Simplex Algorithm - a summary:

Notation:

$\exists$: *"there exists"*

N : set of *nonbasic* columns

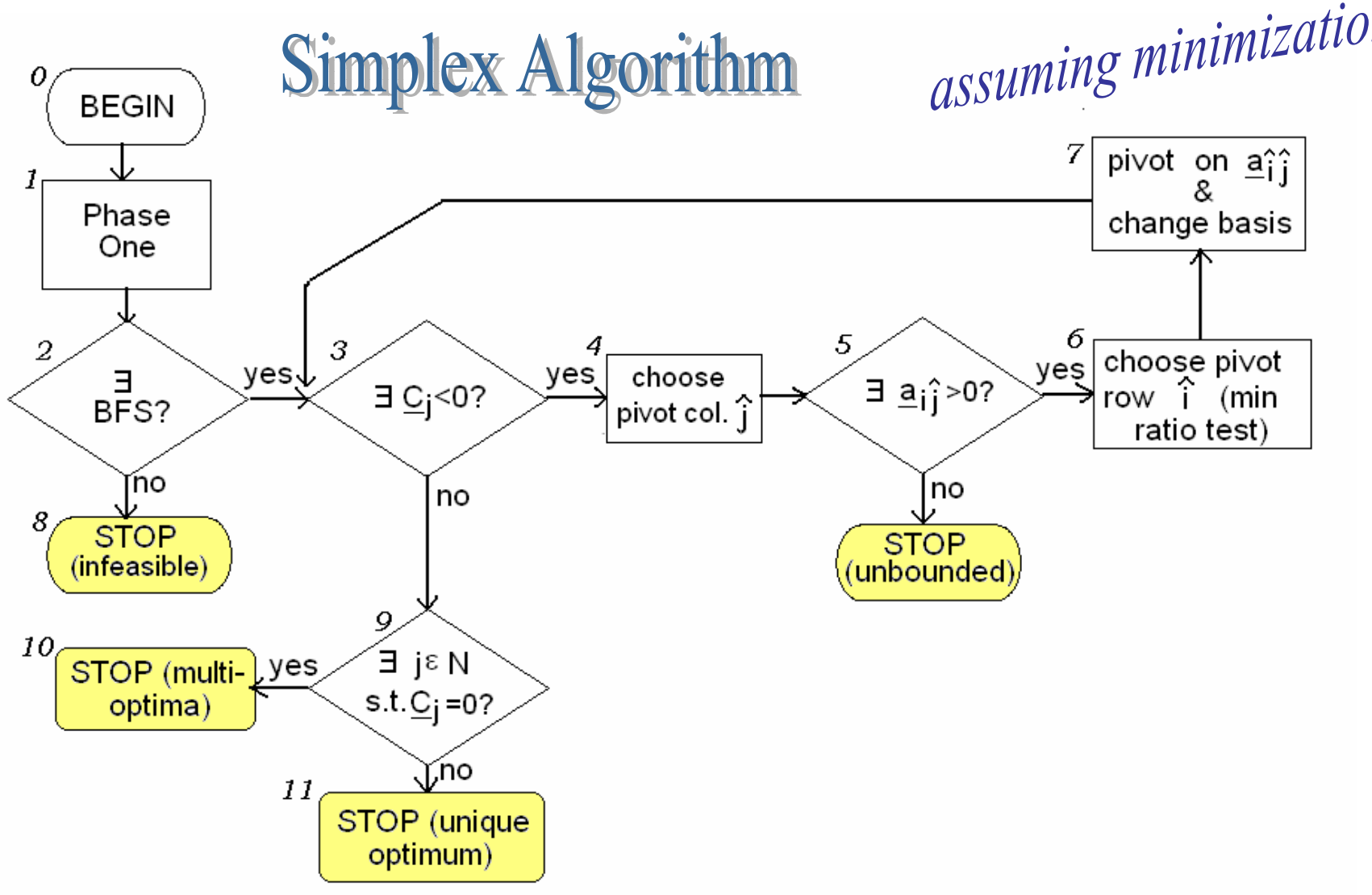
BFS: *"basic feasible solution"*

$\underline{C}_j$: *reduced cost* of column j

$\underline{a}_{ij}$: *substitution rate* of column j for basic variable in row i (*i.e.*, the current tableau entry in constraint row i, column j)

Simplex Algorithm

assuming minimization



Notes:

This flowchart assumes ***minimization*** in the optimality test (*box 3*).

1	Phase One refers to a procedure to identify an initial basic feasible solution *(BFS), a problem which is itself an LP.*
2	There exists a *BFS* if no artificial variable remains in the basis after Phase One.
3	***Optimality test:*** if we were maximizing, we would check whether there is a *positive* $\underline{C}_j$ **.**
4	*Any* column $\hat{j}$ such that $\underline{C}_j$ *<0* may be selected as pivot column (when minimizing).
5	Since the simplex pivot must be performed on a positive element of the tableau, we test whether such a positive element in column $\hat{j}$ exists.

6	If more than one $\underline{a}_{i\hat{j}}$ is positive in column $\hat{j}$, the minimum ratio test $$\min\left\{ \underline{b}_i \Big/ \underline{a}_{i\hat{j}} : \ \underline{a}_{i\hat{j}} > 0 \right\}$$ is used to choose the pivot row $\hat{i}$. (Ties may be broken arbitrarily, but will result in degeneracy at next tableau.)
7	**Pivot**, so that $\underline{a}_{\hat{i}\hat{j}}$ is replaced by 1.000 and the remainder of the column $\hat{j}$ is zero.
8	If Phase One cannot find a feasible solution, the simplex algorithm terminates.
9	The current basis is optimal. If one of the variables not in the basis has a zero reduced cost, then entering that variable into the basis will not cause the objective to change.

10	The simplex algorithm stops with a ***single optimal solution***.
11	The simplex algorithm stops; the current basic solution is ***optimal***, and at least one additional basis is also optimal.