Interest Factors
(time value of money)

Single-payment factors

Let \( P \) = original investment
\( r \) = rate of interest per period
\( n \) = number of periods

\( S_n = \) value of investment after \( n \) periods

Then

\[ S_n = (1 + r)^n P \]

\( \text{spcaf}(r,n) = (1 + r)^n \)

single-payment compound-amount factor
Conversely, a future payment of $S_n$ has an equivalent present worth $P$,

$$P = (1 + r)^{-n} S_n$$

$$\text{sppwf}(r, n) = (1 + r)^{-n}$$

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**Uniform Series of Payments**

Consider a sequence of $n$ uniform periodic payments, $R$, earning interest at rate $r$ per period, compounded at the end of each period. Then the accumulated value after $n$ periods is

$$S_n = \frac{(1 + r)^n - 1}{r} R$$

$$\text{uscaf}(r, n) = \frac{(1 + r)^n - 1}{r}$$

uniform-series compound-amount factor
Conversely, the amount of each payment $R$ required to accumulate a sum $S$ after $n$ periods at interest rate $r$ is

$$R = \frac{r}{1 + r} \frac{S_n}{1 - 1}$$

$sfdf(r,n) = \frac{r}{1 + r} \frac{n}{1 - 1}$

$sinking\text{-}fund\ deposit\ factor$

The sequence of $n$ uniform payments, $R$, can also be expressed as a present worth $P$:

$$P = \frac{1 + r^n}{r(1 + r)^n} R$$

$uspwf(r,n) = \frac{1 + r^n}{r(1 + r)^n}$

$uniform\text{-}series\ present\text{-}worth\ factor$
Finally, expressing a present amount $P$ as an equivalent sequence of $n$ uniform payments $R$ gives

$$R = \frac{r (1 + r)^n}{(1 + r)^n - 1} P$$

**crf(r,n) = \frac{r (1 + r)^n}{(1 + r)^n - 1}**

capital recovery factor

**Summary:**

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<th>Find</th>
<th>by multiplying with the</th>
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<td>$P$</td>
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<td>single-payment compound-amount factor (spcaf)</td>
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<tr>
<td>$S_n$</td>
<td>$P$</td>
<td>single-payment present-worth factor (sppw)</td>
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<tr>
<td>$R$</td>
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<td>$P$</td>
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