# PIVOTS

Suppose that –Z, $X_4$, $X_5$, & $X_6$ are basic in the current tableau:

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 3 | 0 | 0 | 0 | 0 |
| 0 | 9 | 8 | 7 | 1 | 0 | 0 | 12 |
| 0 | 5 | 7 | 4 | 0 | 1 | 0 | 5 |
| 0 | 10 | 4 | 3 | 0 | 0 | 1 | 8 |

If we pivot on "10" in row 4, column 2, the result is

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 9.2 | 2.4 | 0 | 0 | −0.2 | 1.6 |
| 0 | 0 | 4.4 | 4.3 | 1 | 0 | −0.9 | 4.8 |
| 0 | 0 | 5 | 2.5 | 0 | 1 | −0.5 | 1 |
| 0 | 1 | 0.4 | 0.3 | 0 | 0 | 0.1 | 0.8 |

As a result of this pivot, $X_6$ (which was previously basic in row 4) has become nonbasic, replaced by $X_1$.

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This pivot operation consisted of ***4 elementary row operations***:

1. Multiply row #4 by 0.1 to replace the pivot element by 1:

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 3 | 0 | 0 | 0 | 0 |
| 0 | 9 | 8 | 7 | 1 | 0 | 0 | 12 |
| 0 | 5 | 7 | 4 | 0 | 1 | 0 | 5 |
| 0 | 1 | 0.4 | 0.3 | 0 | 0 | 0.1 | 0.8 |

2. Subtract 5 times row 4 from row 3:

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 3 | 0 | 0 | 0 | 0 |
| 0 | 9 | 8 | 7 | 1 | 0 | 0 | 12 |
| 0 | 0 | 5 | 2.5 | 0 | 1 | -0.5 | 1 |
| 0 | 1 | 0.4 | 0.3 | 0 | 0 | 0.1 | 0.8 |

3. Subtract 9 times row 4 from row 2:

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 3 | 0 | 0 | 0 | 0 |
| 0 | 0 | 4.4 | 4.3 | 1 | 0 | -0.9 | 4.8 |
| 0 | 0 | 5 | 2.5 | 0 | 1 | -0.5 | 1 |
| 0 | 1 | 0.4 | 0.3 | 0 | 0 | 0.1 | 0.8 |

4. Subtract 2 times row 4 from row 1:

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 9.2 | 2.4 | 0 | 0 | −0.2 | −1.6 |
| 0 | 0 | 4.4 | 4.3 | 1 | 0 | −0.9 | 4.8 |
| 0 | 0 | 5 | 2.5 | 0 | 1 | −0.5 | 1 |
| 0 | 1 | 0.4 | 0.3 | 0 | 0 | 0.1 | 0.8 |

*After the first row operation, the sequence of the other three is arbitrary!*

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| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 3 | 0 | 0 | 0 | 0 |
| 0 | 9 | 8 | 7 | 1 | 0 | 0 | 12 |
| 0 | 5 | 7 | 4 | 0 | 1 | 0 | 5 |
| 0 | 10 | 4 | 3 | 0 | 0 | 1 | 8 |

Other sequences of elementary row operations can result in 1 in the pivot location and zero elsewhere in the column—they are NOT pivot operations!

Suppose we subtract 0.4 times row 3 from row 1:

| -Z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 7.2 | 1.4 | 0 | -0.4 | 0 | 2 |
| 0 | 9 | 8 | 7 | 1 | 0 | 0 | 12 |
| 0 | 5 | 7 | 4 | 0 | 1 | 0 | 5 |
| 0 | 1 | 0.4 | 0.3 | 0 | 0 | 0.1 | 0.8 |

This gives us the desired "0" in row 1 of the $X_1$ column… but notice what has happened to the column for $X_5$!