LP

FURTHER SIMPLEX EXAMPLES

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Example One

Example Two

Example Three

Example Four

$$\text{Max } z = 2x_1 - x_2 + x_3$$

subject to

$$\begin{cases} 3x_1 + x_2 + x_3 \le 60 \\ x_1 - x_2 + 2x_3 \le 10 \\ x_1 + x_2 - x_3 \le 20 \end{cases}$$

$$x_1,\ x_2,\ x_3 \ge 0$$

$$\text{Max } z = 2x_1 - x_2 + x_3$$

subject to

$$\begin{cases} 3x_1 + x_2 + x_3 + x_4 = 60 \\ x_1 - x_2 + 2x_3 + x_5 = 10 \\ x_1 + x_2 - x_3 + x_6 = 20 \end{cases}$$

$$x_1,\ x_2,\ x_3, x_4,\ x_5,\ x_6 \ge 0$$

Add slack variables to convert the inequalities to equations

	-z	X_1	X_2	X_3	X_4	X_5	X_6	
	1	2	3	4	5	6	7	B
MAX	1	2	-1	1	0	0	0	0
	0	3	1	1	1	0	0	60
	0	1	-1	2	0	1	0	10
	0	1	1	-1	0	0	1	20

	1	2	3	4	5	6	7	B	
		↓							
MAX	1	2	-1	1	0	0	0	0	
	0	3	1	1	1	0	0	60	
	0	(1)	-1	2	0	1	0	10	←
	0	1	1	-1	0	0	1	20	
	*				*	*	*		

⇓

	1	2	3	4	5	6	7	B
MAX	1	0	1	-3	0	-2	0	-20
	0	0	4	-5	1	-3	0	30
	0	1	-1	2	0	1	0	10
	0	0	2	-3	0	-1	1	10
	*	*			*		*	

	1	2	3	4	5	6	7	B	
			↓						
MAX	1	0	1	-3	0	-2	0	-20	
	0	0	4	-5	1	-3	0	30	
	0	1	-1	2	0	1	0	10	
	0	0	(2)	-3	0	-1	1	10	←
	*	*			*		*		

⇓

	1	2	3	4	5	6	7	B
MAX	1	0	0	-1.5	0	-1.5	-0.5	-25
	0	0	0	1	1	-1	-2	10
	0	1	0	0.5	0	0.5	0.5	15
	0	0	1	-1.5	0	-0.5	0.5	5
	*	*	*		*			

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	
	1	2	3	4	5	6	7	B
MAX	1	0	0	¯1.5	0	¯1.5	¯0.5	¯25
	0	0	0	1	1	¯1	¯2	10
	0	1	0	0.5	0	0.5	0.5	15
	0	0	1	¯1.5	0	¯0.5	0.5	5
	*	*	*		*			

optimal tableau!

$$-z = -25 \Rightarrow z = 25$$

basic	*nonbasic*
$z = 25$	$x_3 = 0$
$x_1 = 15$	$x_5 = 0$
$x_2 = 5$	$x_6 = 0$
$x_4 = 10$	

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 \quad &= 15 \\
2x_1 + x_2 - 5x_3 \quad &= 20 \\
x_1 + 2x_2 - x_3 + x_4 &= 10 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}
$$

	-z	x_1	x_2	x_3	x_4	
	1	2	3	4	5	B
MAX	1	1	2	3	¯1	0
	0	1	2	3	0	15
	0	2	1	¯5	0	20
	0	1	2	¯1	1	10
	*				*	

-z and X_4 can serve as the basic variables in the top and bottom rows, respectively.

Minimize $w = x_5 + x_6$

$$-z + x_1 + 2x_2 + 3x_3 - x_4 = 0$$

subject to

$$x_1 + 2x_2 + 3x_3 + x_5 = 15$$
$$2x_1 + x_2 - 5x_3 + x_6 = 20$$
$$x_1 + 2x_2 - x_3 + x_4 = 10$$
$$x_1 , x_2 , x_3 , x_4 \geq 0 , \; x_5 \geq 0, x_6 \geq 0$$

Artificial variables X_5 & X_6 are added to the first two constraints to serve as initial basic variables.

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	1	2	3	−1	0	0	0
	0	0	1	2	3	0	1	0	15
	0	0	2	1	−5	0	0	1	20
	0	0	1	2	−1	1	0	0	10
	*	*							

First, pivot so as to eliminate X_5 & X_6 from the top row and X_4 from the second row.

	1	2	3	4	5	6	7	8	B
MIN	1	0	−3	−3	2	0	0	0	−35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	−5	0	0	1	20
	0	0	1	2	−1	1	0	0	10
	*	*				*	*	*	

We now have a basic "pseudo-feasible" solution with which to begin the Simplex method.

	1	2	3	4	5	6	7	8	B
MIN	1	0	−3	−3	2	0	0	0	−35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	−5	0	0	1	20
	0	0	1	2	−1	1	0	0	10
	*	*				*	*	*	

We are minimizing the Phase-One objective, and select a pivot column having a negative reduced cost in the objective row.

Two columns have a negative reduced cost. Pivoting in either column should reduce the value of the objective.

	1	2	3	4	5	6	7	8	B
MIN	1	0	−3	−3	2	0	0	0	−35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	(2)	1	−5	0	0	1	20
	0	0	1	2	−1	1	0	0	10
	*	*				*	*	*	

Arbitrarily choose X_1 rather than X_2.

Minimum ratio test:

$$\min\left\{\frac{15}{1}, \frac{20}{2}, \frac{10}{1}\right\}$$ *tie!*

1	2	3	4	5	6	7	8	B
1	0	0	−1.5	−5.5	0	0	1.5	−5
0	1	0	3	7	0	0	−1	−10
0	0	0	1.5	5.5	0	1	−0.5	5
0	0	1	0.5	−2.5	0	0	0.5	10
0	0	0	1.5	1.5	1	0	−0.5	0
*	*	*			*	*		

Arbitrarily we select row 4 for the pivot. This introduced a zero on the right-hand-side

(degeneracy!)

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-1.5	-5.5	0	0	1.5	-5
	0	1	0	3	7	0	0	-1	-10
	0	0	0	1.5	5.5	0	1	-0.5	5
	0	0	1	0.5	-2.5	0	0	0.5	10
	0	0	0	1.5	(1.5)	1	0	-0.5	0
	*	*	*			*	*		

Either columns 4 or 5 (X or X) could be selected as pivot column... let's choose column 5.

Minimum Ratio Test:

$$\min\left\{\frac{5}{5.5}, --, \frac{0}{1.5}\right\}$$

The minimum ratio is zero!

1	2	3	4	5	6	7	8	B
1	0	0	4	0	3.667	0	-0.3333	-5
0	1	0	-4	0	-4.667	0	1.333	-10
0	0	0	-4	0	-3.667	1	1.333	5
0	0	1	3	0	1.667	0	-0.3333	10
0	0	0	1	1	0.6667	0	-0.3333	0
*	*	*		*		*		

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	4	0	3.667	0	-0.3333	-5
	0	1	0	-4	0	-4.667	0	1.333	-10
	0	0	0	-4	0	-3.667	1	(1.333)	5
	0	0	1	3	0	1.667	0	-0.3333	10
	0	0	0	1	1	0.6667	0	-0.3333	0
	*	*	*		*		*		

Choose column #8 for the next pivot.

There is only one candidate row for the pivot.

1	2	3	4	5	6	7	8	B
1	0	0	3	0	2.75	0.25	0	-3.75
0	1	0	0	0	-1	-1	0	-15
0	0	0	-3	0	-2.75	0.75	1	3.75
0	0	1	2	0	0.75	0.25	0	11.25
0	0	0	0	1	-0.25	0.25	0	1.25
*	*	*		*			*	

The resulting tableau is optimal (for Phase One), since no column has a negative reduced cost.

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	3	0	2.75	0.25	0	−3.75
	0	1	0	0	0	−1	−1	0	−15
	0	0	0	−3	0	−2.75	0.75	1	3.75
	0	0	1	2	0	0.75	0.25	0	11.25
	0	0	0	0	1	−0.25	0.25	0	1.25
	*	*	*		*			*	

In this tableau, one of the artificial variables remains basic (and positive).

This indicates that the original LP had no feasible solution, since a feasible solution (with artificial variables zero) would be optimal for Phase One, if such a solution exists!

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 - x_3 + x_4 = 10$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

$-z$ & X_4 can serve as basic variables in the first and last rows.

	1	2	3	4	5	B
MAX	1	1	2	3	−1	0
	0	1	2	3	0	15
	0	2	1	5	0	20
	0	1	2	−1	1	10
	*				*	

The second and third rows will require artificial variables to serve as basic variables.

Minimize $w = x_5 + x_6$

$$-z + x_1 + 2x_2 + 3x_3 - x_4 = 0$$

subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 \quad + x_5 \quad &= 15 \\ 2x_1 + x_2 + 5x_3 \quad + x_6 &= 20 \\ x_1 + 2x_2 - x_3 + x_4 \quad &= 10 \end{aligned}$$

$$x_1, x_2, x_3, x_4 \geq 0, \; x_5 \geq 0, x_6 \geq 0$$

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	1	2	3	⁻1	0	0	0
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	1	2	⁻1	1	0	0	10
	*	*							

X_5 & X_6 are artificial variables, and the Phase-One objective is to minimize their sum.

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	1	2	3	⁻1	0	0	0
	0	0	1	2	3	0	(1)	0	15
	0	0	2	1	5	0	0	(1)	20
	0	0	1	2	⁻1	(1)	0	0	10
	*	*				*	*	*	

We must pivot to enter X_4, X_5, and X_6 into the basis (eliminating these variables from the two objective rows).

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	1	2	3	4	5	6	7	8	B
MIN	1	0	⁻3	⁻3	⁻8	0	0	0	⁻35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	1	2	⁻1	1	0	0	10
	*	*				*	*	*	

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	-8	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

Any one of columns 3, 4, and 5 could be selected as the pivot column.
Here, column 3 (X_1) is chosen.

Minimum ratio test:

$$\min\left\{\frac{15}{1}, \frac{20}{2}, \frac{10}{1}\right\}$$ *tie!*

Arbitrarily break the tie

Row 4

Row 5

	1	2	3	4	5	6	7	8	B
MIN	1	0	-3	-3	-8	0	0	0	-35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	(2)	1	5	0	0	1	20
	0	0	1	2	-1	1	0	0	10
	*	*				*	*	*	

Any one of columns 3, 4, and 5 could be selected as the pivot column.
Here, column 3 (X_1) is chosen.
Minimum ratio test:

$$\min\left\{\frac{15}{1}, \frac{20}{2}, \frac{10}{1}\right\}$$ *tie!*

1	2	3	4	5	6	7	8	B
1	0	0	-1.5	-0.5	0	0	1.5	-5
0	1	0	3	-3	0	0	-1	-10
0	0	0	1.5	0.5	0	1	-0.5	5
0	0	1	0.5	2.5	0	0	0.5	10
0	0	0	1.5	-3.5	1	0	-0.5	0
*	*	*			*	*	↵	

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-1.5	-0.5	0	0	1.5	-5
	0	1	0	3	-3	0	0	-1	-10
	0	0	0	1.5	0.5	0	1	-0.5	5
	0	0	1	0.5	2.5	0	0	0.5	10
	0	0	0	(1.5)	-3.5	1	0	-0.5	0
	*	*	*			*	*		

⇓

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	-4	1	0	1	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	4	-1	1	0	5
	0	0	1	0	3.667	-0.3333	0	0.6667	10
	0	0	0	1	-2.333	0.6667	0	-0.3333	0
	*	*	*	*			*		

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	-4	1	0	1	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	(4)	-1	1	0	5
	0	0	1	0	3.667	-0.3333	0	0.6667	10
	0	0	0	1	-2.333	0.6667	0	-0.3333	0
	*	*	*	*			*		

⇓

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	*	*	*	*	*				

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	⁻1	⁻1	0	⁻15
	0	0	0	0	1	⁻0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	⁻0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	⁻0.3333	2.917
	*	*	*	*	*				

This tableau is optimal for the Phase-One objective, and provides us with a basic feasible solution with which to begin Phase Two.

1	2	3	4	5	6	7	8	B
1	0	0	0	0	0	1	1	0
0	1	0	0	0	⁻1	⁻1	0	⁻15
0	0	0	0	1	⁻0.25	0.25	0	1.25
0	0	1	0	0	0.5833	⁻0.9167	0.6667	5.417
0	0	0	1	0	0.08333	0.5833	⁻0.3333	2.917
*	*	*	*	*				

We can now delete the artificial variables (and w) and can delete the Phase-One objective row.

MAX	1	0	0	0	⁻1	⁻15
	0	0	0	1	⁻0.25	1.25
	0	1	0	0	0.5833	5.417
	0	0	1	0	0.08333	2.917
	*	*	*	*		

Since the Phase Two objective is to be minimized, this tableau is optimal!

	−z	X_1	X_2	X_3	X_4	
MAX	1	0	0	0	−1	−15
	0	0	0	1	−0.25	1.25
	0	1	0	0	0.5833	5.417
	0	0	1	0	0.08333	2.917
	*	*	*	*		

$$-z = -15, \text{ i.e., } z = 15$$

Optimal solution

$$X_3 = 1.25$$
$$X_1 = 5.417$$
$$X_2 = 2.917$$
$$X_4 = 0$$

	1	2	3	4	5	6	7	8	B
MIN	1	0	−3	−3	−8	0	0	0	−35
	0	1	2	4	2	0	0	0	10
	0	0	1	2	3	0	1	0	15
	0	0	2	1	5	0	0	1	20
	0	0	(1)	2	−1	1	0	0	10
	*	*				*	*	*	

Here, the pivot is in the bottom row.

The pivot produces a zero on the right-hand-side *(degeneracy)*

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	1	2	3	4	5	6	7	8	B
MIN	1	0	0	3	−11	3	0	0	−5
	0	1	0	0	4	−2	0	0	−10
	0	0	0	0	4	−1	1	0	5
	0	0	0	−3	7	−2	0	1	0
	0	0	1	2	−1	1	0	0	10
	*	*	*				*	*	⏎

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	3	-11	3	0	0	-5
	0	1	0	0	4	-2	0	0	-10
	0	0	0	0	4	-1	1	0	5
	0	0	0	-3	(7)	-2	0	1	0
	0	0	1	2	-1	1	0	0	10
	*	*	*				*	*	

Column 5 is selected for the pivot.

Minimum Ratio Test:

$$\min\left\{\frac{5}{4}, \frac{0}{7}, --\right\} = 0$$

1	2	3	4	5	6	7	8	B
1	0	0	-1.714	0	-0.1429	0	1.571	-5
0	1	0	1.714	0	-0.8571	0	-0.5714	-10
0	0	0	1.714	0	0.1429	1	-0.5714	5
0	0	0	-0.4286	1	-0.2857	0	0.1429	0
0	0	1	1.571	0	0.7143	0	0.1429	10
*	*	*		*		*		

Pivot in row 4

No improvement in the objective!

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	-1.714	0	-0.1429	0	1.571	-5
	0	1	0	1.714	0	-0.8571	0	-0.5714	-10
	0	0	0	(1.714)	0	0.1429	1	-0.5714	5
	0	0	0	-0.4286	1	-0.2857	0	0.1429	0
	0	0	1	1.571	0	0.7143	0	0.1429	10
	*	*	*		*		*		

$$\min\left\{\frac{5}{1.714}, --, \frac{10}{1.571}\right\}$$

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	-1	-1	0	-15
	0	0	0	1	0	0.08333	0.5833	-0.3333	2.917
	0	0	0	0	1	-0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	-0.9167	0.6667	5.417

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	0	0	0	1	1	0
	0	1	0	0	0	¯1	¯1	0	¯15
	0	0	0	1	0	0.08333	0.5833	¯0.3333	2.917
	0	0	0	0	1	¯0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	¯0.9167	0.6667	5.417

This tableau is optimal for Phase One.

	1	2	3	4	5	6	7	8	B
	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~	~~0~~
	0	1	0	0	0	¯1	¯1	0	¯15
	0	0	0	0	1	¯0.25	0.25	0	1.25
	0	0	1	0	0	0.5833	¯0.9167	0.6667	5.417
	0	0	0	1	0	0.08333	0.5833	¯0.3333	2.917
	*	*	*	*	*				

We can now delete the artificial variables (and w) and can delete the Phase-One objective row.

						B
MAX	1	0	0	0	¯1	¯15
	0	0	0	1	¯0.25	1.25
	0	1	0	0	0.5833	5.417
	0	0	1	0	0.08333	2.917
	*	*	*	*		

Since the Phase Two objective is to be minimized, this tableau is optimal!

$$\begin{array}{ll} \textbf{Minimize} & z = 34x_1 + 5x_2 + 19x_3 + 9x_4 \\ \textbf{subject to} & 2x_1 + x_2 + x_3 + x_4 \geq 9 \\ & 4x_1 - 2x_2 + 5x_3 + x_4 \leq 8 \\ & 4x_1 - x_2 + 3x_3 + x_4 \geq 5 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

Convert the inequalities to equations by adding slack & subtracting surplus variables

$$\begin{array}{ll} \textbf{Minimize} & z = 34x_1 + 5x_2 + 19x_3 + 9x_4 \\ \textbf{subject to} & 2x_1 + x_2 + x_3 + x_4 - x_5 = 9 \\ & 4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8 \\ & 4x_1 - x_2 + 3x_3 + x_4 - x_7 = 5 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \end{array}$$

$$\begin{array}{ll} \textbf{Minimize} & z = 34x_1 + 5x_2 + 19x_3 + 9x_4 \\ \textbf{subject to} & 2x_1 + x_2 + x_3 + x_4 - x_5 = 9 \\ & 4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8 \\ & 4x_1 - x_2 + 3x_3 + x_4 - x_7 = 5 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \end{array}$$

The first and last constraint require artificial variables:

$$\begin{array}{ll} \textbf{Minimize} & w = x_8 + x_9 \\ & -z + 34x_1 + 5x_2 + 19x_3 + 9x_4 = 0 \\ \textbf{subject to} & 2x_1 + x_2 + x_3 + x_4 - x_5 + x_8 = 9 \\ & 4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8 \\ & 4x_1 - x_2 + 3x_3 + x_4 - x_7 + x_9 = 5 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0 \end{array}$$

$$\text{Minimize} \quad w = x_8 + x_9$$

$$
\begin{array}{ll}
 & -z + 34x_1 + 5x_2 + 19x_3 + 9x_4 = 0 \\
\text{subject to} & 2x_1 + x_2 + x_3 + x_4 - x_5 + x_8 = 9 \\
 & 4x_1 - 2x_2 + 5x_3 + x_4 + x_6 = 8 \\
 & 4x_1 - x_2 + 3x_3 + x_4 - x_7 + x_9 = 5 \\
 & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0
\end{array}
$$

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	0	0	0	0	0	0	1	1	0
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	⁻1	0	0	1	0	9
	0	0	4	⁻2	5	1	0	1	0	0	0	8
	0	0	4	⁻1	3	1	0	0	⁻1	0	1	5

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	0	0	0	0	0	0	1	1	0
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	⁻1	0	0	(1)	0	9
	0	0	4	⁻2	5	1	0	1	0	0	0	8
	0	0	4	⁻1	3	1	0	0	⁻1	0	(1)	5
	*	*						*		*	*	

⇓

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	⁻6	0	⁻4	⁻2	1	0	1	0	0	⁻14
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	⁻1	0	0	1	0	9
	0	0	4	⁻2	5	1	0	1	0	0	0	8
	0	0	4	⁻1	3	1	0	0	⁻1	0	1	5
	*	*						*		*	*	

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	¯6	0	¯4	¯2	1	0	1	0	0	¯14
	0	1	34	5	19	9	0	0	0	0	0	0
	0	0	2	1	1	1	¯1	0	0	1	0	9
	0	0	4	¯2	5	1	0	1	0	0	0	8
	0	0	(4)	¯1	3	1	0	0	¯1	0	1	5
	*	*						*		*	*	

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	¯1.5	0.5	¯0.5	1	0	¯0.5	0	1.5	¯6.5
	0	1	0	13.5	¯6.5	0.5	0	0	8.5	0	¯8.5	¯42.5
	0	0	0	1.5	¯0.5	0.5	¯1	0	0.5	1	¯0.5	6.5
	0	0	0	¯1	2	0	0	1	1	0	¯1	3
	0	0	1	¯0.25	0.75	0.25	0	0	¯0.25	0	0.25	1.25
	*	*	*					*		*		

	1	2	3	4	5	6	7	8	9	10	11	B
MIN	1	0	0	¯1.5	0.5	¯0.5	1	0	¯0.5	0	1.5	¯6.5
	0	1	0	13.5	¯6.5	0.5	0	0	8.5	0	¯8.5	¯42.5
	0	0	0	(1.5)	¯0.5	0.5	¯1	0	0.5	1	¯0.5	6.5
	0	0	0	¯1	2	0	0	1	1	0	¯1	3
	0	0	1	¯0.25	0.75	0.25	0	0	¯0.25	0	0.25	1.25
	*	*	*					*		*		

1	2	3	4	5	6	7	8	9	10	11	B
1	0	0	0	0	0	0	0	0	1	1	0
0	1	0	0	¯2	¯4	9	0	4	¯9	¯4	¯101
0	0	0	1	¯0.333	0.333	¯0.666	0	0.333	0.6667	¯0.333	4.333
0	0	0	0	1.667	0.333	¯0.666	1	1.333	0.6667	¯1.333	7.333
0	0	1	0	0.666	0.333	¯0.166	0	¯0.166	0.1667	0.166	2.333
*	*	*	*				*				

1	2	3	4	5	6	7	8	9	10	11	B
1	0	0	0	0	0	0	0	0	1	1	0
0	1	0	0	¯2	¯4	9	0	4	¯9	¯4	¯101
0	0	0	1	¯0.333	0.333	¯0.666	0	0.333	0.6667	¯0.333	4.333
0	0	0	0	1.667	0.333	¯0.666	1	1.333	0.6667	¯1.333	7.333
0	0	1	0	0.666	0.333	¯0.166	0	¯0.166	0.1667	0.166	2.333
*	*	*	*				*				

This tableau is optimal for Phase One.

We may now delete the artificial variables and the Phase One objective row to obtain a basic feasible solution with which to begin Phase Two.

1	2	3	4	5	6	7	8	9	10	11	B
~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~	~~0~~
0	1	0	0	¯2	¯4	9	0	4	¯9	¯4	¯101
0	0	0	1	¯0.333	0.333	¯0.666	0	0.333	0.6667	¯0.333	4.333
0	0	0	0	1.667	0.333	¯0.666	1	1.333	0.6667	¯1.333	7.333
0	0	1	0	0.666	0.333	¯0.166	0	¯0.166	0.1667	0.166	2.333
*	*	*	*				*				

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	¯2	¯4	9	0	4	¯101
	0	0	1	¯0.333	0.333	¯0.666	0	0.333	4.333
	0	0	0	1.667	0.333	¯0.666	1	1.333	7.333
	0	1	0	0.666	0.333	¯0.166	0	¯0.166	2.333
	*	*	*				*		

	1	2	3	4	5	6	7	8	B
MIN	1	0	0	¯2	¯4	9	0	4	¯101
	0	0	1	¯0.333	0.333	¯0.666	0	0.333	4.333
	0	0	0	1.667	0.333	¯0.666	1	1.333	7.333
	0	1	0	0.666	0.333	¯0.166	0	¯0.166	2.333
	*	*	*				*		

Tableau is now optimal for Phase Two!

	1	2	3	4	5	6	7	8	B
MIN	1	12	0	6	0	7	0	2	¯73
	0	¯1	1	¯1	0	¯0.5	0	0.5	2
	0	¯1	0	1	0	¯0.5	1	1.5	5
	0	3	0	2	1	¯0.5	0	¯0.5	7
	*		*		*		*		

$-z = -73$, i.e., $z = 73$

$X_2 = 2$

$X_6 = 5$

$X_4 = 7$