PAR, Inc.
golf bag manufacturing

Interpretation of the dual variables

using parametric analysis of right-hand-sides.

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Consider again the LP model of **PAR, Inc.**, which manufactures standard and deluxe golfbags:

X1 = number of **STANDARD** golf bags manufactured per quarter

X2 = number of **DELUXE** golf bags manufactured per quarter

Four operations are required, with the time per golf bag as follows:

OPERATION	STANDARD	DELUXE	Available
Cut-&-Dye	0.7 hr	1.0 hr	630 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1.0 hr	0.6666 hr	708 hrs.
Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$9	

```
MAX 10 X1 + 9 X2
ST
 0.7 X1 + X2 <= 630
 0.5 X1 + 0.866666 X2 <= 600
     X1 + 0.666666 X2 <= 708
 0.1 X1 + 0.25 X2 <= 135
END
```


```
        OBJECTIVE FUNCTION VALUE
        1)      7668.001

  VARIABLE        VALUE          REDUCED COST
        X1       540.000305          0.000000
        X2       251.999771          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          4.375009
        3)       111.600197          0.000000
        4)         0.000000          6.937494
        5)        18.000025          0.000000
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

	OBJ COEFFICIENT RANGES		
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	10.000000	3.500014	3.700000
X2	9.000000	5.285714	2.333340

	RIGHTHAND SIDE RANGES		
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	630.000000	52.363735	134.399994
3	600.000000	INFINITY	111.600197
4	708.000000	192.000000	128.000305
5	135.000000	INFINITY	18.000025

We wish to determine how sensitive the profit is to the number of hours available for the first operation (cutting-&-dyeing).

The current value is **630 hours** (right-hand-side of row #2).

We will change the right-hand-side to **650 hours**, and solve the LP again.

The solution is now

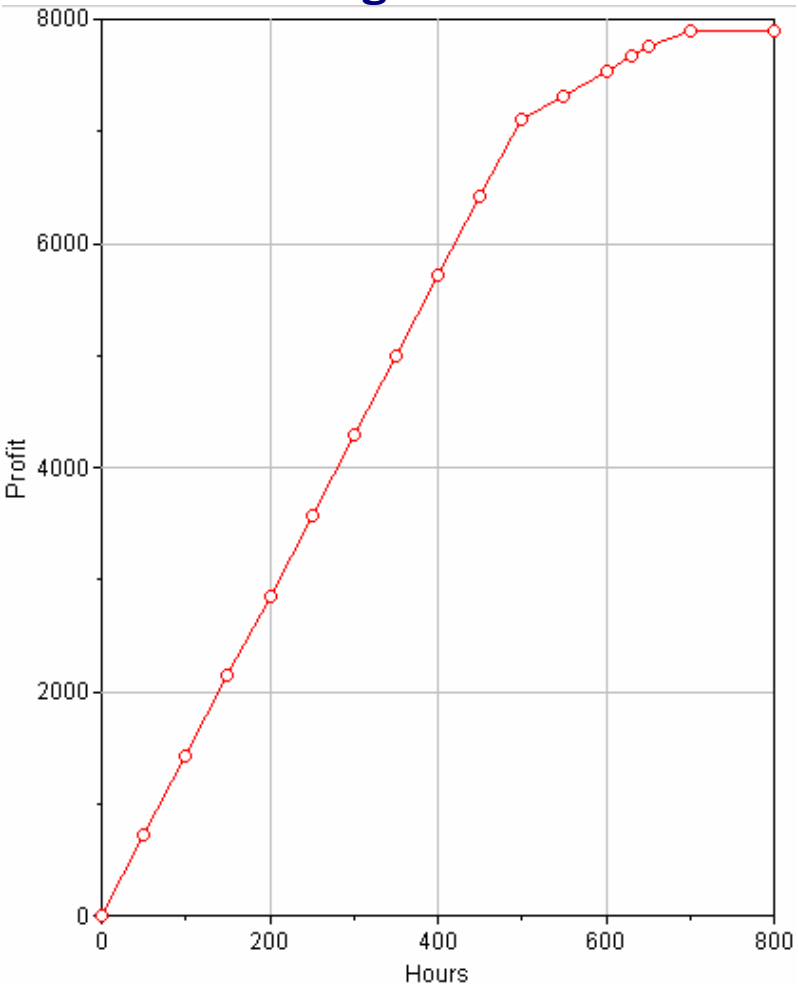
```
        OBJECTIVE FUNCTION VALUE
        1)      7755.501

  VARIABLE         VALUE           REDUCED COST
        X1       515.000366           0.000000
        X2       289.499725           0.000000
```

The profit increases to 7755.501, i.e., at the rate of

$$\frac{\Delta profit}{\Delta hours} = \frac{\$7755.501 - \$7668.001}{650\ hr - 630\ hr} = \frac{\$87.5}{20\ hr} = 4.375\ \$/hr$$

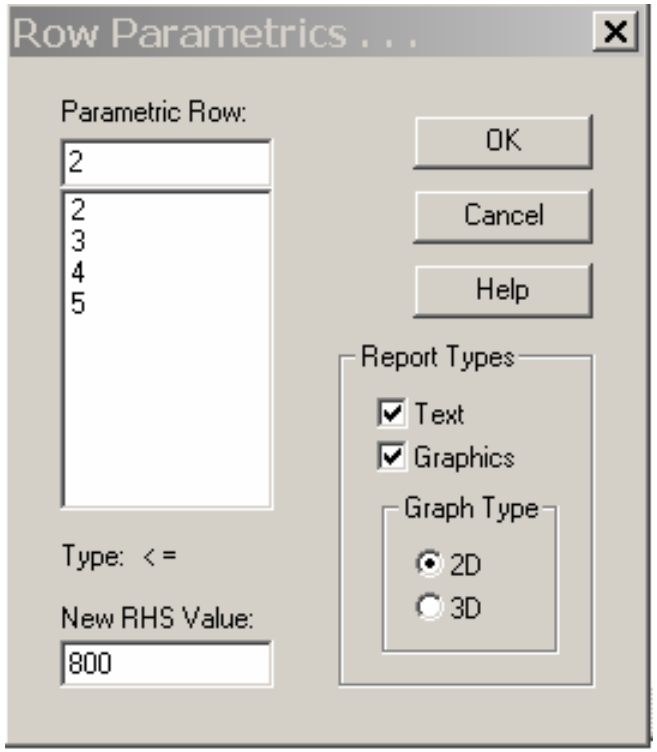
Available hrs in sew-&-dye	Maximum Profit ($)
0	0
50	714.2857
100	1428.571
150	2142.857
200	2857.143
250	3571.428
300	4285.714
350	5000.000
400	5714.286
450	6428.571
500	7099.250
550	7318.000
600	7536.751
630	7668.001
650	7755.501
700	7897.093
750	7897.093
800	7897.093

Results of solving LP for 18 RHSs:



Fortunately, LINDO gives us an easier way to do this!

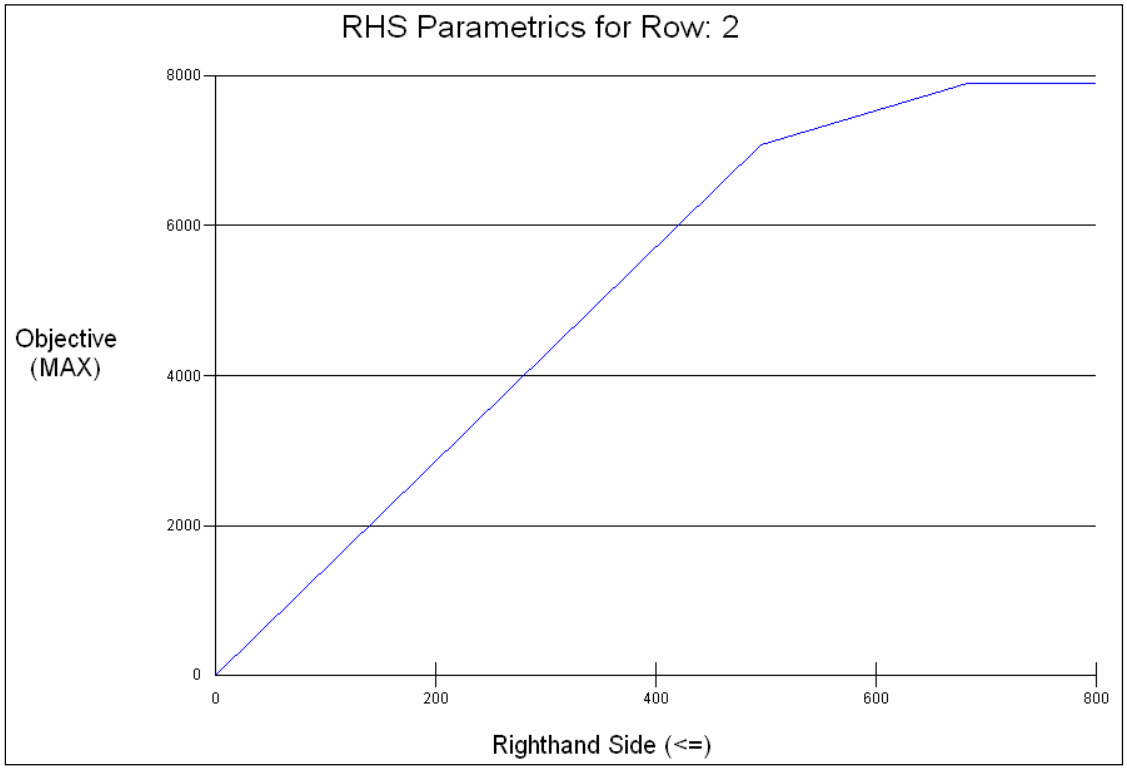
Parametric Analysis of RHS



The right-hand-side of the first constraint (row#2) is varied from zero to 800, while the others are *fixed at their original values*.

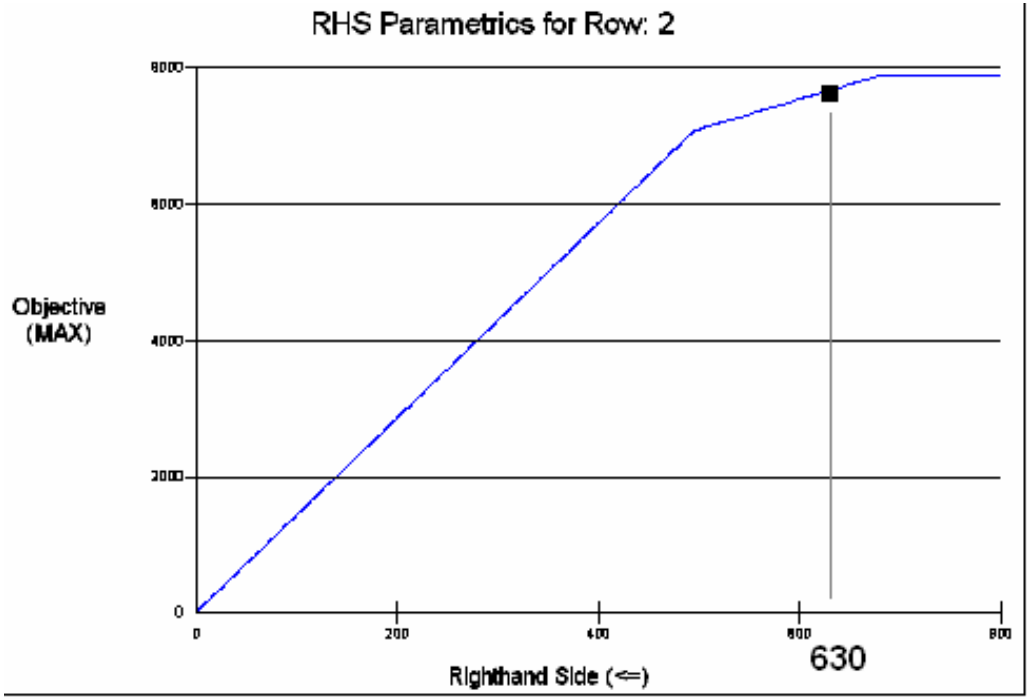
LINDO will give us the (piecewise-linear) plot of the *maximum profit* vs. the *RHS*.

(It is able to do this without solving the LP for every possible right-hand-side!)



RIGHTHANDSIDE PARAMETRICS REPORT FOR ROW: 2

VAR OUT		VAR IN		PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
					0.000	14.2857	0.00
SLK	4		X2	2	495.600	14.2857	7080.00
SLK	5	SLK	2	5	682.364	4.37501	7897.09
					800.000	0.00000	7897.09



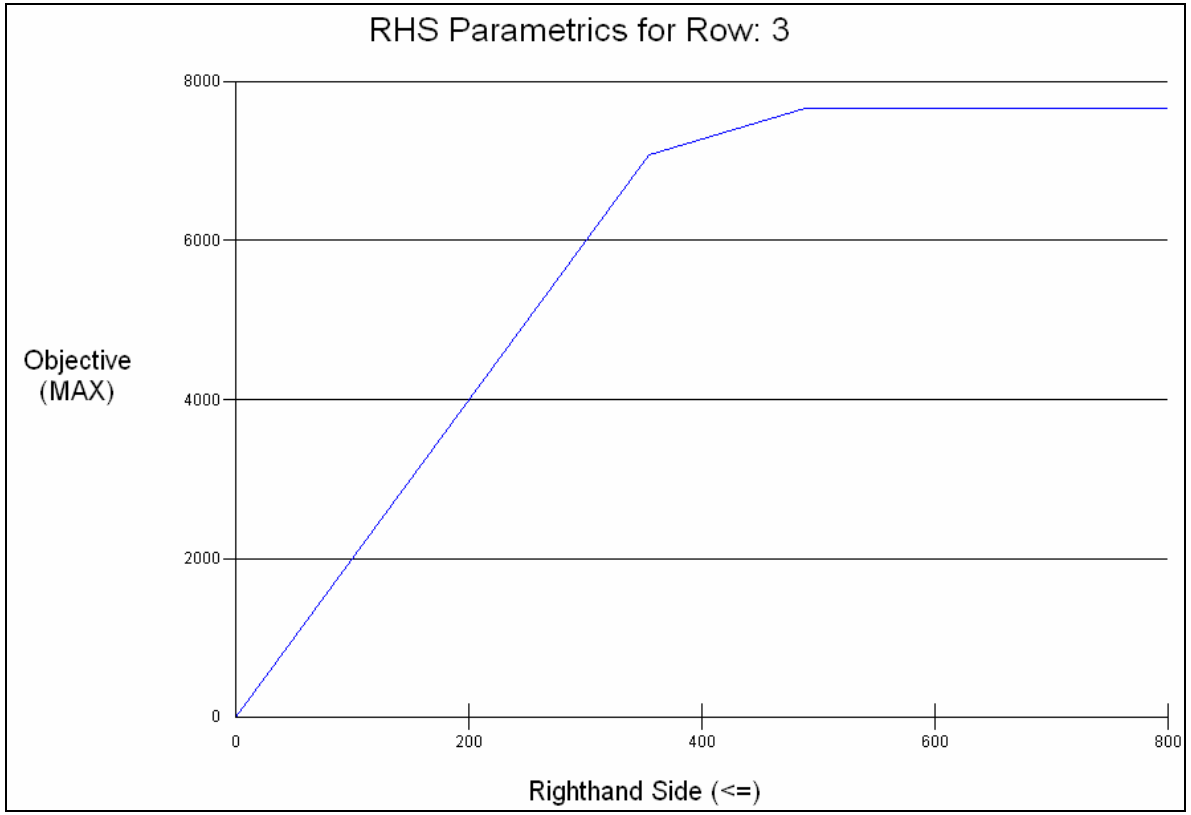
The graph consists of linear segments, i.e., it is ***piecewise-linear***.

Slope is \$4.375/hour at this point! This is reported as the "DUAL PRICE" by **LINDO**.

In addition, **LINDO** gives us the extent of the linear segment to the right & left:

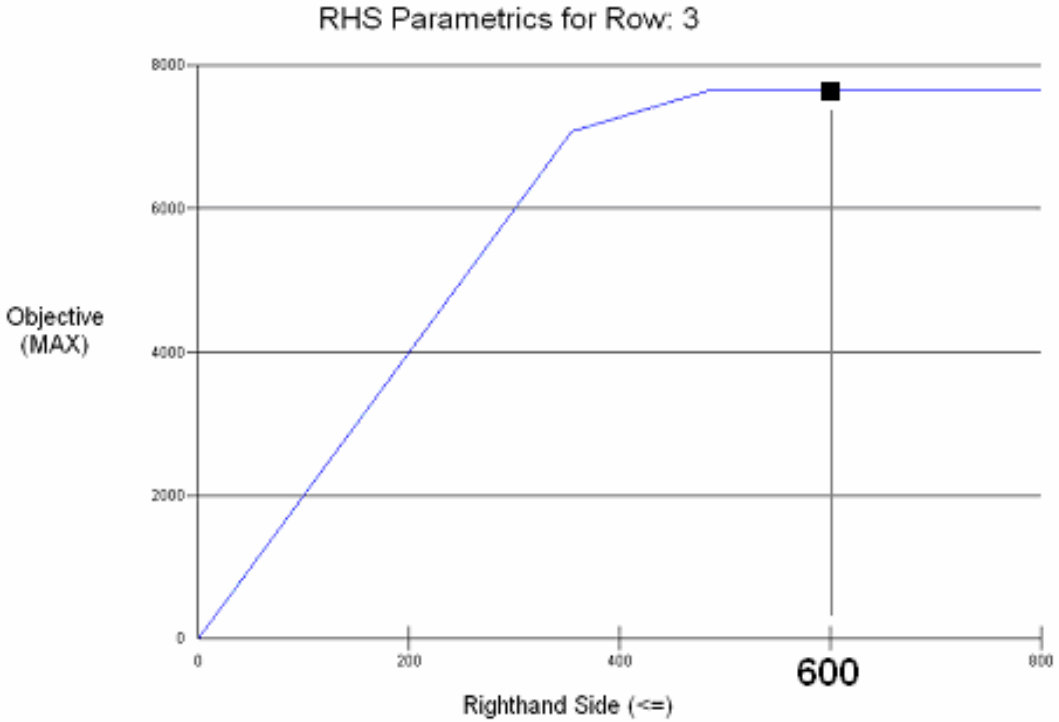
ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	**4.375009**

	RIGHTHAND SIDE RANGES		
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	630.000000	**52.363735**	**134.399994**



RIGHTHANDSIDE PARAMETRICS REPORT FOR ROW: 3

VAR OUT		VAR IN		PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
					0.000	20.0000	0.00
SLK	4		2	4	354.000	20.0000	7080.00
SLK	2	SLK	3	2	488.400	4.3750	7668.00
					800.000	0.0000	7668.00

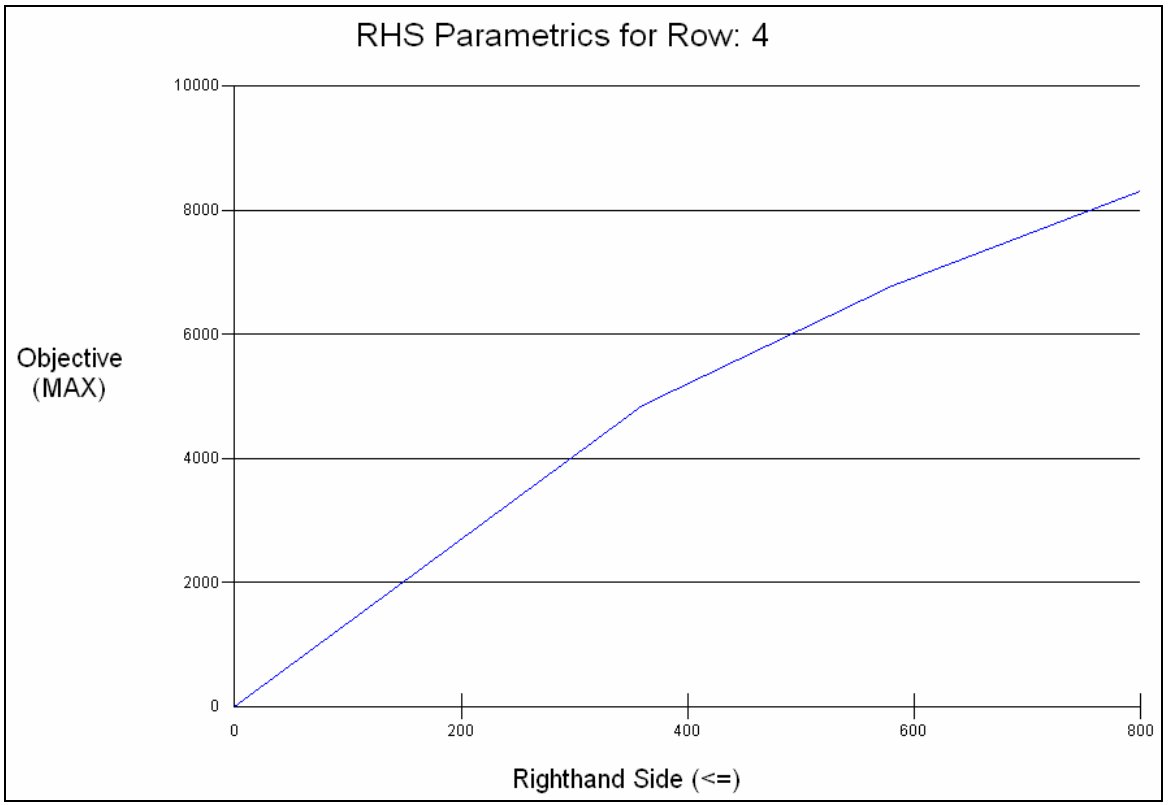


Slope is zero at this point!

Since there is slack (i.e., 111.6 unused hours) in row 3 of the optimal solution, any increase in available hours will gain us no additional profit!

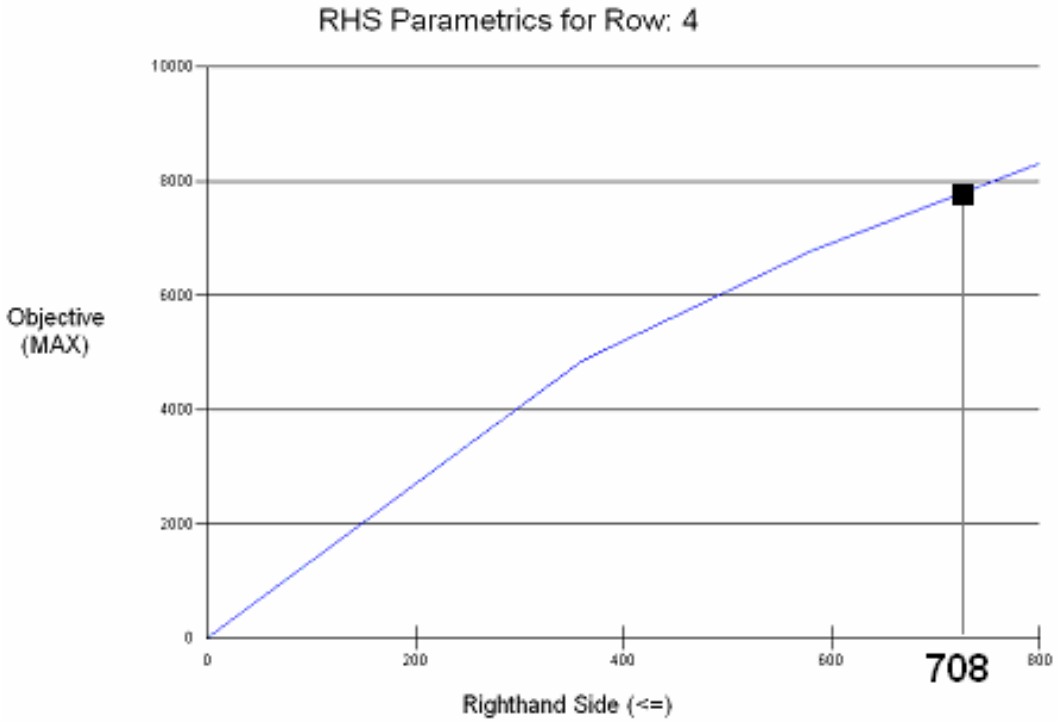
ROW	SLACK OR SURPLUS	DUAL PRICES
3)	**111.600197**	**0.000000**

	RIGHTHAND SIDE RANGES		
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
3	600.000000	**INFINITY**	**111.600197**



RIGHTHANDSIDE PARAMETRICS REPORT FOR ROW: 4

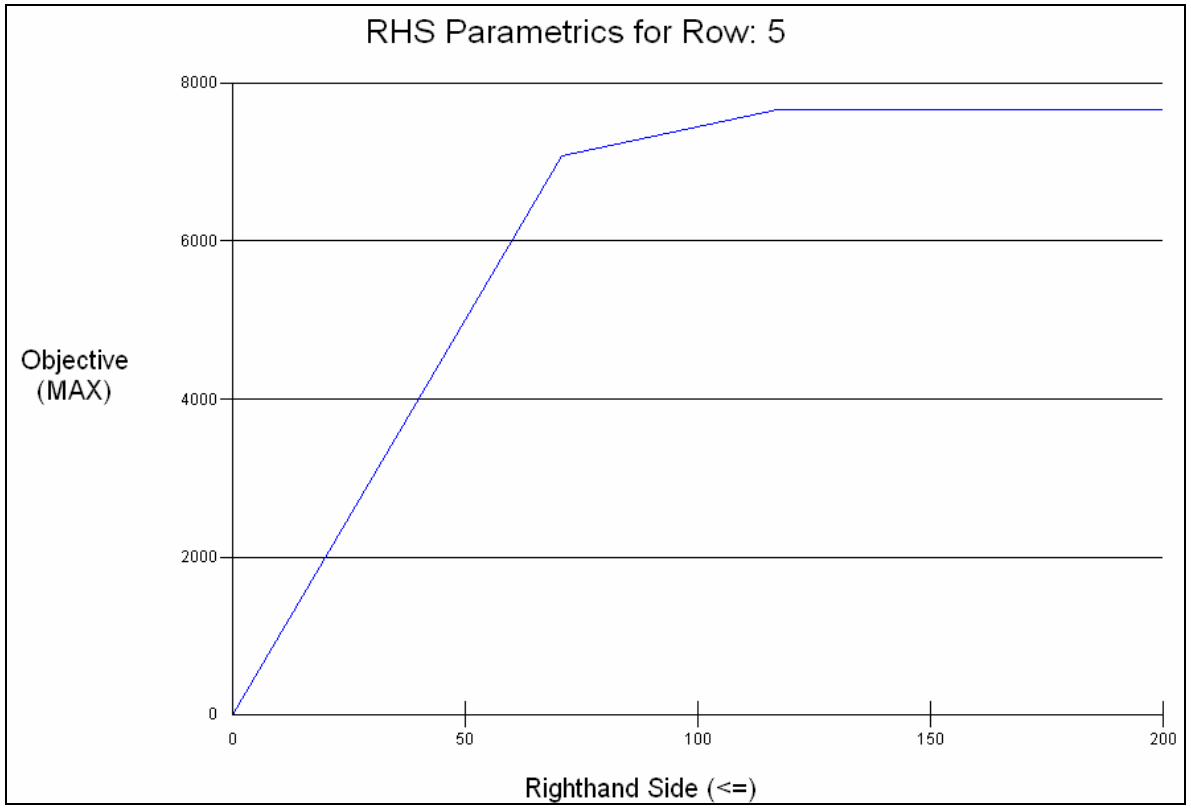
VAR OUT	VAR IN	PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
			0.000	13.5000	0.00
SLK 5	X1	5	360.000	13.5000	4860.00
SLK 2	SLK 5	2	580.000	8.72727	6780.00
			800.000	6.93749	8306.25



The slope is $6.937/hour at this point.

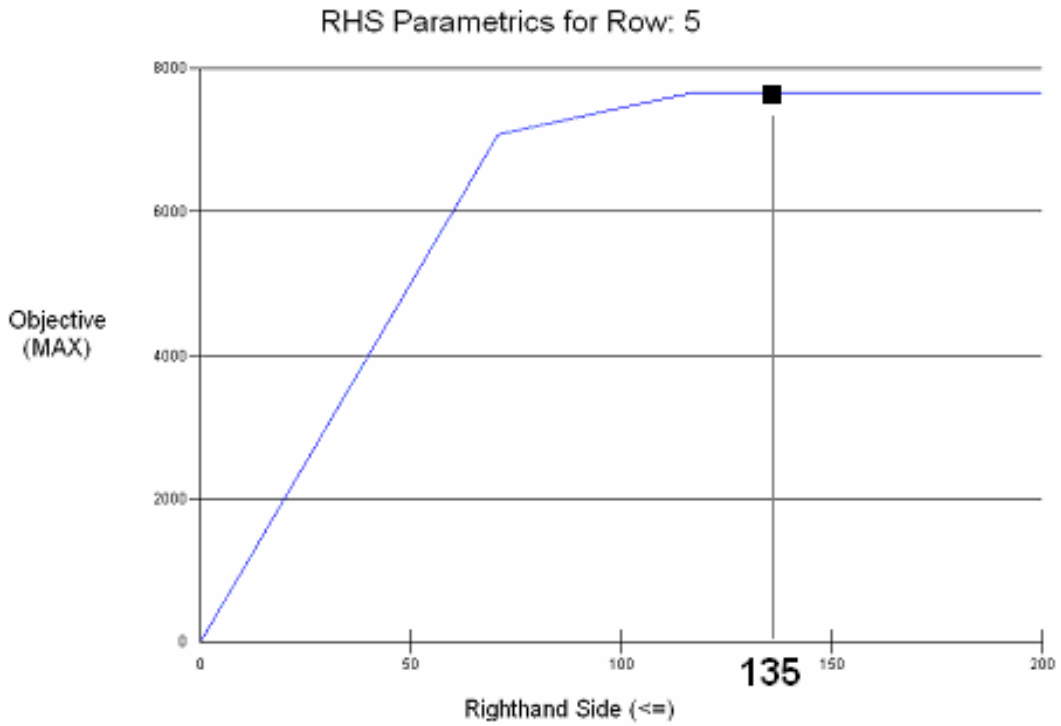
ROW	SLACK OR SURPLUS	DUAL PRICES
4)	0.000000	**6.937494**

	RIGHTHAND SIDE RANGES		
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
4	708.000000	**192.000000**	**128.000305**



RIGHTHANDSIDE PARAMETRICS REPORT FOR ROW: 5

VAR OUT	VAR IN	PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
			0.000	100.000	0.00
SLK 4	X2	4	70.8000	100.000	7080.00
SLK 2	SLK 5	2	117.000	12.727	7668.00
			200.000	0.000	7668.00



Slope is zero at this point!

ROW	SLACK OR SURPLUS	DUAL PRICES
5)	18.000025	**0.000000**

	RIGHTHAND SIDE RANGES		
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
5	135.000000	**INFINITY**	**18.000025**