Never Repair	\$1.7630833	