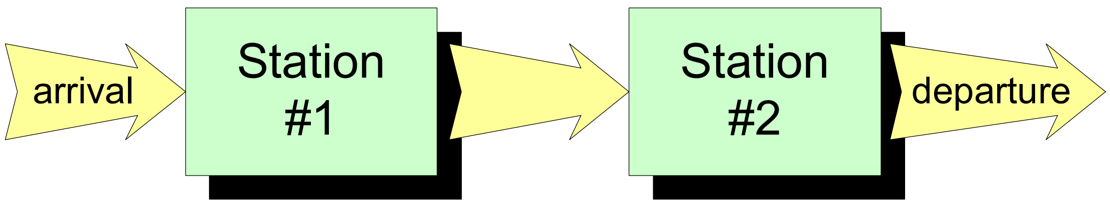
Tandem Stations with Blocking



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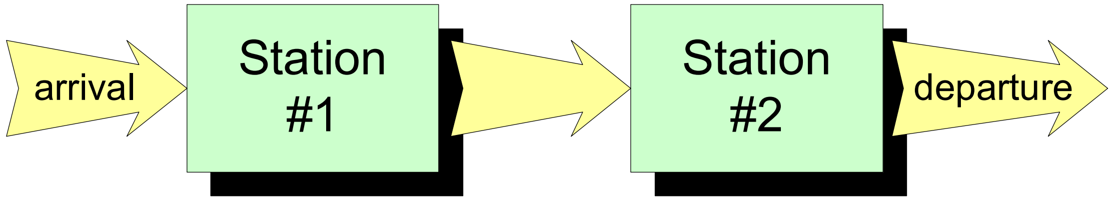
Dept of Mechanical & Industrial Engineering
The University of Iowa



A processing system is composed of **two stations in tandem**.

- The arrival of jobs at station #1 is a Poisson process with a rate of 4/hour, but station #1 can accept jobs only when it is idle.
- The processing time at each station is exponentially distributed with a mean of 10 minutes.
- There is room in the system for only two jobs, one at each station.
- No queueing between stations or before the first station is permitted.

- A job which completes processing at the first station when the second station is busy will remain at the first station, "**blocking** it", i.e., preventing it from accepting a new job.

Compute the steady-state probabilities.

Compute the throughput rate for the system.

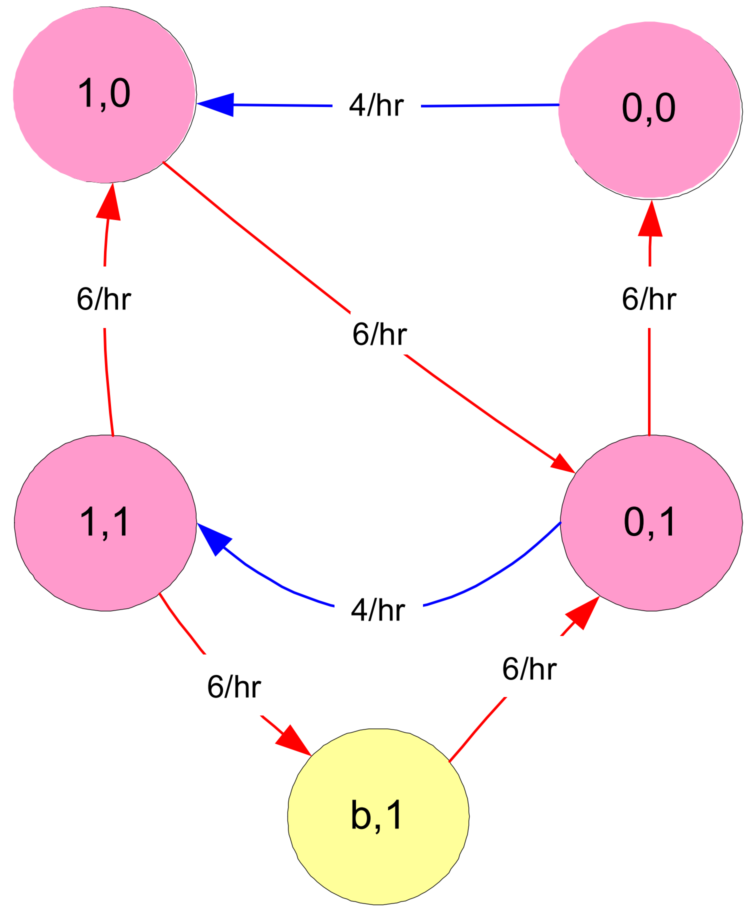
The state of the system is 2-dimensional:

denote the state of each station by

0) idle
1) busy

and, for station #1,

b) "blocked".



The possible states of the system are therefore

1. (0,0)
2. (1,0)
3. (0,1)
4. (1,1)
5. (b,1)

Transition Rate Matrix

	00	10	01	11	b1
00	−4	4	0	0	0
10	0	−6	6	0	0
01	6	0	−10	4	0
11	0	6	0	−12	6
b1	0	0	6	0	−6

Steady-state equations:

$$\begin{cases} -4\pi_{00} + 6\pi_{01} = 0 \\ 4\pi_{00} - 6\pi_{10} + 6\pi_{11} = 0 \\ 6\pi_{10} - 10\pi_{01} + 6\pi_{b1} = 0 \\ 4\pi_{01} - 12\pi_{11} = 0 \\ 6\pi_{11} - 6\pi_{b1} = 0 \end{cases}$$

& $\pi_{00} + \pi_{10} + \pi_{01} + \pi_{11} + \pi_{b1} = 1$

Steady-state Distribution

i	state	π_i
1	00	0.3333
2	10	0.2963
3	01	0.2222
4	11	0.0741
5	b1	0.0741

What is the average throughput of the system?

Jobs are completed at the rate **6/hr** when system is in states 3, 4, & 5, having total probability **0.3704**.

Therefore the **throughput** is $0.3704 \times 6/\text{hr}$ = **2.222/hr.**

(or **4/hr** $\times (\pi_{00}+\pi_{01})$ = **2.222/hr.**)