System Reliability

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System reliability is the *probability* that the system will *not fail* to perform

- within specified limits
- for a specified length of time
- in a specified environment

Define the *random variable*

T_{sys} = effective lifetime of the system

with cumulative distribution function (CDF):

$F_{sys}(t) = P\{T_{sys} \leq t\}$

The *reliability function* is the complement of F_{sys}, i.e.,

$$R_{sys}(t) = 1 - F_{sys}(t)$$

$$= P\{T_{sys} > t\}$$

$$= P\{\text{system is functioning properly at time } t\}$$

System Reliabilty

Suppose that we know the reliabilities of the individual components of a system.

How do we estimate the reliability of the system?

SYSTEM RELIABILTY

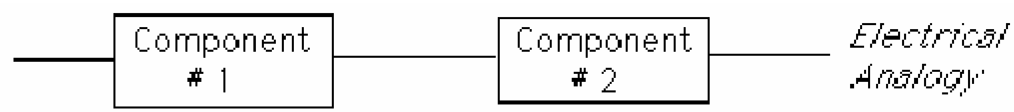
- Components in series
- Components in parallel (redundancy)
- Partially redundant systems (m-out-of-n)
  - Examples: hybrid systems with both series & parallel components
- Standby system
  - With perfect switching
  - With imperfect switching
  - With failures in standby mode

Etc.

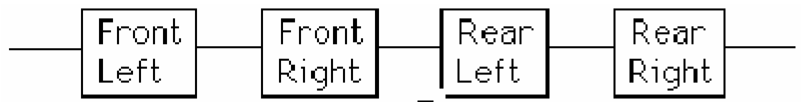
Series

Assume

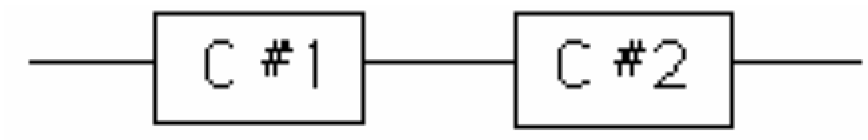
- Any component may fail, independently of any other
- When one component fails, the system (device) fails.



Example: an automobile "fails" when any one of its tires fail:



Note that the tires are not physically arranged in series (i.e., tandem)!



$R_s(t)$ = reliability of device

= P{all components survive until time t}

= P{component#1 survives until time t }

× P{component #2 survives until time t}

$= R_1(t) \times R_2(t)$

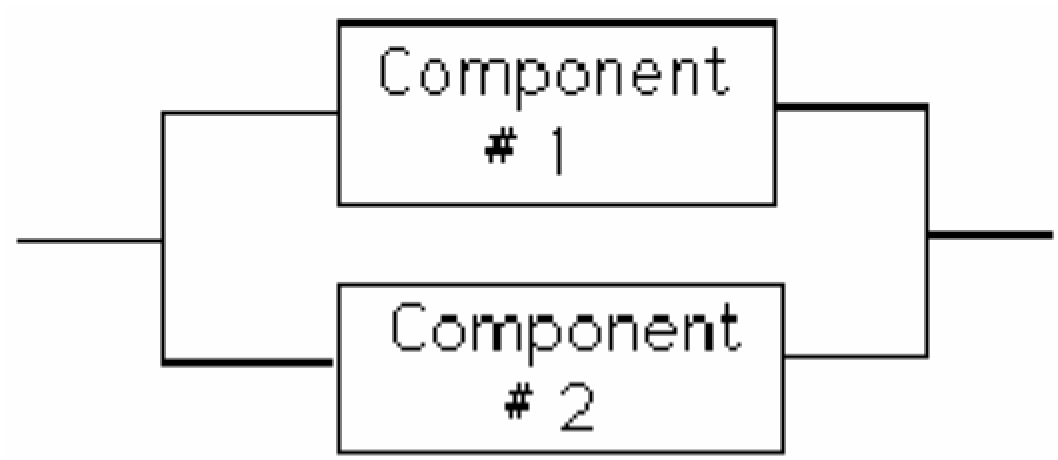
In general, for n components in series,

$$R_s(t) = R_1(t) \times R_2(t) \times \ldots \times R_n(t)$$

PARALLEL COMPONENTS

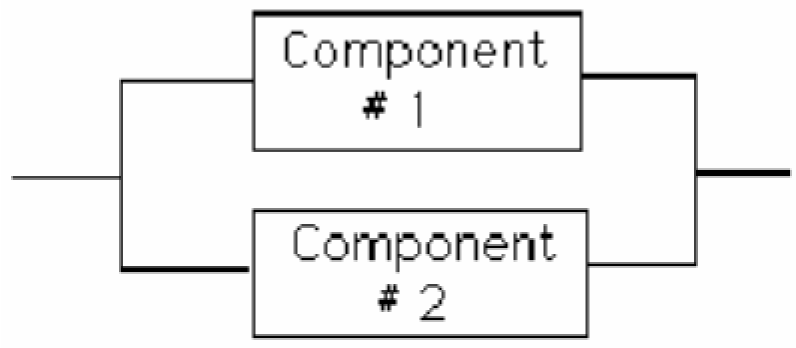
Assume:

- Components operate simultaneously (not "standby")
- All components must fail in order that the system fails



Example: a 2-engine plane fails only if *both* of its engines fail.

PARALLEL COMPONENTS



Let $F_i(t) = 1 - R_i(t) = P\{$ component #i fails before time t $\}$

(CDF of the component's lifetime)

$F_p(t) = P\{$ parallel system fails before time t $\}$

$= P\{$ all components fail before time t $\}$

$= P\{$ component #1 fails before time t$\}$

$\times P\{$component #2 fails before time t$\}$

$= F_1(t) \times F_2(t) = [1 - R_1(t)] \times [1 - R_2(t)]$

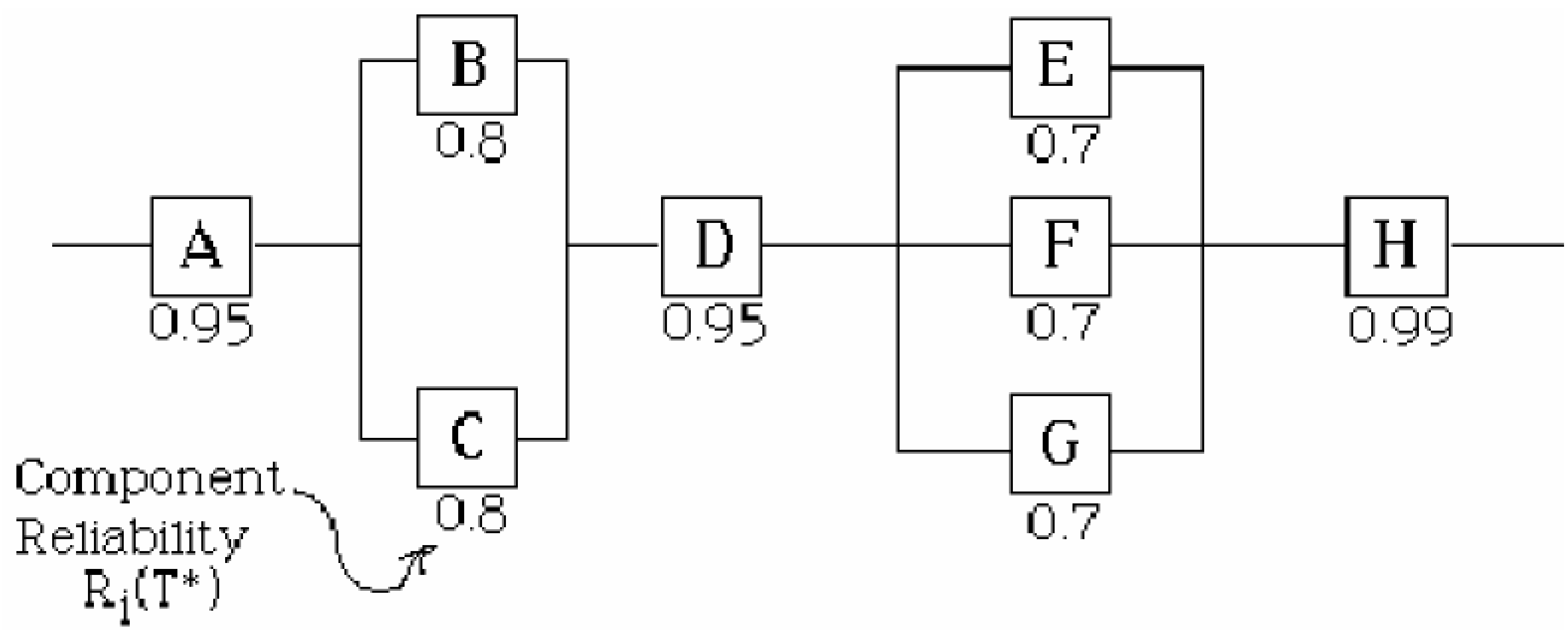
So

$$R_p(t) = 1 - F_p(t) = 1 - [1 - R_1(t)] \times [1 - R_2(t)]$$

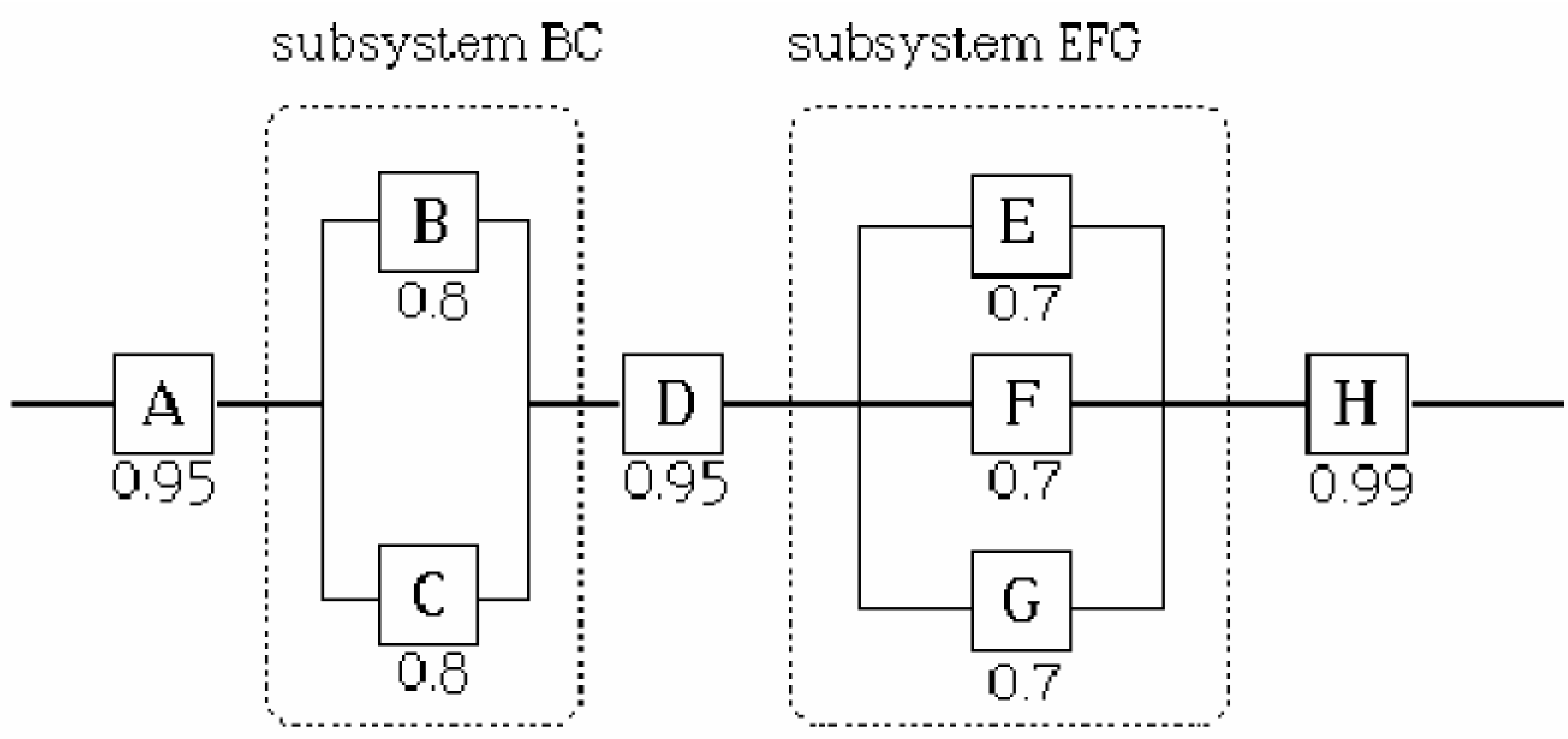
HYBRID SYSTEM

Suppose that a system is designed to fulfill its mission for a planned lifetime T*.

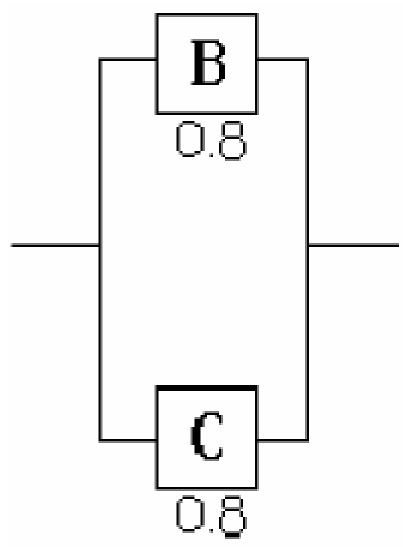
Find the reliability of the system, given the component reliabilities:



Decompose the system into series &/or parallel subsystems:



subsystem BC:



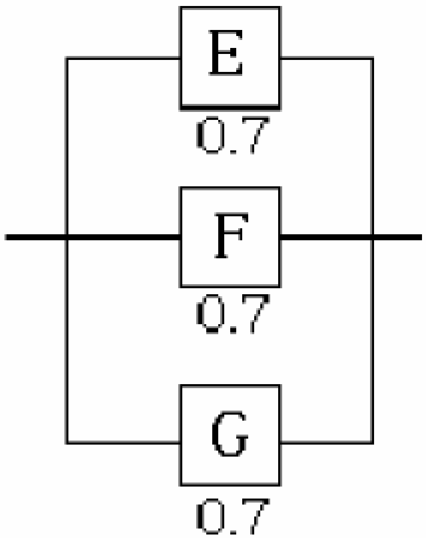
$$R_{BC} = 1 - [1 - R_B]\times[1 - R_C]$$

$$= 1 - 0.2\times0.2$$

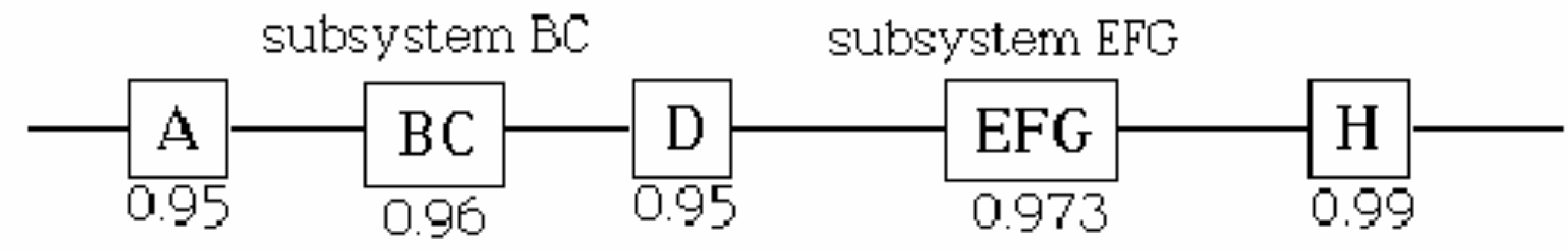
$$= 1 - 0.04$$

$$= 0.96$$

Subsystem EFG:



$$R_{EFG} = 1 - [1-R_E] \times [1-R_F] \times [1-R_G]$$
$$= 1 - 0.3 \times 0.3 \times 0.3$$
$$= 1 - 0.027$$
$$= 0.973$$



Reliability of series of subsystems:

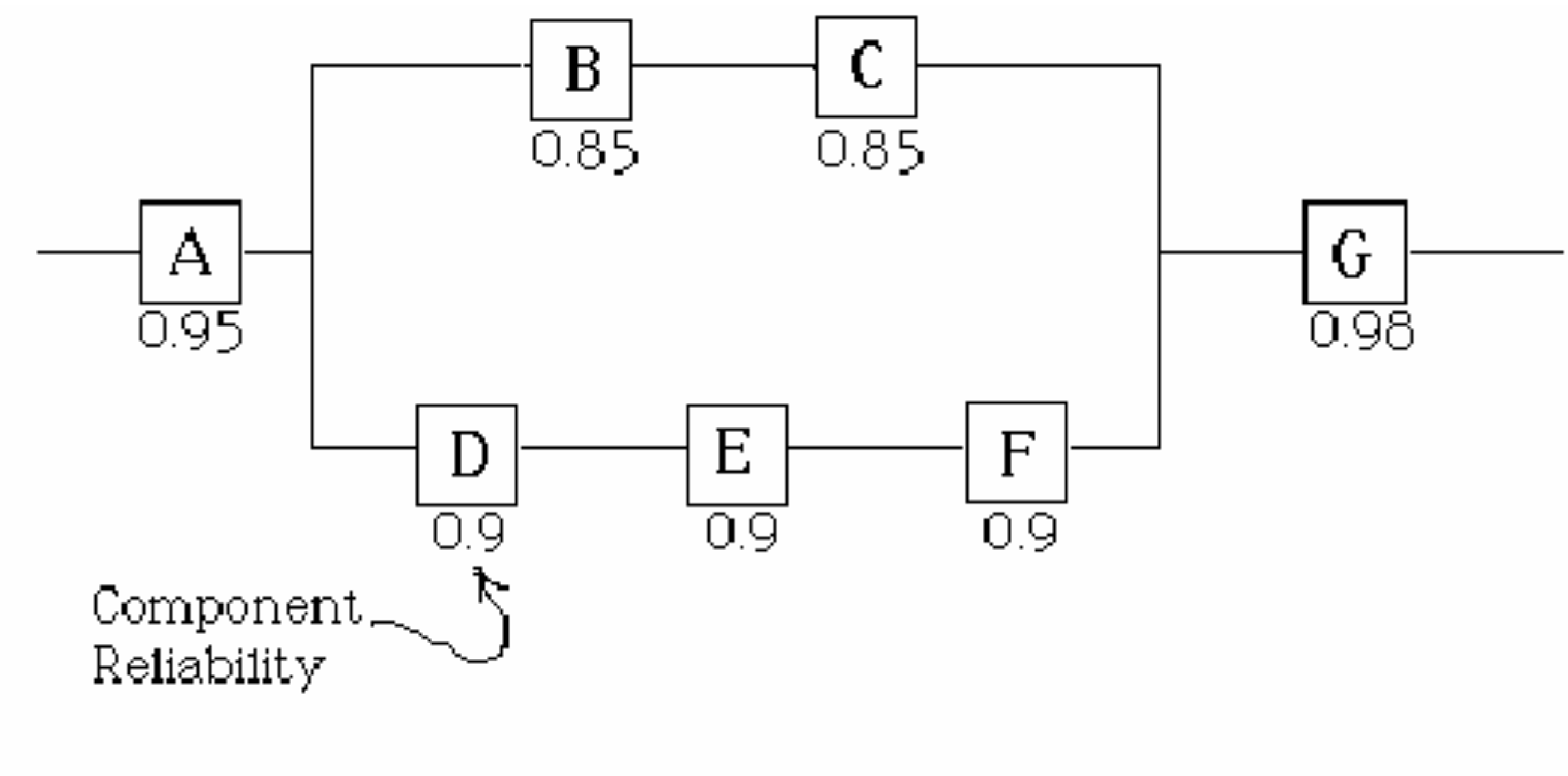
$$R_{system} = R_A \times R_{BC} \times R_D \times R_{EFG} \times R_H$$

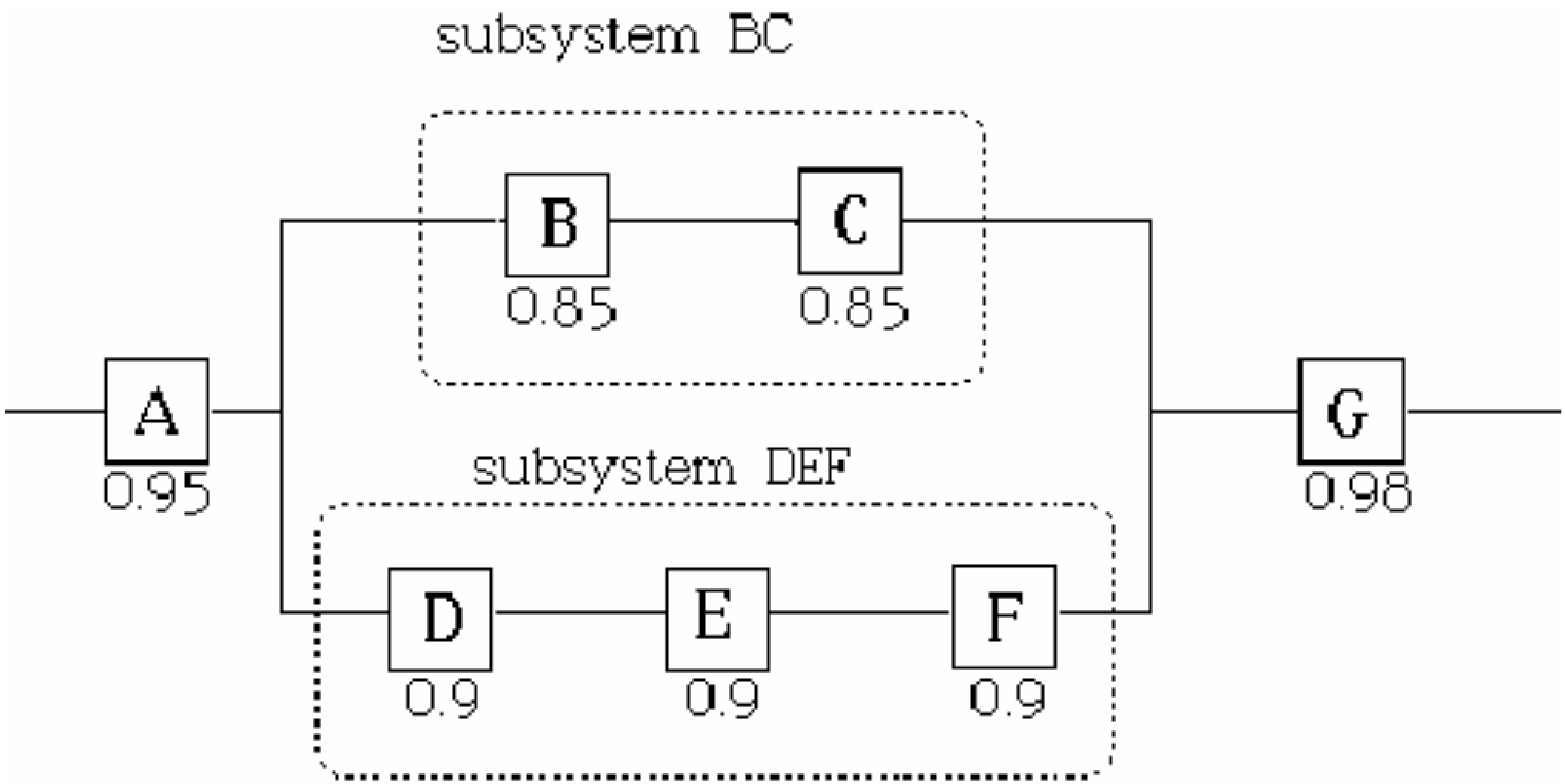
$$= (0.95)(0.96)(0.95)(0.973)(0.99)$$

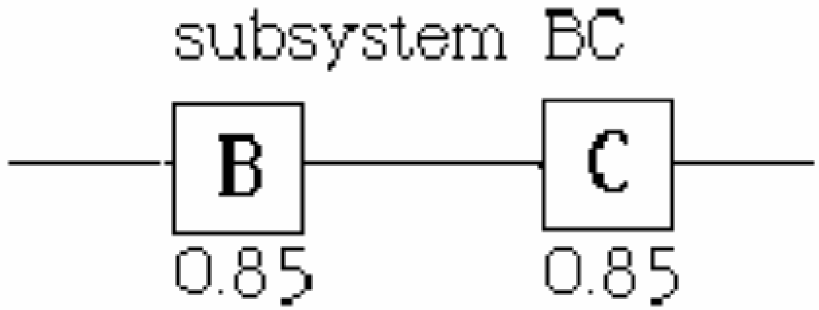
$$= 0.834577$$

Another Example:

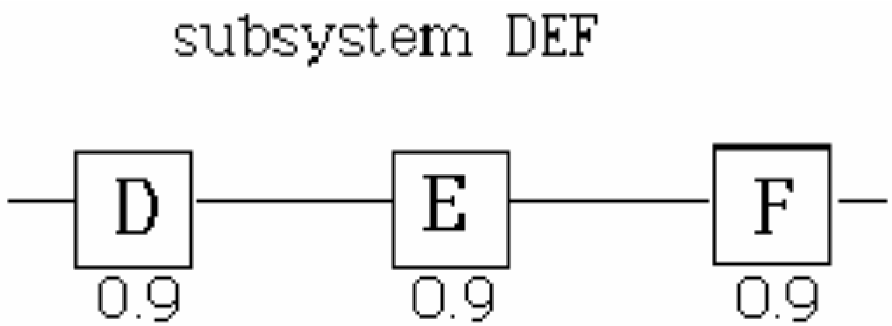
Find the reliability of the system:



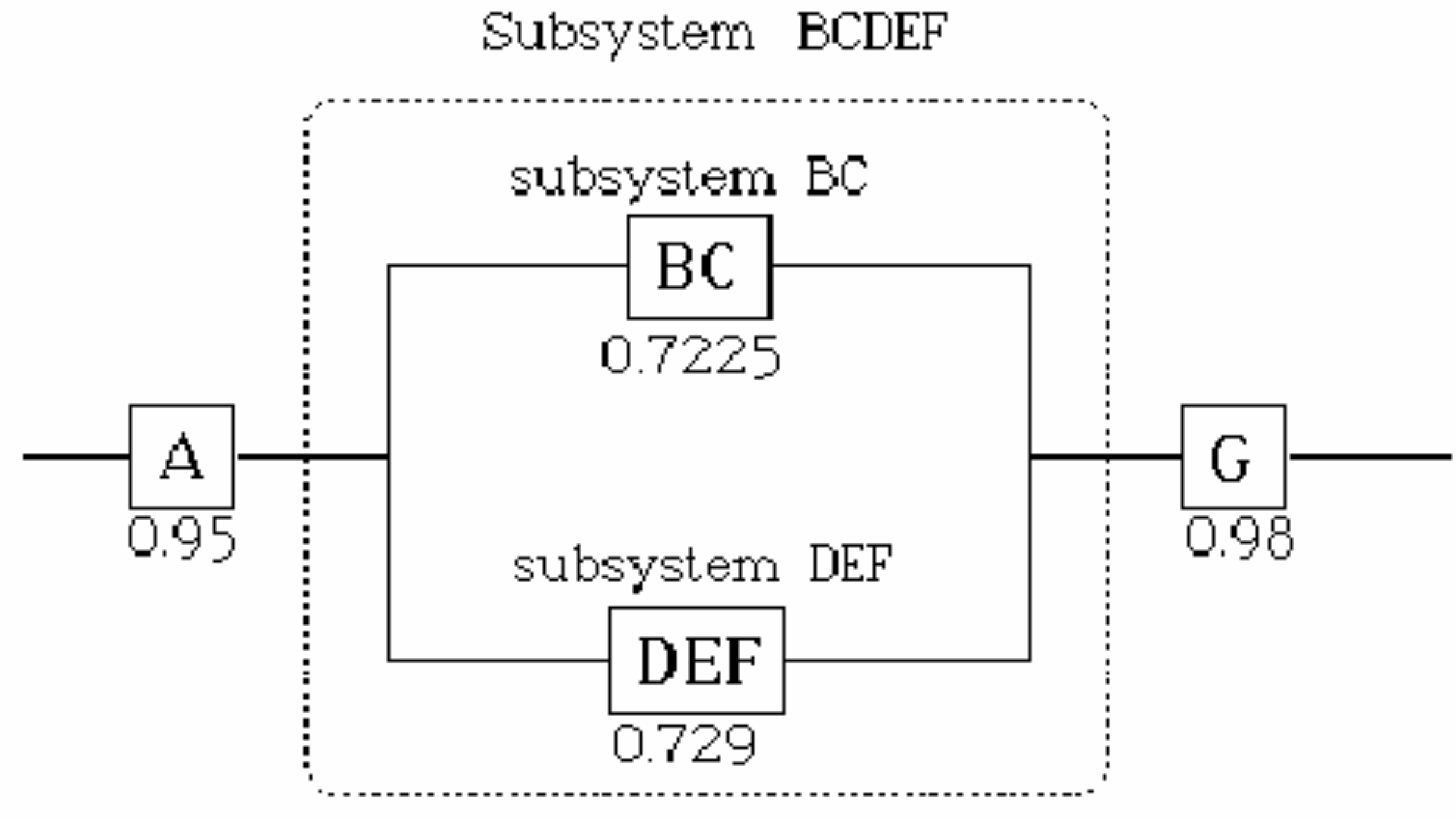

subsystem BC
B
C
0.85
0.85
A
0.95
G
0.98
subsystem DEF
D
E
F
0.9
0.9
0.9



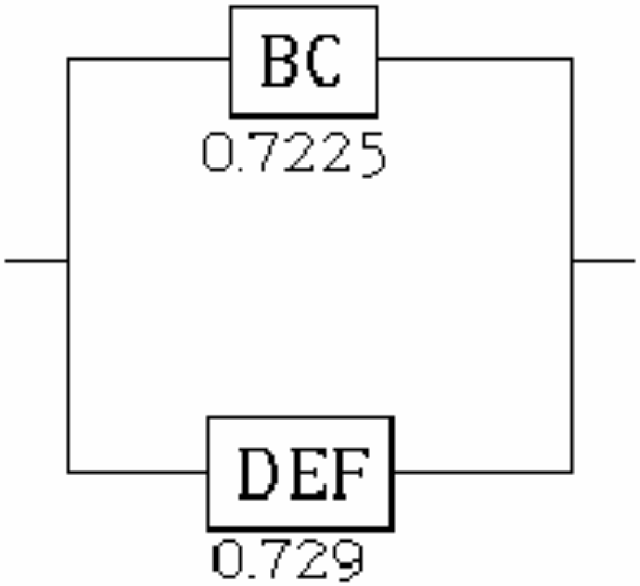
$$R_{BC} = R_B \times R_C$$
$$= 0.85 \times 0.85$$
$$= 0.7225$$



$$R_{DEF} = R_D \times R_E \times R_F$$
$$= 0.9 \times 0.9 \times 0.9$$
$$= 0.729$$


Subsystem BCDEF
subsystem BC
BC
0.7225
A
0.95
G
0.98
subsystem DEF
DEF
0.729

Subsystem BCDEF



$$R_{BCDEF} = 1 - [1-R_{BC}]\times[1-R_{DEF}]$$
$$= 1 - (1-0.7225)(1-0.729)$$
$$= 1 - (0.2775)(0.271)$$
$$= 1 - 0.075202$$
$$= 0.924798$$

$$R_{system} = R_A \times R_{BCDEF} \times R_G$$

$$= (0.95)(0.924798)(0.98)$$

$$= 0.860987$$