

Operations Research – In-class activity (LP Sensitivity Analysis)

Recent federal regulations strongly encourage the assignment of students to schools in a city so that the racial composition of any school approximates the racial composition of the entire city. Consider the case of the Greenville city schools. The city can be considered as composed of five areas with the following characteristics:

Area	Percent minority	Number of students
1	20%	1200
2	10%	900
3	85%	1700
4	60%	2000
5	90%	2500

The ruling handed down for Greenville is that a school can have neither more than 75% nor less than 30% minority enrollment. There are three schools in Greenville with the following capacities:

<u>School</u>	<u>Capacity</u>
Bond	3900
Pocahontas	3100
Pierron	2100

The objective is to design an assignment of students to schools so as to stay within the capacity of each school and satisfy the composition constraints, while minimizing the total distance traveled by students (and therefore the average distance traveled by students). The distances in kilometers between areas and schools are:

<u>School</u>	<u>Area</u>				
	1	2	3	4	5
Bond	2.7	1.4	2.4	1.1	0.5
Pocahontas	0.5	0.7	2.9	0.8	1.9
Pierron	1.6	2.0	0.1	1.3	2.2

There is an additional condition that no student can be transported more than 2.6 kilometers. Find the number of students which should be assigned to each school from each area. Assume that any group of students from an area have the same ethnic mix as the whole area.

Define variables:

X_{ij} = number of students from area i assigned to school j
 where $i = 1, 2, 3, 4, 5$ and $j = 1, 2, 3$, except for the cases $i=1 \& j=1$ and $i=3 \& j=2$
 (because the distances traveled exceed the maximum allowed.)

LINDO output:

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MIN      1.4 X21 + 2.4 X31 + 1.1 X41 + 0.5 X51 + 0.5 X12
+ 0.7 X22 + 0.8 X42 + 1.9 X52 + 1.6 X13 + 2 X23 + 0.1 X33
+ 1.3 X43 + 2.2 X53
SUBJECT TO
2)      X12 + X13 =      1200      (all students from area 1 must be
                                     assigned to a school)
3)      X21 + X22 + X23 =      900      (all students from area 2 must be
                                     assigned to a school)
4)      X31 + X33 =      1700      (all students from area 3 must
                                     be assigned to a school)
5)      X41 + X42 + X43 =      2000      (all students from area 4 must be
                                     assigned to a school)
6)      X51 + X52 + X53 =      2500      (all students from area 5 must be
                                     assigned to a school)
7)      X21 + X31 + X41 + X51 <=  3900      (capacity of school 1)
8)      X12 + X22 + X42 + X52 <=  3100      (capacity of school 2)
9)      X13 + X23 + X33 + X43 + X53 <=  2100 (capacity of school 3)
10)     - 0.55 X21 + 0.1 X31 - 0.15 X41 + 0.15 X51 <=  0
11)     0.2 X21 - 0.55 X31 - 0.3 X41 - 0.6 X51 <=  0
12)     - 0.55 X12 - 0.65 X22 - 0.15 X42 + 0.15 X52 <=  0
13)     0.1 X12 + 0.2 X22 - 0.3 X42 - 0.6 X52 <=  0
14)     - 0.55 X13 - 0.65 X23 + 0.1 X33 - 0.15 X43 + 0.15 X53 <=  0
15)     0.1 X13 + 0.2 X23 - 0.55 X33 - 0.3 X43 - 0.6 X53 <=  0
END

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LP OPTIMUM FOUND AT STEP 12

OBJECTIVE FUNCTION VALUE

1) 5014.364

VARIABLE	VALUE	REDUCED COST
X21	542.727295	0.000000
X31	0.000000	2.212727
X41	148.181824	0.000000
X51	2500.000000	0.000000
X12	890.909119	0.000000
X22	357.272736	0.000000
X42	1851.818237	0.000000
X52	0.000000	1.460000
X13	309.090912	0.000000
X23	0.000000	0.032727
X33	1700.000000	0.000000
X43	0.000000	0.069091
X53	0.000000	1.830909

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.680000
3)	0.000000	-0.880000
4)	0.000000	-0.267273
5)	0.000000	-0.980000
6)	0.000000	-0.620000
7)	709.090881	0.000000
8)	0.000000	0.180000

9)	90.909088	0.000000
10)	0.000000	0.800000
11)	1435.909058	0.000000
12)	1000.000000	0.000000
13)	395.000000	0.000000
14)	0.000000	1.672727
15)	904.090881	0.000000

NO. ITERATIONS= 12

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
X21	1.400000	0.600000	0.316666
X31	2.400000	INFINITY	2.212727
X41	1.100000	0.073077	0.138462
X51	0.500000	1.460000	INFINITY
X12	0.500000	0.920000	0.027692
X22	0.700000	0.034615	0.600000
X42	0.800000	0.138462	0.400000
X52	1.900000	INFINITY	1.460000
X13	1.600000	0.027692	0.920000
X23	2.000000	INFINITY	0.032727
X33	0.100000	2.212727	INFINITY
X43	1.300000	INFINITY	0.069091
X53	2.200000	INFINITY	1.830909

ROW	CURRENT RHS	RIGHTHAND SIDE RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	1200.000000	709.090881	113.986023
3	900.000000	607.692322	113.986023
4	1700.000000	76.923080	1699.999878
5	2000.000000	709.090881	113.986023
6	2500.000000	493.939392	1809.090942
7	3900.000000	INFINITY	709.090881
8	3100.000000	113.986023	709.090881
9	2100.000000	INFINITY	90.909088
10	0.000000	271.363647	74.090912
11	0.000000	INFINITY	1435.909058
12	0.000000	INFINITY	1000.000000
13	0.000000	INFINITY	395.000000
14	0.000000	170.000000	49.999996
15	0.000000	INFINITY	904.090881

Solution:

School	Area					total
	1	2	3	4	5	
Bond	_____	_____	_____	_____	_____	_____
Pocahontas	_____	_____	_____	_____	_____	_____
Pierron	_____	_____	_____	_____	_____	_____

What is the average distance traveled by students, according to this plan? _____ km.

Suppose that a temporary classroom building are available which could be erected at any one of the school locations, which would increase the school's capacity by 100 students.

At which school should the building be erected? _____

How much decrease in the average distance traveled would result? _____ km.