Variations of the Simplex Tableau

Summary: The simplex tableau as shown in various textbooks and these notes may differ, and cause you some confusion.
Let’s assume the LP model

\[
\text{Minimize } \quad z = \sum_{j=1}^{n} c_j x_j
\]

\[
\text{s.t. } \quad \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad \forall i = 1, \ldots, m
\]

\[
x_j \geq 0, \quad j = 1, \ldots, n
\]

We can write the objective equation in either of two ways:

\[
z - \sum_{j=1}^{n} c_j x_j = 0 \quad \text{or} \quad -z + \sum_{j=1}^{n} c_j x_j = 0
\]

The objective row can be written either as the \textit{first} or \textit{last row} of the tableau. In my notes, it will appear as the first row.

In my notes, I will use \textquotedblleft$-z$\textquotedblright as the basic variable in the objective row.
For example, the tableau might appear as

<table>
<thead>
<tr>
<th></th>
<th>$-z$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>$-3$</td>
<td>0</td>
<td>0</td>
<td>$-4$</td>
<td>0</td>
<td>$-22$</td>
</tr>
<tr>
<td>0</td>
<td>$-1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$-2$</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

where $X_2$, $X_4$, and $X_6$ (along with $-z$) are basic.

In this tableau, the values in the objective row are the **reduced costs**. Thus,

- increasing $X_1$ will **increase** the objective at the rate of 2 units of cost per unit of $X_1$,
- increasing $X_3$ will **decrease** the objective at the rate of 3 units of cost per unit of $X_3$.

If **minimizing**, then any column with **negative** value in objective row can be chosen as the pivot column.
Since the first column never changes, some textbooks don’t explicitly include it in the tableau:

\[
\begin{array}{ccccccc}
-\text{z} & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & \text{RHS} \\
1 & 2 & 0 & -3 & 0 & -4 & 0 & -22 \\
0 & -1 & 0 & 0 & 1 & 3 & 0 & 5 \\
0 & 4 & 1 & 2 & 0 & 0 & 0 & 2 \\
0 & 2 & 0 & 1 & 0 & -2 & 1 & 3 \\
\end{array}
\]

i.e.,

\[
\begin{array}{ccccccc}
X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & \text{RHS} \\
2 & 0 & -3 & 0 & -4 & 0 & -22 \\
-1 & 0 & 0 & 1 & 3 & 0 & 5 \\
4 & 1 & 2 & 0 & 0 & 0 & 2 \\
2 & 0 & 1 & 0 & -2 & 1 & 3 \\
\end{array}
\]
In some textbooks, \( z \) (rather than \(-z\)) is used as the basic variable in the objective row. The earlier tableau would then appear as

\[
\begin{array}{cccccccc}
\text{z} & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & \text{RHS} \\
1 & -2 & 0 & 3 & 0 & 4 & 0 & 22 \\
0 & -1 & 0 & 0 & 1 & 3 & 0 & 5 \\
0 & 4 & 1 & 2 & 0 & 0 & 0 & 2 \\
0 & 2 & 0 & 1 & 0 & -2 & 1 & 3 \\
\end{array}
\]

If this convention is used, then the signs of all values in the objective row are reversed, and the rule for choosing the variable to enter the basis is changed:

If \textit{minimizing}, then any column with \textbf{positive} value in objective row can be chosen as the pivot column.

If \textit{maximizing}, then any column with \textbf{negative} value in objective row can be chosen as the pivot column.
Knowing the basic variables in each row is enough to know the contents of each row:

<table>
<thead>
<tr>
<th></th>
<th>z</th>
<th>X₁</th>
<th>X₂</th>
<th>X₃</th>
<th>X₄</th>
<th>X₅</th>
<th>X₆</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>−1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>−2</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, some textbooks don’t explicitly show the columns of the basic variables in the tableau, but indicate the basic variable for each row, to get a more **compact tableau**:

<table>
<thead>
<tr>
<th></th>
<th>X₁</th>
<th>X₃</th>
<th>X₅</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>−z</td>
<td>2</td>
<td>−3</td>
<td>−4</td>
<td>−22</td>
</tr>
<tr>
<td>X₄</td>
<td>−1</td>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>X₂</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>X₆</td>
<td>2</td>
<td>1</td>
<td>−2</td>
<td>3</td>
</tr>
</tbody>
</table>

*(When using this form of the tableau, the formula for performing a pivot will be quite different, since the pivot column is replaced by the result of performing the pivot on the column which was previously basic in the pivot row.)*