Nearest Insertion Algorithm for the Traveling Salesman Problem

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The "Nearest Insertion" heuristic algorithm constructs a tour, starting with an arbitrary node. Each step begins with a subtour, and selects the node which is *nearest* to the set of nodes on the subtour to be added to the subtour. After selecting the node $k$ to be added, an edge $(i,j)$ is selected and the edges $(i,k)$ and $(k,j)$ then replace the edge $(i,j)$. The edge $(i,j)$ is selected so as to minimize the increase in the length of the subtour, i.e.,

$$d_{ik} + d_{kj} - d_{ij}$$

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The "Nearest Insertion" heuristic constructs a tour for the TSP as follows:

step 0: Select an initial node $\hat{i}$.

Let $N'$ denote the set of nodes $N - \{\hat{i}\}$

Let $T = \{ (\hat{i}, \hat{i}) \}$

step 1: Let $\hat{j} = \arg\min_{j \in N'} \left[ \min_{i \in T} \{d_{ij}\} \right]$

step 2: Let $(i', i'') = \arg\min_{(i_1, i_2) \in T} \{d_{i_1j} + d_{ji_2} - d_{i_1i_2} \}$
step 3: Replace arc \((i_1, i_2)\) in the tour \(T\) with the pair of arcs \((i_1, \hat{j})\) and \((\hat{j}, i_2)\). Let \(N' = N' - \{\hat{j}\}\) and \(i = \hat{j}\).

step 4: If \(N' = \emptyset\), STOP. Else return to step 1.
Example

Random Symmetric TSP
(seed = 133398)
## Distances

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Let’s arbitrarily begin the tour with node *1*, i.e., \( T = \{1\} \), and \( N' = \{2,3,4,5,6,7,8,9,10,11,12\} \).

The nearest node to \( T=\{1\} \) is node 2.
Nearest Insertion

(Starting with node #1)

Insert node 2

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The nearest node to the subtour $T = \{1, 2\}$ is node #3.
The nearest node to $T = \{1, 2, 3\}$ is node *4.*

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Node *4* can be inserted in the tour in 3 different ways:

1 → 4 → 2
2 → 4 → 3
3 → 4 → 1
We insert node #4 in such a way as to minimize the increase in tour length:

\[
\begin{array}{c|cccc}
\text{distances} & 1 & 2 & 3 & 4 \\
1 & 0 & 19 & 30 & 50 \\
2 & 19 & 0 & 16 & 34 \\
3 & 30 & 16 & 0 & 41 \\
4 & 50 & 34 & 41 & 0 \\
\end{array}
\]

Increase in tour length:

- \[50 + 34 - 19 = 65\]
- \[34 + 41 - 16 = 59\]

\[\text{minimum!}\]

- \[41 + 50 - 30 = 61\]

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Insert node 5
Insert node 6... etc.
Nearest Insertion Tour: 6 7 8 10 9 11 3 2 1 12 4 5 6, with length 321

Note that the final tour varies according to the initial node in the tour!

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