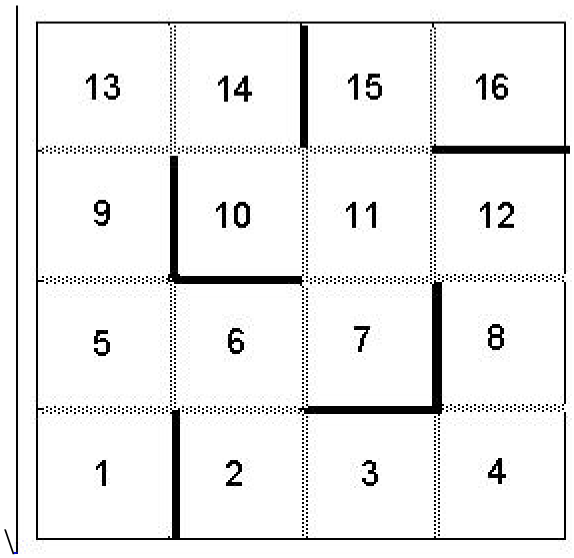
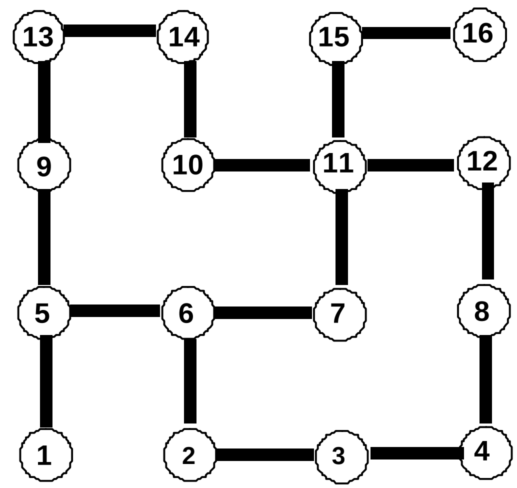
Rats!!

We wish to model the passage of a rat through a maze. Consider a maze in the form of a 4x4 array of boxes, such as the one below on the left:





The solid lines represent walls, the shaded lines represent doors. We will assume that a rat is placed into box #1. While in any box, the rat is assumed to be equally likely to choose each of the doors leaving the box (including the one by which he entered the box). For example, when in box #2 above, the probability of going next to boxes 3 and 6 are each $^1/_2$, regardless of the door by which he entered the box. *This assumption implies that no learning takes place if the rat tries the maze several times!*

a. On the diagram representing the Markov chain, write the transition probabilities on each transition in each direction.

b. Compute the steady-state distribution of the rat's location.

c. Which box will be visited most frequently by the rat?

d. Suppose that in box #16 a reward (e.g. food) is placed. What is the expected number of moves of the rat required to reach this reward?

e. Count the minimum number of moves (M) required to reach the reward. What is the probability that the rat reaches the reward in **exactly** this number of moves?

f. What is the probability that the rat reaches the reward with no more than 4 unnecessary moves?

g. Simulate five times the first 2M moves made by the rat. Did he reach the reward in any simulation?

h. Briefly discuss the utility of this model in testing a hypothesis that a *real* rat is able to learn, thereby finding the reward in relatively few moves after he has made several trial runs through the maze.

i. Briefly discuss how you might modify your Markov chain model if the rat will never exit a box by the same door through which he entered, unless he has reached a "dead end".

Transition Probabilities

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1)	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2)	0	0	0.5	0	0	0.5	0	0	0	0	0	0	0	0	0	0
3)	0	0.5	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0
4)	0	0	0.5	0	0	0	0	0.5	0	0	0	0	0	0	0	0
5)	0.333	0	0	0	0	0.333	0	0	0.333	0	0	0	0	0	0	0
6)	0	0.333	0	0	0.333	0	0.333	0	0	0	0	0	0	0	0	0
7)	0	0	0	0	0	0.5	0	0	0	0	0.5	0	0	0	0	0
8)	0	0	0	0.5	0	0	0	0	0	0	0	0.5	0	0	0	0
9)	0	0	0	0	0.5	0	0	0	0	0	0	0	0.5	0	0	0
10)	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0.5	0	0
11)	0	0	0	0	0	0	0.25	0	0	0.25	0	0.25	0	0	0.25	0
12)	0	0	0	0	0	0	0	0.5	0	0	0.5	0	0	0	0	0
13)	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0.5	0	0
14)	0	0	0	0	0	0	0	0	0	0.5	0	0	0.5	0	0	0
15)	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0.5
16)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

First-Passage Probabilities

Probability of Visit

n	$f_{1,16}$	$P\{X_n=16 \mid X_o=1\}$
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	0.00694	0.00694
7	0	0
8	0.0126	0.0161
9	0	0
10	0.0165	0.025
11	0	0
12	0.0189	0.0326
13	0	0
14	0.0203	0.0388
15	0	0
16	0.021	0.0437
17	0	0
18	0.0213	0.0474
19	0	0
20	0.0213	0.0503
21	0	0
22	0.0211	0.0524
23	0	0
24	0.0208	0.0541
25	0	0
26	0.0204	0.0553
27	0	0
28	0.02	0.0562
29	0	0
30	0.0196	0.0568
sum	0.241	0.536

Steadystate Probabilities

i	P{i}
1	0.0294
2	0.0588
3	0.0588
4	0.0588
5	0.0882
6	0.0882
7	0.0588
8	0.0588
9	0.0588
10	0.0588
11	0.118
12	0.0588
13	0.0588
14	0.0588
15	0.0588
16	0.0294

Results of five simulations of 30 moves:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	5	1	5	6	2	6	5	1	5	9	5	1	5	1	5	9	13	9	13	14	10	14	10	11	10	11	15	11	15	16
1	5	6	7	11	15	16	15	11	12	8	4	8	4	8	12	8	12	8	12	11	12	11	10	14	10	14	10	14	13	9
1	5	6	5	9	13	9	13	14	10	11	15	16	15	16	15	11	10	11	12	11	10	14	10	11	12	8	12	11	15	16
1	5	6	7	11	7	6	7	6	5	6	5	9	5	1	5	9	13	14	13	9	5	6	7	11	12	8	12	11	12	8
1	5	1	5	1	5	1	5	1	5	6	7	11	15	11	10	11	12	8	12	8	4	3	4	8	4	3	2	6	7	6

Mean First Passage Times

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1)	34	31	43.6	49.1	1	11.3	23.1	47.6	19.6	33	23.3	39	31.1	35.6	54.3	87.3
2)	59.7	17	18.3	29.5	26.7	8.67	20.5	33.6	39.6	36	20.7	30.7	45.5	44.3	51.7	84.7
3)	65.2	11.2	17	15.7	32.2	15.3	23.7	24.4	44	37	20.5	26	48.7	46.4	51.5	84.5
4)	68.7	20.4	13.7	17	35.7	20	25	13.2	46.4	36	18.4	19.3	50	46.5	49.4	82.4
5)	33	30	42.6	48.1	11.3	10.3	22.1	46.6	18.6	32	22.3	38	30.1	34.6	53.3	86.3
6)	52.1	20.8	34.5	41.2	19.1	11.3	15.2	40.8	33.2	33	18.8	33.3	40.2	40.1	49.8	82.8
7)	60.7	29.4	39.7	43	27.7	12	17	39.2	38.4	28	10.4	28.3	42	38.5	41.4	74.4
8)	70.3	27.6	25.5	16.3	37.3	22.7	24.3	17	46.8	33	14.3	10.7	49.3	44.7	45.3	78.3
9)	43.9	35.2	46.7	51.1	10.9	16.7	25.1	48.4	17	27	21.9	38.7	16.1	25.1	52.9	85.9
10)	64.5	38.8	46.9	47.9	31.5	23.7	21.9	41.8	34.2	17	8.47	28.7	29.9	18.5	39.5	72.5
11)	67.3	36	42.9	42.8	34.3	22	16.8	35.6	41.6	21	8.5	21.3	41.8	34.9	31	64
12)	69.8	32.8	35.2	30.5	36.8	23.3	21.5	18.8	45.2	28	8.13	17	46.5	40.8	39.1	72.1
13)	52.7	38.4	48.7	52	19.7	21	26	48.2	13.4	20	19.4	37.3	17	13.5	50.4	83.4
14)	59.6	39.6	48.8	50.9	26.6	23.3	24.9	46	24.8	11	14.9	34	15.9	17	45.9	78.9
15)	70.3	39	45.9	45.8	37.3	25	19.8	38.6	44.6	24	3	24.3	44.8	37.9	17	33
16)	71.3	40	46.9	46.8	38.3	26	20.8	39.6	45.6	25	4	25.3	45.8	38.9	1	34