Reliability of Components

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Component Reliability is the probability that the component will ***not fail*** to perform

- within specified limits
- for a specified period of time
- in a specified environment

Suppose that

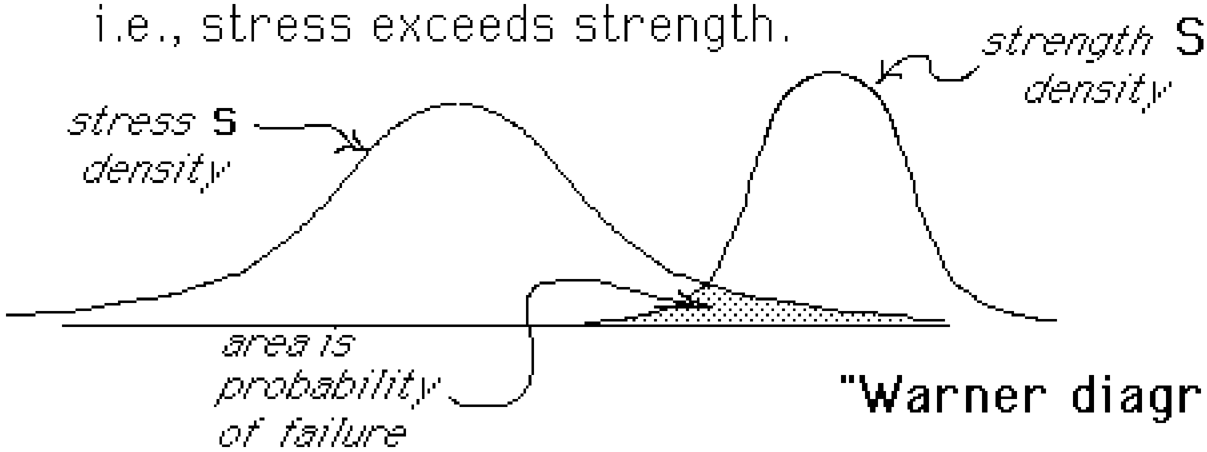
S = strength of component

s = stress on component

are both ***random variables***

The component fails if $s > S$,

i.e., stress exceeds strength.



"Warner diagram"

Component reliability when strength & stress each have Normal distribution.

S (strength) $\sim N(\overline{S}, \sigma_S)$

s (stress) $\sim N(\overline{s}, \sigma_s)$

Let $z = S - s$. Then z is $N(\overline{z}, \sigma_z)$,

where $\overline{z} = \overline{S} - \overline{s}$

$$\sigma_z = \sqrt{\sigma_S^2 + \sigma_s^2}$$

Then $P\{S > s\} = P\{z > 0\} = P\left\{\dfrac{z - \overline{z}}{\sigma_z} > \dfrac{0 - \overline{z}}{\sigma_z}\right\} = 1 - \Phi\left(\dfrac{-\overline{z}}{\sigma_z}\right)$

CDF of standard N(0,1) distribution

Example Suppose that strength (S) and stress (s) are both Normally-distributed random variables:

$S \sim N(27000, 4000)$

$s \sim N(13000, 3000)$

Then $z = S - s$ is Normally-distributed,

with mean $\bar{z} = \bar{S} - \bar{s} = 27000-13000=14000$

and std.deviation $\sigma_z = \sqrt{\sigma_S^2 + \sigma_s^2}$

$$\frac{\bar{z}}{\sigma_z} = 2.8$$

Probability of failure ($z \le 0$) $\Phi(-2.8) = 0.002505$

Monte Carlo technique to estimate reliability

When strength &/or stress are not Normally-distributed, analytical computation of reliability is generally impossible.

At each trial of the Monte Carlo technique, values of strength and stress are generated randomly with the specified distributions, and are compared to determine whether failure occurs.

Reliability is estimated by $1 - \dfrac{\#\text{failures}}{\#\text{trials}}$

Example Yield values for a certain grade of steel have a sample mean of 25000 psi and standard deviation of 8000 psi.

Maximum loads are found to have a mean of 10000 psi and standard deviation of 7500 psi

Assume that the yield strength has the ***Weibull*** distribution, and that the load has the ***Gumbel*** distribution.

Estimate the reliability of the steel, i.e., P{Yield value > maximum load}

Randomly Generating Random Variables

Both the Weibull and Gumbel distribution functions can be analytically inverted

Weibull $F_W(y) = X \Longrightarrow F_W^{-1}(X) = y$

Gumbel $F_G(s) = X \Longrightarrow F_G^{-1}(X) = s$

To generate a random variable by the Inverse Transformation technique, we generate a value for X uniformly distributed in [0, 1] and compute the CDF inverse $F^{-1}(X)$

Randomly Generating Random Variables

$$F_W(y) = 1 - e^{-(y/u)^k} = \hat{X} \leftarrow$$

$$e^{-(y/u)^k} = 1 - \hat{X} = X \leftarrow$$

uniformly distributed in interval [0,1]

$$-(y/u)^k = \ln X$$

$$\frac{y}{u} = (-\ln X)^{1/k}$$

Weibull

$$\Longrightarrow \boxed{F_W^{-1}(\hat{X}) = y = u(-\ln x)^{1/k}}$$

Randomly Generating Random Variables

$$F_G(s) = \exp\{-e^{-\alpha(s-u)}\} = X \leftarrow$$

uniformly distributed in interval [0,1]

Gumbel

$$-e^{-\alpha(s-u)} = \ln X$$

$$-\alpha(s-u) = \ln(-\ln X)$$

$$\Longrightarrow \boxed{F_G^{-1}(X) = s = u - \frac{\ln(-\ln X)}{\alpha}}$$

Computation of Weibull Parameters

Secant Method, solving equation:

$$0.32 = \frac{\sigma_Y}{\mu_Y} = \sqrt{\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1}$$

k	error
1.000000	0.680000
2.000000	0.202724
2.424751	0.119778
3.038111	0.039357
3.338278	0.010214
3.443479	0.001129
3.456552	0.000036
3.456984	0.000000

$$\Longrightarrow k = 3.456984$$ ← *shape parameter*

$$u = \frac{\mu_Y}{\Gamma\left(1+\frac{1}{k}\right)} \Longrightarrow u = 27803.718$$ ← *scale parameter*

Computation of Gumbel Parameters

$$\sigma_Y \approx \frac{1.282}{\alpha} \Rightarrow \alpha \approx \frac{1.282}{\sigma_Y} = \frac{1.282}{7500}$$

$$\boxed{\alpha = 1.709 \times 10^{-4}}$$

$$\mu_Y \approx u + \frac{0.577}{\alpha} \Rightarrow u \approx \mu_Y - \frac{0.577}{\alpha}$$

$$\Rightarrow u = 10000 - \frac{0.577}{1.709 \times 10^{-4}}$$

$$\boxed{u = 6624.415}$$

Generating Random Yield Strength

$$F_W^{-1}(\hat{X}) = y = u(-\ln x)^{1/k}$$

yield strength

$$y = 2.7803.718(-\ln x)^{0.289269}$$

Randomly generate X uniformly distributed in [0, 1]
Then compute y, which will have Weibull dist'n!

Generating Random Load

$$F_G^{-1}(X) = s = u - \frac{\ln(-\ln X)}{\alpha}$$

load (stress)

$$s = 66.24.415 - \frac{\ln(-\ln x)}{1.709\times 10^{-4}}$$

Randomly generate X uniformly distributed in [0, 1]
Then compute s, which will have Gumbel dist'n!

Result of 30 trials

i	Strength	Stress	i	Strength	Stress
1	28713.927	8054.9772	16	21474.919	8350.95
2	20173.594	3412.3496	17	20800.798	20506.287
3	15681.922	16530.259	18	30356.377	5064.2124
4	31294.302	28131.322	19	21814.9	6562.7207
5	35479.562	10067.631	20	32437.055	16530.259
6	15567.975	3526.0751	21	30431.954	4492.4061
7	39262.022	3335.6286	22	32870.706	10440.191
8	33598.851	1668.2489	23	28465.444	11161.577
9	18501.784	13387.878	24	23216.254	1318.1163
10	31878.472	25336.589	25	35662.53	11614.067
11	21994.128	4389.1246	26	27691.991	6004.9445
12	17987.175	2753.7447	27	21337.37	8483.6973
13	39356.132	7875.7278	28	15175.55	8567.0796
14	22282.57	8317.885	29	27302.49	23107.953
15	24190.804	23212.771	30	20393.457	14387.751

Estimated reliability = 96.666667%

Stress exceeded strength in only 1 of 30 trials!

When the experiment was repeated, but with 300 trials, stress exceeded strength in 26 (i.e., 8.6666%) of the trials, so that the estimated reliability was 91.333%

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