

**Power Plant
Capacity Planning**



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A power company is doing long-range planning, and has projected the following needs for additional power plants:

year	1	2	3	4	5	6
#plants req'd	1	2	4	6	7	8
cost/plant (\$million)	5.4	5.6	5.8	5.7	5.5	5.2

cumulative!

There is a fixed cost of \$1.5 million if any plants are added in a year
 At most 3 plants can be added in one year

When should each of the 8 needed plants be added?

(\$ might be saved by adding some plants in advance of the year in which they're needed, thereby avoiding the \$1.5million fixed cost each year.)

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DP Model

Stage: n = # of years remaining in the planning period

State: S_n = total # of plants which have been added

Decision: X_n = # of plants to be added in the current year

(Note that the stages are being numbered in reverse chronological order! That is, stage 6 is the first year in the planning period, etc.)

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Immediate cost of stage n

$$g_n(S_n, X_n) = \begin{cases} +\infty & \text{if } S_n < R_{n+1} \\ 0 & \text{if } X_n = 0 \quad \& S_n \geq R_n \\ 1.5 + C_n X_n & \text{if } X_n = 1, 2, \text{ or } 3 \quad \& S_n \geq R_n \end{cases}$$

assures that sufficient plants have been added in previous year.

fixed cost *cost per plant*

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Transition Function

If the decision X_n is made, when the current state is S_n , the resulting state will be

cumulative number of plants at beginning of next year →

$T(S_n, X_n) = S_n + X_n$

current number of plants *number of plants added*

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Objective



Minimizing Total Cost



Minimizing *Present Value* of Total Cost

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Definition of Optimal Value

$f_n(S_n)$ = minimum total cost of the current & remaining stages ($n, n-1, \dots, 0$) if stage n is entered in state S_n

$$f_n(S_n) = \begin{cases} \text{minimum}_{(R_n - S_n)^+ \leq X_n \leq 3} \{ g_n(S_n, X_n) + f_{n-1}(S_n + X_n) \} & \text{if } n \geq 1 \\ 0 & \text{if } n=0 \text{ \& } S_0 \geq R_1 \\ +\infty & \text{if } n=0 \text{ \& } S_0 < R_1 \end{cases}$$

$$z^+ \equiv \max\{0, z\}$$

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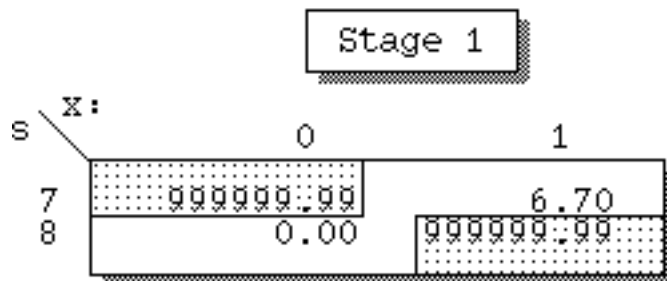
APL code

Power Plant Capacity Planning

```

Z←F N;t;S;G
R
R      Optimal Value Function
R      Power Plant Capacity Planning problem
R
→End IF N=0
R      Cost of adding plants in current year
G←((ρS)ρ0) °.+ (x>0)×1.5+COST[N]×x
R      Cumulative # of plants added at beginning of next yr.
S← s°.+x
R      Recursive definition of optimal value
Z← MIN G + (F N-1)[TRANSITION S+BIG×(S<RQMT[N])]
→0
End:z←((ρS)ρ0),BIG
    
```

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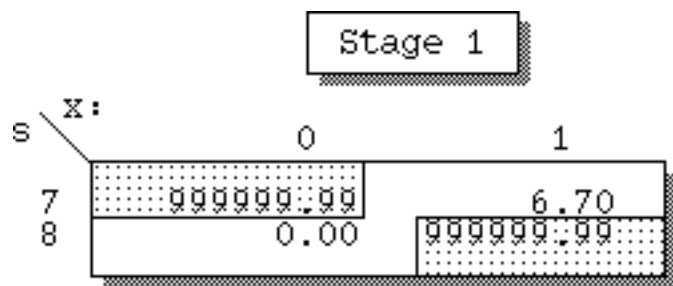
Cost per plant = $C_1 = 5.2$

Req'd # of plants = $R_1 = 8$

$R_2 = 7$

999999.99 represents infinity
(infeasible decision).

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State	Optimal Values	Optimal Decisions	Resulting State
7	6.70	1	8
8	0.00	0	8

$f_1(S_1)$ X_1^* S_0^*

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Stage 2

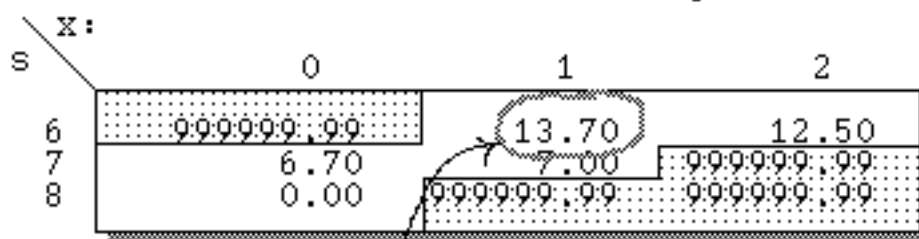
from computations just completed:

S_1	$f_1(S_1)$
7	6.70
8	0.00

$$f_2(S_2) = \text{minimum}_{(R_2 - S_2) \leq X_2 \leq 3} g_2(S_2, X_2) + f_1(S_2 + X_2)$$

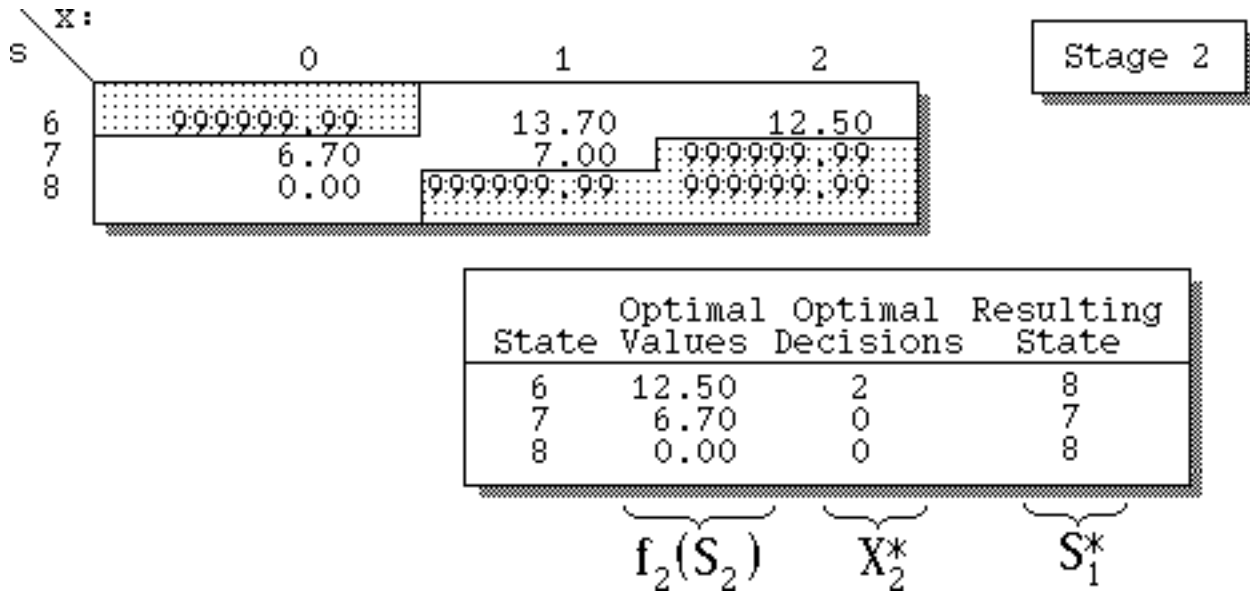
Cost per plant = $C_2 = 5.5$

Req'd number of plants = $R_2 = 7$

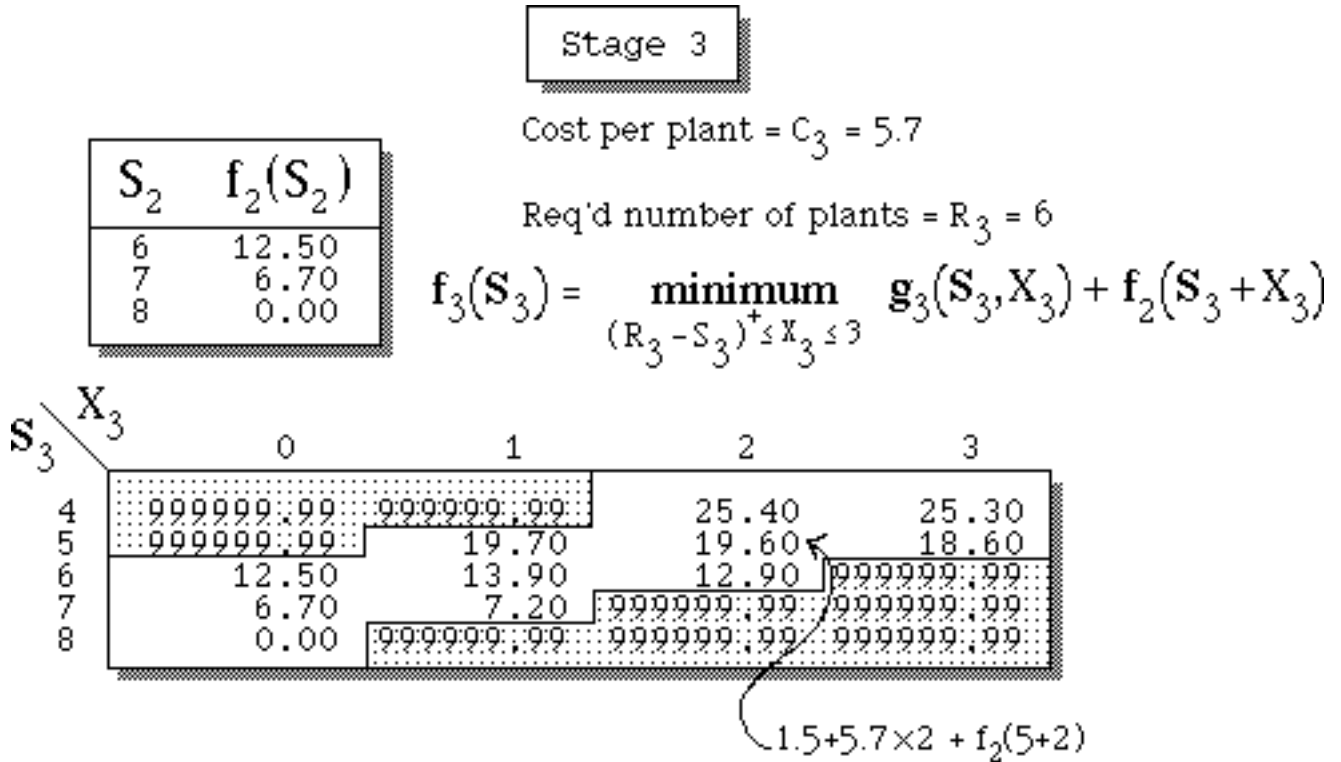


$1.5 + 5.5 \times 1 + f_1(6+1)$

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S \ X:		0				1				2				3			
		9999999.99				9999999.99				25.40				25.30			
4		9999999.99				19.70				19.60				18.60			
5		12.50				13.90				12.90				9999999.99			
6		6.70				7.20				9999999.99				9999999.99			
7		0.00				9999999.99				9999999.99				9999999.99			
8		9999999.99				9999999.99				9999999.99				9999999.99			

Stage 3

	Optimal State Values	Optimal Decisions	Resulting State
4	25.30	3	7
5	18.60	3	8
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

$f_3(S_3)$ X_3^* S_2^*

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S_3	$f_3(S_3)$
4	25.30
5	18.60
6	12.50
7	6.70
8	0.00

Stage 4

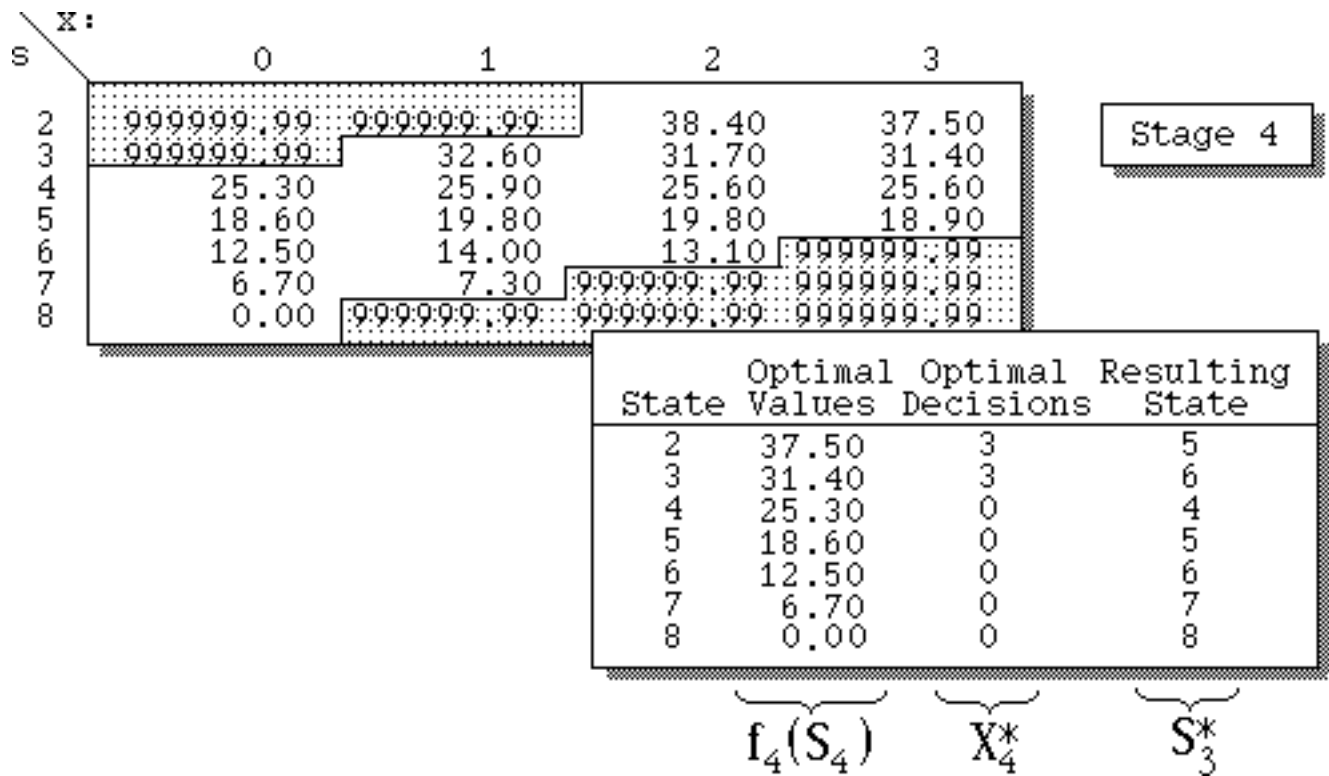
Cost per plant = $C_4 = 5.8$

Req'd number of plants = $R_4 = 4$

$$f_4(S_4) = \text{minimum}_{(R_4 - S_4) \leq X_4 \leq 3} g_4(S_4, X_4) + f_3(S_4 + X_4)$$

S \ X:		0				1				2				3			
		9999999.99				9999999.99				38.40				37.50			
2		9999999.99				32.60				31.70				31.40			
3		25.30				25.90				25.60				25.60			
4		18.60				19.80				19.80				18.90			
5		12.50				14.00				13.10				9999999.99			
6		6.70				7.30				9999999.99				9999999.99			
7		0.00				9999999.99				9999999.99				9999999.99			
8		9999999.99				9999999.99				9999999.99				9999999.99			

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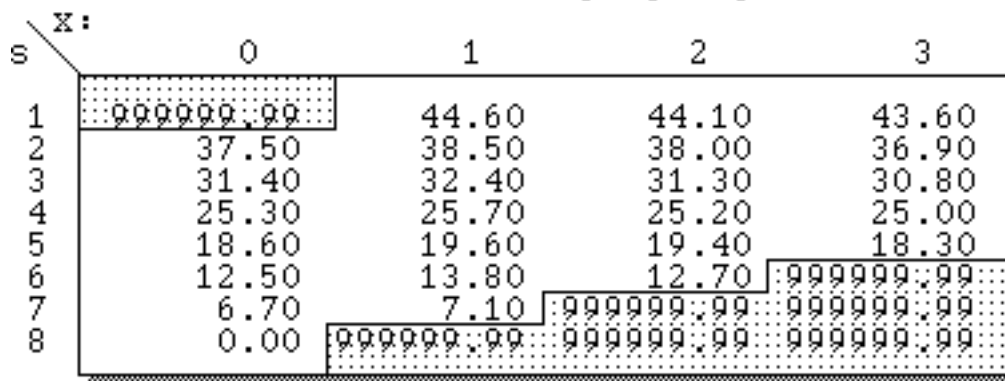
S_4	$f_4(S_4)$
2	37.50
3	31.40
4	25.30
5	18.60
6	12.50
7	6.70
8	0.00

Stage 5

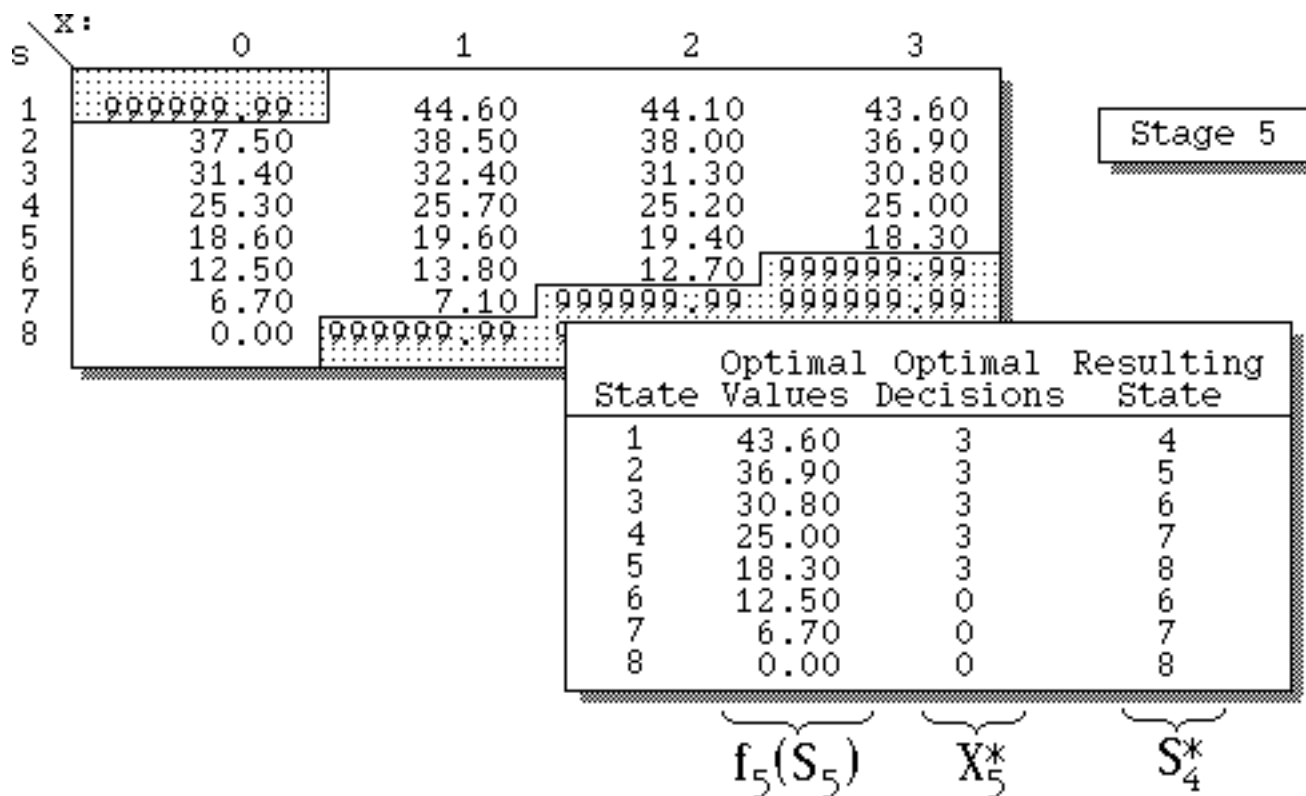
Cost per plant = $C_5 = 5.6$

Req'd number of plants = $R_5 = 2$

$$f_5(S_5) = \text{minimum}_{(R_5 - S_5) \leq X_5 \leq 3} g_5(S_5, X_5) + f_4(S_5 + X_5)$$



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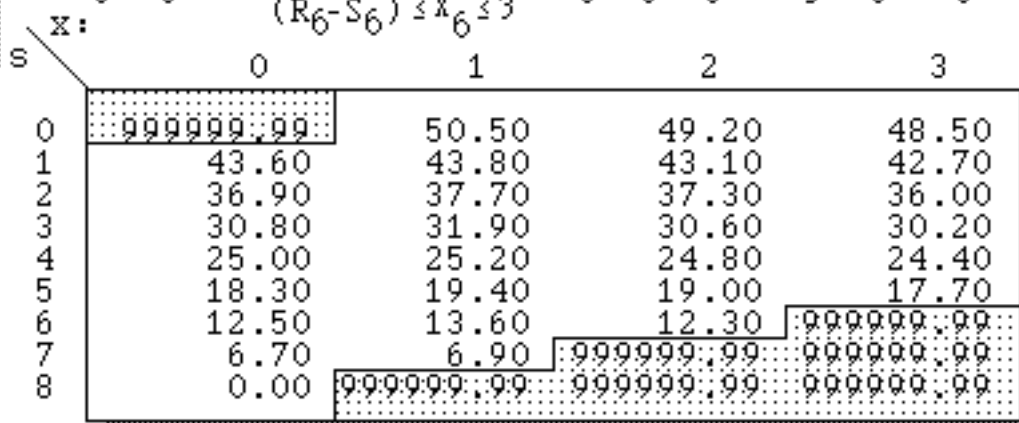
S_5	$f_5(S_5)$
1	43.60
2	36.90
3	30.80
4	25.00
5	18.30
6	12.50
7	6.70
8	0.00

Stage 6

Cost per plant = $C_6 = 5.4$

Req'd number of plants = $R_6 = 1$

$$f_6(S_6) = \text{minimum}_{(R_6 - S_6)^+ \leq X_6 \leq 3} g_6(S_6, X_6) + f_5(S_6 + X_6)$$



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S \ X:	0	1	2	3	
0	999999.99	50.50	49.20	48.50	Stage 6
1	43.60	43.80	43.10	42.70	
2	36.90	37.70	37.30	36.00	
3	30.80	31.90	30.60	30.20	
4	25.00	25.20	24.80	24.40	
5	18.30	19.40	19.00	17.70	
6	12.50	13.60	12.30	999999.99	
7	6.70	6.90	999999.99	999999.99	
8	0.00	999999.99	999999.99	999999.99	

	Optimal State	Optimal Values	Optimal Decisions	Resulting State
0	48.50	3	3	
1	42.70	3	4	
2	36.00	3	5	
3	30.20	3	6	
4	24.40	3	7	
5	17.70	3	8	
6	12.30	2	8	
7	6.70	0	7	
8	0.00	0	8	

$f_6(S_6)$ X_6^* S_5^*

cont

Optimal Returns & Decisions

Stage 6:

	Optimal State	Optimal Values	Optimal Decisions	Resulting State
0	48.50	3	3	
1	42.70	3	4	
2	36.00	3	5	
3	30.20	3	6	
4	24.40	3	7	
5	17.70	3	8	
6	12.30	2	8	
7	6.70	0	7	
8	0.00	0	8	

Stage 5:

State	Optimal Values	Optimal Decisions	Resulting State
1	43.60	3	4
2	36.90	3	5
3	30.80	3	6
4	25.00	3	7
5	18.30	3	8
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
2	37.50	3	5
3	31.40	3	6
4	25.30	0	4
5	18.60	0	5
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

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Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
4	25.30	3	7
5	18.60	3	8
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
6	12.50	2	8
7	6.70	0	7
8	0.00	0	8

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
7	6.70	1	8
8	0.00	0	8

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Optimal Solution

The initial state (entering stage 6) is 0

Power Plant Capacity Planning

*** Optimal value is 48.5 ***

STAGE	STATE	DECISION
6	0	3
5	3	3
4	6	0
3	6	0
2	6	2
1	8	0
0	8	



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Discounting Future Costs

Let r = rate of interest

Then

\$1 invested now has the same value as
 $\$1(1+r)^n$ after n periods,

and

\$1 paid n periods hence is equivalent to

$$\$1 \frac{1}{(1+r)^n} = \$1 \beta^n \quad \text{where } \beta = \frac{1}{1+r} \quad \text{discount factor}$$

paid now.



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Minimizing total discounted future costs,
 i.e., present value of future costs,
 in a DP model:

In the power plant capacity planning model:

$$f_n(S_n) = \begin{cases} \text{minimum}_{(R_n - S_n)^+ \leq X_n \leq 3} \{ g_n(S_n, X_n) + \beta f_{n-1}(S_n + X_n) \} & \text{if } n \geq 1 \\ 0 & \text{if } n=0 \text{ \& } S_0 \geq R_1 \\ +\infty & \text{if } n=0 \text{ \& } S_0 < R_1 \end{cases}$$

discount factor

= minimum present value of costs of last n stages
 ("present" \longleftrightarrow stage n)

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Power Plant Capacity Planning

State Vector

i	1	2	3	4	5	6	7	8	9
s(i)	0	1	2	3	4	5	6	7	8

Decision Vector

i	1	2	3	4
x(i)	0	1	2	3

Suppose that $r=20\%$, so that $\beta = \frac{1}{1.2} = 0.83333$

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Power Plant Capacity Planning

```

Z←F N;t;S;G
R
R      Optimal Value Function
R      Power Plant Capacity Planning problem
R      (minimizing present value of future costs, with
R      discount factor beta)
R
→End IF N=0

R      Cost of adding plants in current year
G←((ρS)ρ0)°.(x>0)×1.5+COST[N]×x

R      Cumulative # of plants added at beginning of next yr
S←s°.+x
R      Recursive definition of optimal value
Z← MIN G + beta×(F N-1)[TRANSITION S+BIG×(S<RQMT[N])]
→0
End:Z←((ρS)ρ0),BIG
    
```

Note: recursion is backward

APL Code

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Power Plant Capacity Planning

Recursion type: backward

— Stage 1 —

s \ x:	0	1	2	3
5	9999.99	9999.99	9999.99	17.10
6	9999.99	9999.99	11.90	9999.99
7	9999.99	6.70	9999.99	9999.99
8	0.00	9999.99	9999.99	9999.99

State	Optimal Values	Optimal Decisions	Resulting State
5	17.10	3	8
6	11.90	2	8
7	6.70	1	8
8	0.00	0	8

$f_1(S_1)$

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s \ x:	0	1	2	3
4	9999.99	9999.99	9999.99	23.58
5	9999.99	9999.99	18.08	18.00
6	9999.99	12.58	12.50	9999.99
7	5.58	7.00	9999.99	9999.99
8	0.00	9999.99	9999.99	9999.99

S_1	$f_1(S_1)$
5	17.10
6	11.90
7	6.70
8	0.00

---Stage 2---

State	Optimal Values	Optimal Decisions	Resulting State
4	23.58	3	7
5	18.00	3	8
6	12.50	2	8
7	5.58	0	7
8	0.00	0	8

$f_2(S_2)$

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s \ x:	0	1	2	3
3	9999.99	9999.99	9999.99	29.02
4	9999.99	9999.99	23.32	23.25
5	9999.99	17.62	17.55	18.60
6	10.42	11.85	12.90	9999.99
7	4.65	7.20	9999.99	9999.99
8	0.00	9999.99	9999.99	9999.99

S_2	$f_2(S_2)$
4	23.58
5	18.00
6	12.50
7	5.58
8	0.00

---Stage 3---

State	Optimal Values	Optimal Decisions	Resulting State
3	29.02	3	6
4	23.25	3	7
5	17.55	2	7
6	10.42	0	6
7	4.65	0	7
8	0.00	0	8

$f_3(S_3)$

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s	x:	0	1	2	3
1		9999.99	9999.99	9999.99	38.28
2		9999.99	9999.99	32.48	33.53
3		9999.99	26.68	27.73	27.58
4		19.38	21.93	21.78	22.78
5		14.63	15.98	16.98	18.90
6		8.68	11.18	13.10	9999.99
7		3.88	7.30	9999.99	9999.99
8		0.00	9999.99	9999.99	9999.99

S_3	$f_3(S_3)$
3	29.02
4	23.25
5	17.55
6	10.42
7	4.65
8	0.00

---Stage 4---

State	Optimal Values	Optimal Decisions	Resulting State
1	38.28	3	4
2	32.48	2	4
3	26.68	1	4
4	19.38	0	4
5	14.63	0	5
6	8.68	0	6
7	3.88	0	7
8	0.00	0	8

$f_4(S_4)$

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s	x:	0	1	2	3
0		9999.99	9999.99	39.76	40.53
1		9999.99	34.16	34.93	34.45
2		27.06	29.33	28.85	30.49
3		22.23	23.25	24.89	25.53
4		16.15	19.29	19.93	21.53
5		12.19	14.33	15.93	18.30
6		7.23	10.33	12.70	9999.99
7		3.23	7.10	9999.99	9999.99
8		0.00	9999.99	99	99

S_4	$f_4(S_4)$
1	38.28
2	32.48
3	26.68
4	19.38
5	14.63
6	8.68
7	3.88
8	0.00

---Stage 5---

State	Optimal Values	Optimal Decisions	Resulting State
0	39.76	2	2
1	34.16	1	2
2	27.06	0	2
3	22.23	0	3
4	16.15	0	4
5	12.19	0	5
6	7.23	0	6
7	3.23	0	7
8	0.00	0	8

$f_5(S_5)$

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<u>S</u>	<u>X:</u>	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>S₅</u>	<u>f₅(S₅)</u>
0		999999.99	35.37	34.85	36.23	0	39.76
1		28.47	29.45	30.83	31.16	1	34.16
2		22.55	25.43	25.76	27.86	2	27.06
3		18.53	20.36	22.46	23.73	3	22.23
4		13.46	17.06	18.33	20.39	4	16.15
5		10.16	12.93	14.99	17.70	5	12.19
6		6.03	9.59	12.30	9999.99	6	7.23
7		2.69	6.90	9999.99	9999.99	7	3.23
8		0.00	9999.99	9999.99	9999.99	8	0.00

--Stage 6---

State	Optimal Values	Optimal Decisions	Resulting State
0	34.85	2	2
1	28.47	0	1
2	22.55	0	2
3	18.53	0	3
4	13.46	0	4
5	10.16	0	5
6	6.03	0	6
7	2.69	0	7
8	0.00	0	8

$f_6(S_6)$

Power Plant Capacity Planning

*** Optimal value is 34.85369084 ***

STAGE	STATE	DECISION
6	0	2
5	2	0
4	2	2
3	4	3
2	7	0
1	7	1
0	8	

