

Developed by Frank Russell Company and The Yasuda Fire & Marine Insurance Company.

Decisions are made on how best to invest in assets to meet a random liability stream over time, with random investment returns.

Goal is to produce a high-income return to pay annual interest on savings-type insurance policies while maximizing the long-term wealth of the firm.

Handles complex regulations imposed by Japanese insurance laws and practices.

Model is multistage stochastic LP with recourse
Random Variables

\[ rp_{jt} = \text{price return of asset } j \text{ in period } t \]

\[ ri_{jt} = \text{income return of asset } j \text{ in period } t \]

\[ F_t = \text{deposit inflow in period } t \]

\[ P_t = \text{principal payout in period } t \]

\[ I_t = \text{income payout in period } t \]

\[ g_t = \text{rate of interest paid on policies in period } t \]

\[ L_t = \text{liability valuation at end of period } t \]

Decision Variables

\[ w_{jt} = \text{market value held in asset } j \text{ in period } t \]

\[ W_t = \text{total fund market value in period } t \]

\[ u_t = \text{income shortfall in period } t \]

\[ v_t = \text{income surplus in period } t \]
Objective:

Maximize \[ E\left[ W_H - \sum_{t=1}^{H} c_t(u_t) \right] \]

where \( c_t(u_t) \) is a piecewise-linear concave function which specifies the penalties for income shortfalls.

*(Converts to LP with introduction of additional variables.)*
**Constraints** include:

\[
\sum_{j=1}^{J} w_{jt} = W_t
\]

\[
W_{t+1} - \sum_{j=1}^{J} \left(1 + r p_{j,t+1} + r_i_{j,t+1}\right) w_{jt} = F_{t+1} - P_{t+1} - I_{t+1}
\]

\[
\sum_{j=1}^{J} r_i_{j,t+1} w_{jt} + u_{t+1} - v_{t+1} = g_{t+1} L_t
\]

\[
w_{jt} \geq 0, \quad u_t \geq 0, \quad v_t \geq 0
\]
Periods are of varying length:
  8 branches in period 1 (first quarter)
  4 branches in period 2 (remainder of first year)
  4 branches in period 3 (year 2)
  2 branches in period 4 (years 3-5)
  1 branch in period 6 (terminal conditions)

Total number of scenarios: 8×4×4×2×1=256
Size of Problem (# asset classes=7)

  # rows: 263
  # columns: 431

Size of Deterministic Equivalent Problem

  # rows ~ 31,000
  # columns ~ 44,000

Nested Benders' Decomposition was used.

Three hours of computation was required to solve full model.

Contributed 79 million US$ in first two years of use.