Branch & Bound Algorithms for the Traveling Salesman Problem

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A TSP tour has the properties:

- it is a *connected* subgraph of the network
- the degree of every node is 2

The solution of the *Assignment Problem* satisfies the second property, but not always the first. The solution of the *minimum spanning 1-tree* problem satisfies the first property, but not always the second.
A 1-tree is constructed by adding a single edge to a tree.
Note that a tour is a 1-tree:
Minimize \[ \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}X_{ij} \]

subject to

\[ \sum_{i=1}^{n} X_{ij} = 1 \quad \forall \quad j=1, \ldots, n \]  \hspace{1cm} \text{Assignment constraints}

\[ \sum_{j=1}^{n} X_{ij} = 1 \quad \forall \quad i=1, \ldots, n \]

\[ X \in \mathcal{T} = \text{set of all 1-trees} \]
If either the assignment or the 1–tree constraints are relaxed, the resulting problem (which is easy to solve) provides a lower bound on the length of the optimal tour.

- Relaxation of 1–tree constraints
- Relaxation of Assignment constraints