

# Stochastic Processes



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## Stochastic Process

For each  $t$ ,  $t \in T$ , let  $X_t$  be a random variable.  
Then the collection of random variables

$$\{X_t, t \in T\}$$

is a stochastic process.

Generally,  $t$  represents a time parameter.

A stochastic process is classified as

*discrete-parameter*

if the index set  $T = \{0, 1, 2, 3, \dots\}$

and

*continuous-parameter*

if  $T = [0, +\infty)$ , i.e., the set of non-negative real numbers.

The "State Space" of the process is the set of possible values that  $X_t$  may assume.

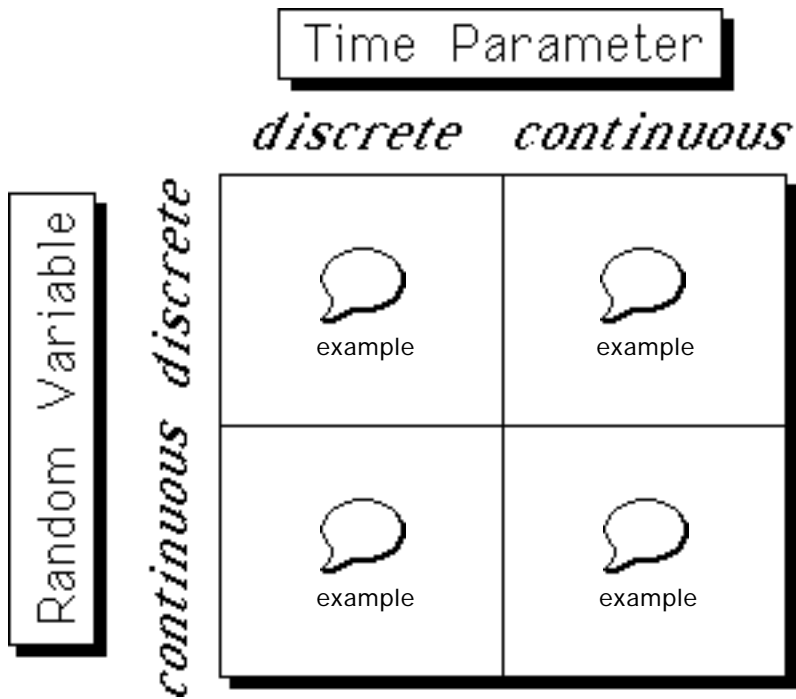
The process is classified as

*discrete-valued*

if the state space is a discrete set (e.g., the integers), and

*continuous-valued*

otherwise (e.g., if  $X_t$  may be any non-negative real number.)



## Common Stochastic Processes

- ☞ Discrete-time Markov Chains
- ☞ Continuous-time Markov Chains
- ☞ Bernoulli Process
- ☞ Poisson Process
- ☞ Birth-death Process

*Examples:***Discrete-parameter, discrete-valued process:**

Let the index set  $T$  refer to customer numbers,

$$T = \{1, 2, 3, \dots, n, \dots\}$$

and let the random variable  $X_n$  be the number of customers in the system when service is completed for the  $n^{\text{th}}$  customer.



## Continuous-parameter, discrete-valued process

Let the index set  $T$  refer to time (continuous)

$$T = [0, +\infty)$$

and let the random variable  $X_t$  be the number of customers in the system at time  $t$ .





## Discrete-parameter, continuous-valued process

Let the index set  $T$  refer to customer number,

$$T = \{1, 2, 3, \dots, n, \dots\}$$

and let the random variable  $X_n$  be the waiting time of the  $n^{\text{th}}$  customer prior to service, so that

$$X_n \in [0, +\infty)$$



## Continuous-parameter, continuous-valued process

Let the index set  $T$  refer to time (continuous), and let the random variable  $X_t$  be the amount of service (in minutes) which has been provided to the customer currently being served.

