An easier method for computing reduced costs:

Let u_i = dual variable (simplex multiplier) for the supply constraint: $\sum_{j=1}^{n} X_{ij} = S_i$

v_j = dual variable (simplex multiplier) for the demand constraint: $\sum_{i=1}^{m} X_{ij} = D_j$

Then the reduced cost, as in the revised simplex method, is computed by: $\overline{C}_{ij} = C_{ij} - (u_i + v_j)$

Then the reduced cost, as in the revised simplex method, is computed by:

reduced cost

$$\overline{C}_{ij} = C_{ij} - [u_1, u_2, \ldots u_m, v_1, \ldots v_n] \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \leftarrow \text{row } i \\ \\ \\ \\ \\ \\ \\ \leftarrow \text{row } m+j \\ \\ \\ \\ \end{matrix}$$

cost of x_{ij}

Column of tableau for x_{ij}

$$= C_{ij} - (u_i + v_j)$$

which is much simpler than identifying the cycles!

Computing the simplex multipliers

Recall that the simplex multipliers are the values of the dual variables.

We will next write the dual constraints, and use "Complementary Slackness" to compute the dual variables.

The Primal LP

X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}		
.95	1.05	.80	.15	1.00	0	.35	1.80	1.40	.80	.30	0	.90	1.80	1.60	.70	.85	0		MIN
1	1	1	1	1	1													=	12
						1	1	1	1	1	1							=	7
												1	1	1	1	1	1	=	15
1						1						1						=	5
	1						1						1					=	4
		1						1						1				=	4
			1						1						1			=	11
				1						1						1		=	8
					1						1						1	=	2

The Dual LP

Max $12u_1 + 7u_2 + \ldots + 2v_6$

subject to

$u_1 + v_1 \leq 0.95$

$u_1 + v_2 \leq 1.05$

$u_1 + v_3 \leq 0.80$

$u_1 + v_4 \leq 0.15$

$u_1 + v_5 \leq 1.00$

⋮

$u_3 + v_5 \leq 0.85$

$u_3 + v_6 \leq 0$

(u_i & v_j unrestricted in sign)

The dual constraints are

$$u_i + v_j \le C_{ij} \quad \text{for all } i \,\&\, j$$

Complementary Slackness implies that

$$X_{ij} > 0 \implies u_i + v_j = C_{ij}$$

This provides us with (m+n-1) equations

↳ *# of basic variables*

from which we can compute the (m+n) unknowns.

(Because the system of equations is "overdetermined", we can assign an arbitrary value, e.g., zero, to one of the dual variables.)

u_i \ v_j	u_i	ATLANTA	L.A.	DALLAS	CHGO.	N.Y.	EXCESS CAP.
HOME CITY	0	.95	1.05	1 / .80	11 / .15	1.00	0
BRANCH #1		.35	1.80	1.40	.80	7 / .30	0
BRANCH #2		5 / .90	4 / 1.80	3 / 1.60	.70	1 / .85	2 / 0

Let's arbitrarily set $u_1 = 0$

Then complementary slackness implies that

$$u_1 + v_3 = 0.80 \quad \Rightarrow v_3 = 0.80$$

and

$$u_1 + v_4 = 0.15 \quad \Rightarrow v_4 = 0.15$$

u_i \ v_j		ATLANTA	L.A.	DALLAS	CHGO.	N.Y.	EXCESS CAP.
				.8	.15		
HOME CITY	0	.95	1.05	1 / .80	11 / .15	1.00	0
BRANCH #1		.35	1.80	1.40	.80	7 / .30	0
BRANCH #2		5 / .90	4 / 1.80	3 / 1.60	.70	1 / .85	2 / 0

Now we can use Complementary Slackness to obtain

$$u_3 + v_3 = 1.60$$
$$u_3 + .8 = 1.60$$
$$\Rightarrow u_3 = .8$$

u_i \ v_j	ATLANTA	L.A.	DALLAS	CHGO.	N.Y.	EXCESS CAP.
			.8	.15		
HOME CITY 0	.95	1.05	1 / .80	11 / .15	1.00	0
BRANCH #1	.35	1.80	1.40	.80	7 / .30	0
BRANCH #2 .8	5 / .90	4 / 1.80	3 / 1.60	.70	1 / .85	2 / 0

Now we can use complementary slackness to obtain v_1, v_2, v_5, and v_6

$$u_3 + v_1 = 0.90 \Rightarrow v_1 = 0.10$$

$$u_3 + v_2 = 1.80 \Rightarrow v_2 = 1.00$$

$$u_3 + v_5 = 0.85 \Rightarrow v_5 = 0.05$$

$$u_3 + v_6 = 0 \Rightarrow v_6 = -0.8$$

u_i \ v_j		ATLANTA	L.A.	DALLAS	CHGO.	N.Y.	EXCESS CAP.
		.1	1.0	.8	.15	.05	-.8
HOME CITY	0	.95	1.05	1 / .80	11 / .15	1.00	0
BRANCH #1		.35	1.80	1.40	.80	7 / .30	0
BRANCH #2	.8	5 / .90	4 / 1.80	3 / 1.60	.70	1 / .85	2 / 0

Finally, we can use v_4 to compute u_2 :

$$u_2 + v_4 = 0.30$$
$$\Rightarrow u_2 = 0.25$$

u_i \ v_j		ATLANTA	L.A.	DALLAS	CHGO.	N.Y.	EXCESS CAP.
		.1	1.0	.8	.15	.05	-.8
HOME CITY	0	[.95]	[1.05]	1 [.80]	11 [.15]	[1.00]	[0]
BRANCH #1	.25	[.35]	[1.80]	[1.40]	[.80]	7 [.30]	[0]
BRANCH #2	.8	5 [.90]	4 [1.80]	3 [1.60]	[.70]	1 [.85]	2 [0]

Now let's use the simplex multipliers to compute the reduced costs, using the formula: $\overline{C}_{ij} = C_{ij} - (u_i + v_j)$

$\overline{C}_{11} = 0.95 - (0 + .1) = 0.85$

$\overline{C}_{24} = 0.80 - (0.25 + 0.15) = 0.40$

$\overline{C}_{34} = 0.70 - (0.8 + 0.15) = -0.25$

These are in agreement with the earlier computations!