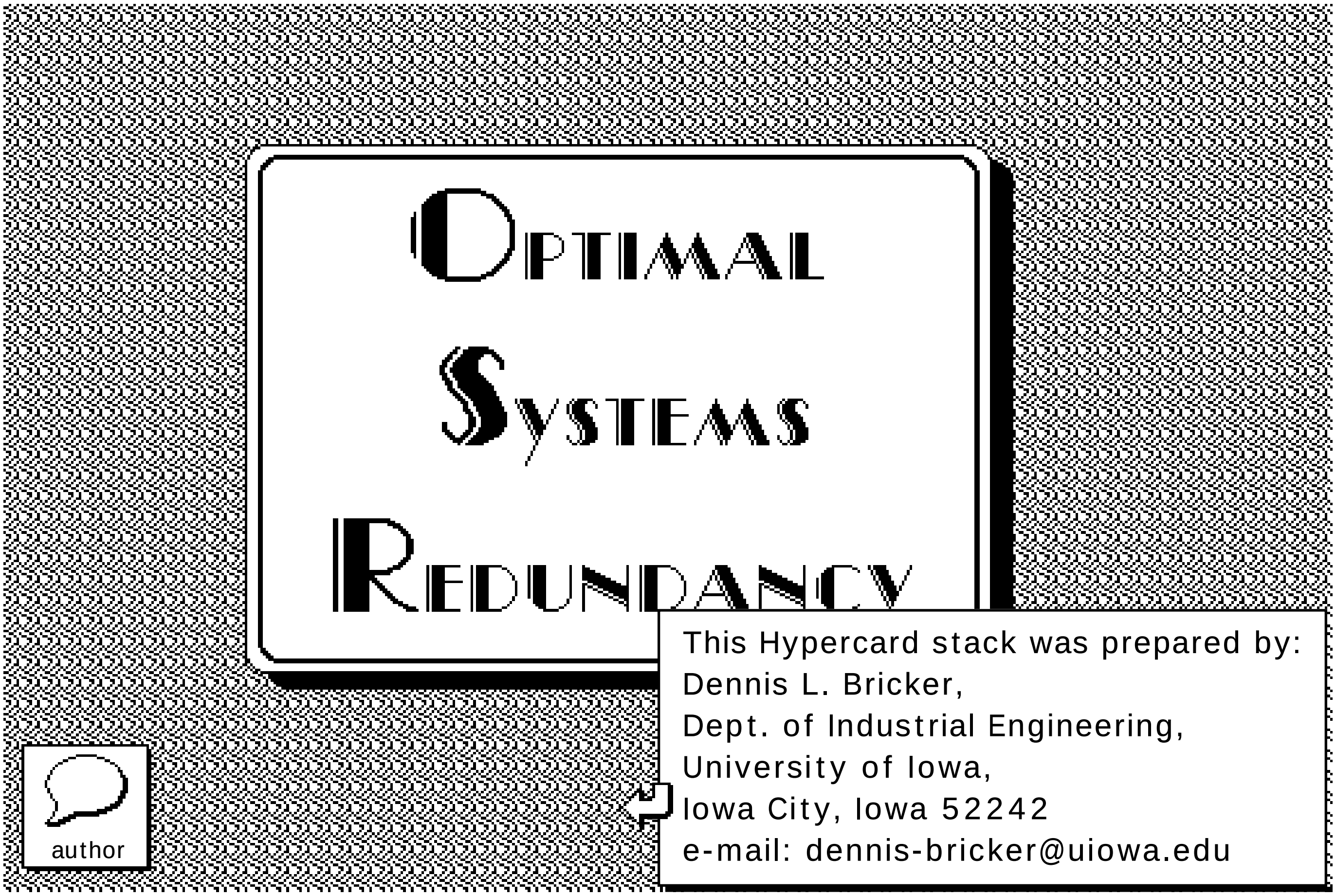
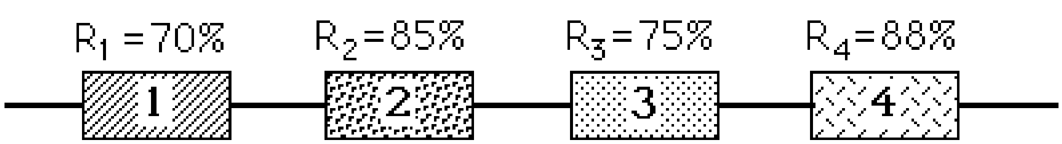


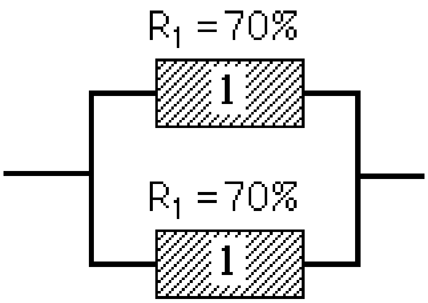
One of the systems of a communications satellite consists of 4 electronic devices. The system will fail whenever one or more of these devices fails.



Reliability of system = P{all devices are functional}

$$= R_1 \times R_2 \times R_3 \times R_4$$

$$= 39.27\%$$



The reliability of a device can be increased by the use of "backup" units of the same type, in parallel.

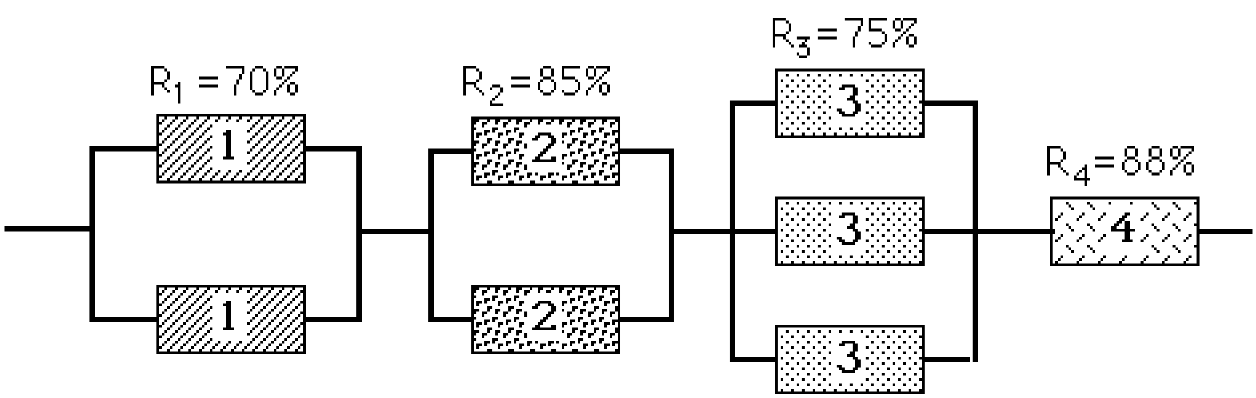
The backup devices are controlled by a monitoring system, so that, if a device fails, a backup unit will immediately be switched into use.

Reliability of Device #1 subsystem with two units

= 1 – P{both units fail}

$= 1 - (0.30)^2$

= 91%



By using backup units of each device, the system reliability can be dramatically increased.
In the system above,

$$\text{System Reliability} = \left[1 - 0.3^2\right] \times \left[1 - 0.15^2\right] \times \left[1 - 0.25^3\right] \times 0.88$$

$$= (0.91)(0.9775)(0.984375)(0.88)$$

$$= 77.0551\%$$

The problem faced by the designer is to maximize the system reliability, subject to a limit on the total weight of the system:

Total weight must not exceed 12 kg.

device i	1	2	3	4
Wt[i]	1	2	1	3

(One unit of each device totals 7 kg., leaving 5 kg. available for the backup units.)

Data

device i	1	2	3	4
Wt[i]	1	2	1	3

Reliability (%) vs # redundant units

i	1	2	3
1	70	91	97.3
2	85	97.75	99.6625
3	75	93.75	98.4375
4	88	98.56	99.8272

DP Model

Stage: n = device type

State: S_n = number of kg. available

Decision: X_n = number of units of device n to be included in system

We impose a sequential decision-making structure on the problem, by supposing that we consider the devices individually in sequence, deciding how many units to include.

Optimal Value Function

$f_n(S_n)$ = maximum reliability of the subsystem consisting of devices n, n-1, ... 1, if S_n kg. of available capacity may be allocated.

Recursive Definition

$$f_n(S_n) = \underset{X_n \le S_n/W_n}{\text{maximum}} \left\{ \left(1 - p_n^{X_n}\right) \times f_{n-1}(S_n - W_n X_n) \right\}$$

for n=4, 3, 2, 1

$$f_0(S_0) = 1$$

Stage 1

Device #1:
Weight = 1
Reliability = 70%

s \ x	1	2	3
1	0.70	-99.99	-99.99
2	0.70	0.91	-99.99
3	0.70	0.91	0.97
4	0.70	0.91	0.97
5	0.70	0.91	0.97
6	0.70	0.91	0.97
7	0.70	0.91	0.97
8	0.70	0.91	0.97
9	0.70	0.91	0.97
10	0.70	0.91	0.97
11	0.70	0.91	0.97
12	0.70	0.91	0.97

Stage 1

s \ x	1	2	3
1	0.70	-99.99	-99.99
2	0.70	0.91	-99.99
3	0.70	0.91	0.97
4	0.70	0.91	0.97
5	0.70	0.91	0.97
6	0.70	0.91	0.97
7	0.70	0.91	0.97
8	0.70	0.91	0.97
9	0.70	0.91	0.97
10	0.70	0.91	0.97
11	0.70	0.91	0.97
12	0.70	0.91	0.97

$f_1(S_1)$

State	Optimal Values	Optimal Decisions	Resulting State
1	0.70	1	0
2	0.91	2	0
3	0.97	3	0
4	0.97	3	1
5	0.97	3	2
6	0.97	3	3
7	0.97	3	4
8	0.97	3	5
9	0.97	3	6
10	0.97	3	7
11	0.97	3	8
12	0.97	3	9

Stage 2

s \ x	1	2	3
3	0.60	-99.99	-99.99
4	0.77	-99.99	-99.99
5	0.83	0.68	-99.99
6	0.83	0.89	-99.99
7	0.83	0.95	0.70
8	0.83	0.95	0.91
9	0.83	0.95	0.97
10	0.83	0.95	0.97
11	0.83	0.95	0.97
12	0.83	0.95	0.97

Device #2:
Weight = 2 kg.
Reliability = 85%

S_1	$f_1(S_1)$
1	0.70
2	0.91
3	0.97
4	0.97
5	0.97
6	0.97
7	0.97
8	0.97
9	0.97
10	0.97
11	0.97
12	0.97

Stage 2

s \ x	1	2	3
3	0.60	-99.99	-99.99
4	0.77	-99.99	-99.99
5	0.83	0.68	-99.99
6	0.83	0.89	-99.99
7	0.83	0.95	0.70
8	0.83	0.95	0.91
9	0.83	0.95	0.97
10	0.83	0.95	0.97
11	0.83	0.95	0.97
12	0.83	0.95	0.97

$f_2(S_2)$ → Optimal Values

State	Optimal Values	Optimal Decisions	Resulting State
3	0.60	1	1
4	0.77	1	2
5	0.83	1	3
6	0.89	2	2
7	0.95	2	3
8	0.95	2	4
9	0.97	3	3
10	0.97	3	4
11	0.97	3	5
12	0.97	3	6

Stage 3

Device #3:
Weight = 1 kg.
Reliability = 75%

s \ x	1	2	3
4	0.45	-99.99	-99.99
5	0.58	0.56	-99.99
6	0.62	0.73	0.59
7	0.67	0.78	0.76
8	0.71	0.83	0.81
9	0.71	0.89	0.88
10	0.73	0.89	0.94
11	0.73	0.91	0.94
12	0.73	0.91	0.95

S_2	$f_2(S_2)$
3	0.60
4	0.77
5	0.83
6	0.89
7	0.95
8	0.95
9	0.97
10	0.97
11	0.97
12	0.97

Stage 3

s \ x	1	2	3
4	0.45	-99.99	-99.99
5	0.58	0.56	-99.99
6	0.62	0.73	0.59
7	0.67	0.78	0.76
8	0.71	0.83	0.81
9	0.71	0.89	0.88
10	0.73	0.89	0.94
11	0.73	0.91	0.94
12	0.73	0.91	0.95

$f_3(S_3)$ (→ Optimal Values)

State	Optimal Values	Optimal Decisions	Resulting State
4	0.45	1	3
5	0.58	1	4
6	0.73	2	4
7	0.78	2	5
8	0.83	2	6
9	0.89	2	7
10	0.94	3	7
11	0.94	3	8
12	0.95	3	9

Stage 4

Device #4:
Weight = 3 kg.
Reliability = 88%

s \ x	1	2	3
7	0.39	¯99.99	¯99.99
8	0.51	¯99.99	¯99.99
9	0.64	¯99.99	¯99.99
10	0.68	0.44	¯99.99
11	0.73	0.57	¯99.99
12	0.78	0.71	¯99.99

S_3	$f_3(S_3)$
4	0.45
5	0.58
6	0.73
7	0.78
8	0.83
9	0.89
10	0.94
11	0.94
12	0.95

Stage 4

s \ x	1	2	3
7	0.39	¯99.99	¯99.99
8	0.51	¯99.99	¯99.99
9	0.64	¯99.99	¯99.99
10	0.68	0.44	¯99.99
11	0.73	0.57	¯99
12	0.78	0.71	¯99

State	Optimal Values	Optimal Decisions	Resulting State
7	0.39	1	4
8	0.51	1	5
9	0.64	1	6
10	0.68	1	7
11	0.73	1	8
12	0.78	1	9

The maximum reliability is 78.4664%

Weight	Reliability
7	39.27
8	51.051
9	63.8137
10	68.2316
11	73.3858
12	78.4664

Stage 4

State	Optimal Values	Optimal Decisions	Resulting State
7	0.39	1	4
8	0.51	1	5
9	0.64	1	6
10	0.68	1	7
11	0.73	1	8
12	0.78	1	9

Stage 3

State	Optimal Values	Optimal Decisions	Resulting State
4	0.45	1	3
5	0.58	1	4
6	0.73	2	4
7	0.78	2	5
8	0.83	2	6
9	0.89	2	7
10	0.94	3	7
11	0.94	3	8
12	0.95	3	9

Stage 2

State	Optimal Values	Optimal Decisions	Resulting State
3	0.60	1	1
4	0.77	1	2
5	0.83	1	3
6	0.89	2	2
7	0.95	2	3
8	0.95	2	4
9	0.97	3	3
10	0.97	3	4
11	0.97	3	5
12	0.97	3	6

Stage 1

State	Optimal Values	Optimal Decisions	Resulting State
1	0.70	1	0
2	0.91	2	0
3	0.97	3	0
4	0.97	3	1
5	0.97	3	2
6	0.97	3	3
7	0.97	3	4
8	0.97	3	5
9	0.97	3	6
10	0.97	3	7
11	0.97	3	8
12	0.97	3	9

Optimal Redundancy Report

Weight available: 12

Component number	# used	Reliability each	Net Reliability
1	3	70.00%	97.30%
2	2	85.00%	97.75%
3	2	75.00%	93.75%
4	1	88.00%	88.00%

What if only 11 kg. were available?

Optimal Redundancy Report

Weight available: 11

Component number	# used	Reliability each	Net Reliability
1	2	70.00%	91.00%
2	2	85.00%	97.75%
3	2	75.00%	93.75%
4	1	88.00%	88.00%