Heuristic Algorithms for the Traveling Salesman Problem

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Most heuristics are of one of two types:

- **Construction Heuristics**
  Add one arc after another to a tour, until the tour includes all nodes.

- **Improvement Heuristics**
  Given an initial tour, find an improved tour
Construction Heuristics

- Nearest Neighbor Algorithm
- Nearest Insertion Algorithm
- Farthest Insertion Algorithm
- Space-filling Curve Algorithm
Improvement Heuristics

Exchange Algorithm

Simulated Annealing
Example

30 CITIES

Random Symmetric TSP (seed= 185204)

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Spacefilling Curve Heuristic

Random Symmetric TSP (seed= 185204)

Length: 664
Nearest Neighbor Heuristic

Random Symmetric TSP (seed= 185204)

Starting Node: #1
Length: 684

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Nearest Neighbor Heuristic

Random Symmetric TSP (seed= 185204)

Starting Node: #10
Length: 633

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Farthest Insertion Heuristic

Random Symmetric TSP (seed= 185204)

Starting Node: #1
Length: 513
Farthest Insertion Heuristic

Random Symmetric TSP (seed = 185204)

Starting Node: #10
Length: 516

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Farthest Insertion Heuristic

Random Symmetric TSP (seed= 185204)

Starting Node: #20
Length: 508

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Nearest Insertion Heuristic

Random Symmetric TSP (seed= 185204)

Starting Node: #10
Length: 610
Lin's 2-Opt Heuristic

Random Symmetric TSP (seed= 185204)

Starting Tour

Nearest Neighbor Tour, Originating at Node #5
Length: 620

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Lin's 2-Opt Heuristic

Random Symmetric TSP (seed= 185204)

2-Optimal Tour
Length: 549

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Lin’s 2-Opt Heuristic

Random Symmetric TSP (seed= 185204)

2-Optimal Tour
Length: 524

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Lin’s 2-Opt Heuristic

Tour Lengths
Lin's 3-Opt Heuristic

Random Symmetric TSP (seed= 185204)

3-Optimal Tour
Length: 503

Starting with Farthest Insertion Heuristic
(origin: Node #20)
Iteration 1
Lower Bound: 445
Sum of excess degrees: 8

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Iteration 2
Lower Bound: 451
Sum of excess degrees: 5
Iteration 3
Lower Bound: 452.7
Sum of excess degrees: 8

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Iteration 4
Lower Bound: 463.545
Sum of excess degrees: 6
Iteration 5
Lower Bound: 472.0270833
Sum of excess degrees: 8
Iteration 6
Lower Bound: 475.0625781
Sum of excess degrees: 8

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Iteration 7
Lower Bound: 477.417609
Sum of excess degrees: 4

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<th>Iteration</th>
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</table>
2-Exchange Heuristic

Nearest Neighbor Tour, originating at Node #15
Length: 632
2-Exchange Heuristic

2-Optimal Tour Length: 548

(Starting search with Nearest Neighbor Tour originating at #15)

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2-Exchange Heuristic

Tour Lengths

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Simulated Annealing

Final Tour
Length: 613

(Beginning with tour of length 632)
Simulated Annealing

\[ P(\text{Accept } = 25) \]

\[ \text{Iteration #} \]

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