Dual ascent example

Lagrangian Dual of SCP

(Solved via dual ascent)

Set(s) 23 24 25 removed from problem
(P = 16 17 22 > 14 = incumbent!)
# sets remaining is 22
Iteration # 1

Current multipliers:

\[
\begin{array}{c|cccccccc}
  i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
----------------------
w[i] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Solving Lagrangian relaxation:

*** Dual value is 0 ***
Sets in cover: #
Points not covered: 1 2 3 4 5 6 7 8

Heuristic solution cost is 12
*** New incumbent! *** Cover is 2 4 7
with cost 12

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Dual ascent step

Selecting multiplier \( w[8] = 0 \) with subgradient \( = 1 \)
Sets not in solution which cover #8 are:
\[
1 \ 6 \ 7 \ 10 \ 13 \ 14 \ 18 \ 20 \ 21 \ 22
\]

\( X[1] \) enters when \( w[8] \) is 2
\( X[6] \) enters when \( w[8] \) is 4

Updated \( w[8] = 2 = \max\{0, (1- \alpha)x2 + \alpha \times 4\} \) where \( \alpha = 0 \)
Anticipated improvement: 2

Point #8 is arbitrarily selected as one of the uncovered points.
Its multiplier should be increased to provide incentive for a set to cover it.

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Iteration # 2

Current multipliers:

\[
\begin{array}{c|cccccccc}
  i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  \hline
  w[i] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
\end{array}
\]

Solving Lagrangian relaxation:

*** Dual value is 2 ***
Sets in cover: # 1
Points not covered: 1 3 4 5 6 7

Set(s) 16 17 19 22 removed from problem
(P= 13 13 14 14 > 12 = incumbent!)
# sets remaining is 18
Dual ascent step

Selecting multiplier \( w[3] = 0 \) with subgradient = 1
Sets not in solution which cover #3 are: 2 3 4 7 14

\( X[2] \) enters when \( w[3] \) is 3
\( X[3] \) enters when \( w[3] \) is 3

Updated \( w[3] = 3 = \text{Max} \{ 0, (1-\alpha) \times 3 + \alpha \times 3 \} \) where \( \alpha = 0 \)
Anticipated improvement: 3

\[ \Phi \]

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Iteration # 3

Current multipliers:

<table>
<thead>
<tr>
<th>i</th>
<th>1 2 3 4 5 6 7 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>w[i]</td>
<td>0 0 3 0 0 0 0 2</td>
</tr>
</tbody>
</table>

Solving Lagrangian relaxation:

*** Dual value is 5 ***  (Improvement: 3)

Sets in cover: # 1 2
Points not covered: 5 6

Set(s) 18 20 21 removed from problem
(P = 14 15 15 > 12 = incumbent!)
# sets remaining is 15
Dual ascent step

Selecting multiplier \( w[6] = 0 \) with subgradient = 1

Sets not in solution which cover #6 are: 4 8 13 15

\( X[4] \) enters when \( w[6] \) is 1
\( X[8] \) enters when \( w[6] \) is 5

Updated \( w[6] = 1 = \max\{0, (1-\alpha) \times 1 + \alpha \times 5\} \) where \( \alpha = 0 \)

Anticipated improvement: 1

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Iteration # 4

Current multipliers:

<table>
<thead>
<tr>
<th>i</th>
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</tr>
</thead>
<tbody>
<tr>
<td>w[i]</td>
<td>0 0 3 0 0 1 0 2</td>
</tr>
</tbody>
</table>

Solving Lagrangian relaxation:

*** Dual value is 6 ***  (Improvement: 1)

Sets in cover: # 1 2 4
Points not covered: 5