Variations of the Simplex Tableau

Summary: The simplex tableau as shown in various textbooks and these notes may differ, and cause you some confusion.

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Let's assume the LP model

$$\textit{Minimize} \quad z = \sum_{j=1}^{n} c_j x_j$$

$$s.t. \qquad \sum_{j=1}^{n} a_{ij} x_j = b_i, \qquad \forall i = 1,...m$$

$$x_j \geq 0, \qquad j = 1,...n$$

We can write the objective equation in either of two ways:

$$z - \sum_{j=1}^{n} c_j x_j = 0 \qquad \text{or} \qquad -z + \sum_{j=1}^{n} c_j x_j = 0$$

The objective row can be written either as the ***first*** or ***last row*** of the tableau. In my notes, it will appear as the first row.
In my notes, I will use "$-z$" as the basic variable in the objective row.

For example, the tableau might appear as

$-z$	X_1	X_2	X_3	X_4	X_5	X_6	RHS
1	2	0	–3	0	–4	0	–22
0	–1	0	0	1	3	0	5
0	4	1	2	0	0	0	2
0	2	0	1	0	–2	1	3

where X_2, X_4, and X_6 (along with –z) are basic.

In this tableau, the values in the objective row are the ***reduced costs***. Thus,

- increasing X_1 will ***increase*** the objective at the rate of 2 units of cost per unit of X_1,
- increasing X_3 will ***decrease*** the objective at the rate of 3 units of cost per unit of X_3.

If ***minimizing***, then any column with ***negative*** value in objective row can be chosen as the pivot column.

Since the first column never changes, some textbooks don't explicitly include it in the tableau:

–z	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**RHS**
1	2	0	–3	0	–4	0	–22
0	–1	0	0	1	3	0	5
0	4	1	2	0	0	0	2
0	2	0	1	0	–2	1	3

i.e.,

X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**RHS**
2	0	–3	0	–4	0	–22
–1	0	0	1	3	0	5
4	1	2	0	0	0	2
2	0	1	0	–2	1	3

In some textbooks, z (rather than $-z$) is used as the basic variable in the objective row. The earlier tableau would then appear as

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	–2	0	3	0	4	0	22
0	–1	0	0	1	3	0	5
0	4	1	2	0	0	0	2
0	2	0	1	0	–2	1	3

If this convention is used, then the signs of all values in the objective row are reversed, and the rule for choosing the variable to enter the basis is changed:

> If ***minimizing***, then any column with ***positive*** value in objective row can be chosen as the pivot column.
> If ***maximizing***, then any column with ***negative*** value in objective row can be chosen as the pivot column.

Knowing the basic variables in each row is enough to know the contents of each row:

z	**X_1**	X_2	**X_3**	X_4	**X_5**	X_6	**RHS**
1	–2	0	3	0	4	0	22
0	–1	0	0	1	3	0	5
0	4	1	2	0	0	0	2
0	2	0	1	0	–2	1	3

Therefore, some textbooks don't explicitly show the columns of the basic variables in the tableau, but indicate the basic variable for each row, to get a more ***compact tableau***:

	X_1	**X_3**	**X_5**	**RHS**
–z	2	–3	–4	–22
X_4	–1	0	3	5
X_2	4	2	0	2
X_6	2	1	–2	3

(When using this form of the tableau, the formula for performing a pivot will be quite different, since the pivot column is replaced by the result of performing the pivot on the column which was previously basic in the pivot row.)