Lot-Sizing Algorithms

In the classical EOQ model, it is assumed that demand is constant and known.

We now consider the case in which demand is known, but variable, i.e., it varies from one period to the next.

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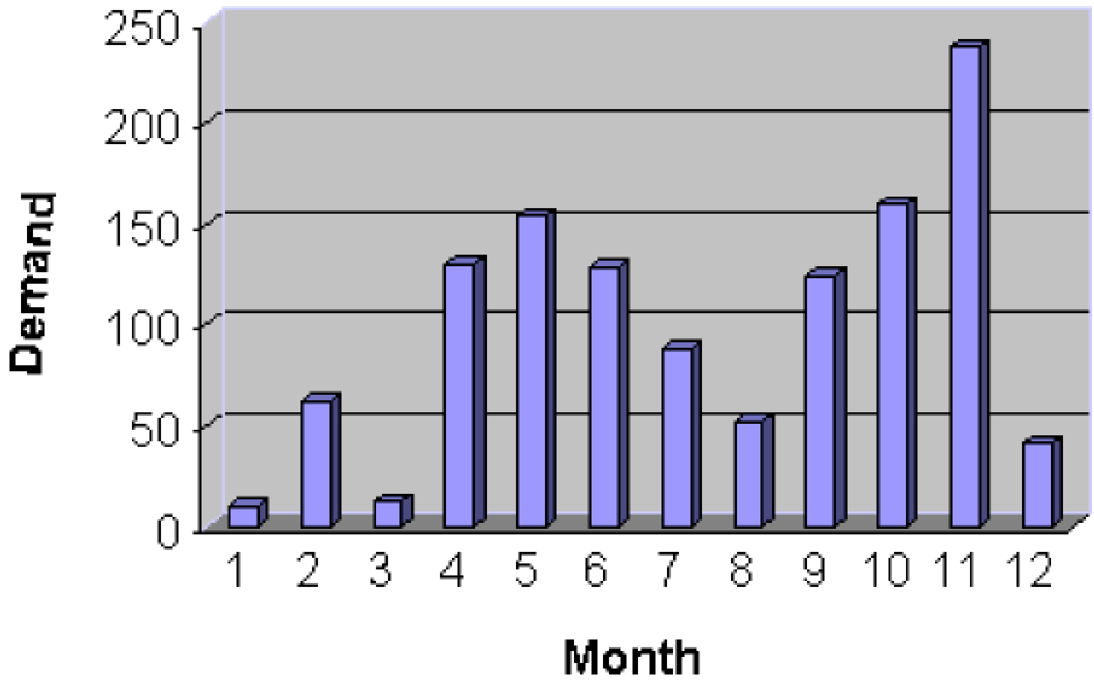
Example:

Period	Month	Requirement
1	January	10
2	February	62
3	March	12
4	April	130
5	May	154
6	June	129
7	July	88
8	August	52
9	September	124
10	October	160
11	November	238
12	December	41

Setup cost: $54 Carrying cost: $0.40/unit/month



Demand has two peaks:
one in late spring,
another in autumn.

This type of problem arises in several situations:

- **manufacturing operations with a schedule for completion of finished products, which are in turn "exploded" by a MRP system to production requirements in earlier periods.**
- **Production to fill contracts with customers.**
- **Items having a seasonal demand pattern.**
- **Replacement parts for a product being phased out.**
- **Parts for preventive maintenance where the maintenance schedule is known.**

We make the assumptions:

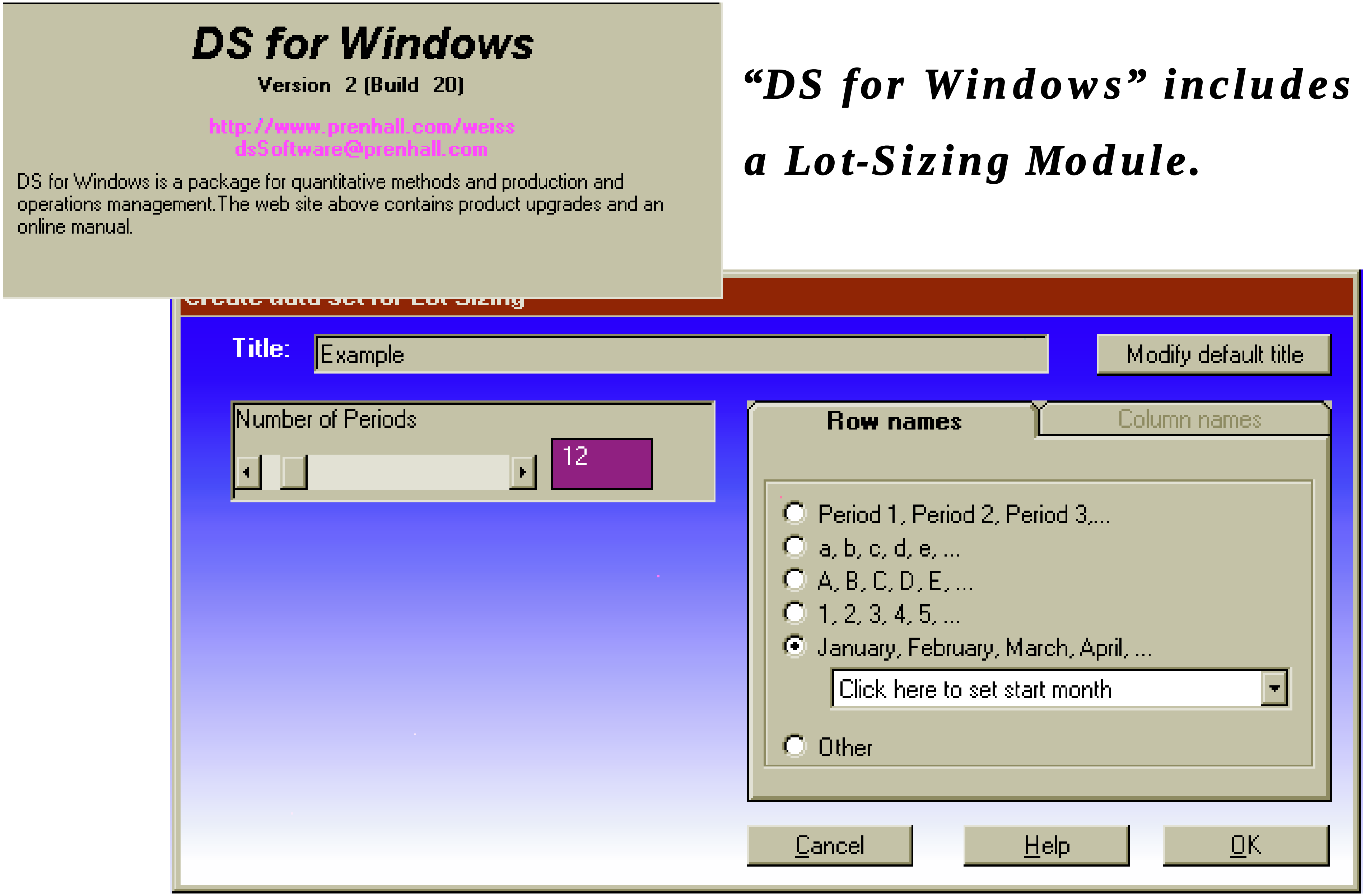
- no uncertainty in demand or lead times
- any replenishment must arrive at beginning of period
- no quantity discounts in price
- costs do not vary with time
- no shortages are allowed

Objective, as in EOQ model, is to minimize the sum of

- ordering costs
- inventory holding costs

(ignoring variable cost of production, since we are making the assumption that all demand is satisfied, which means this cost component will be the same for all lot-sizing decisions!)

“DS for Windows” includes a Lot-Sizing Module.



Data Entry:

Method: Lot for lot

Instruction: Choose the method that you wish to use by clicking on it.

Example

Period	Demand		Parameter	Value
January	10		Holding Cost	0.4
February	62		Setup Cost	54.
March	12		Stockout cost	99,999.
April	130		Initial Inventory	0.
May	154		Lead time	0.
June	129			
July	88			
August	52			
September	124			
October	160			
November	238			
December	41			

Stockout cost was assigned an arbitrary large value to avoid any stockouts!

“Lot-for-Lot” algorithm:

Produce each period a lot to satisfy *only* that period’s demand.

Period	Demand	Order receipt	Inventory	Holding Cost $.40	Setup Cost $54.00
Initial Inventory			0.		
January	10.	10.	0.		54.
February	62.	62.	0.		54.
March	12.	12.	0.		54.
April	130.	130.	0.		54.
May	154.	154.	0.		54.
June	129.	129.	0.		54.
July	88.	88.	0.		54.
August	52.	52.	0.		54.
September	124.	124.	0.		54.
October	160.	160.	0.		54.
November	238.	238.	0.		54.
December	41.	41.	0.		54.
Totals	1,200.	1,200.		0.	648.
Average demand	100.				
Total cost =	648.				

The simplest approach, and the approach generally used by MRP.

Use of the EOQ model

One simple approach would be to compute the EOQ formula

$$Q^* = \sqrt{\frac{2AD}{h}}$$

where the *average* demand is used for $\boldsymbol{D}$.

Lots of size Q* are produced or ordered.

Example:

- Average demand = **D** = 100 / month.
- Ordering cost = **A** = $54
- holding cost = **h** = $0.40 / month

(Caution: express D and h using same time units!)

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 54 \times 100}{0.40}} = \sqrt{27000} = 164.3$$

which we round to **164**.

When the inventory is not sufficient to satisfy the next month's demand, an order of size **164** is generated.

EOQ algorithm

Period	Demand	Order receipt	Inventory	Holding Cost $.40	Setup Cost $54.00
Initial Inventory			0.		
January	10.	164.	154.	61.6	54.
February	62.		92.	36.8	
March	12.		80.	32.	
April	130.	164.	114.	45.6	54.
May	154.	164.	124.	49.6	54.
June	129.	164.	159.	63.6	54.
July	88.		71.	28.4	
August	52.		19.	7.6	
September	124.	164.	59.	23.6	54.
October	160.	164.	63.	25.2	54.
November	238.	328.	153.	61.2	54.
December	41.		112.	44.8	
Totals	1,200.	1,312.		480.	378.
Average demand	100.	EOQ =	164.		
Total cost =	858.				

It can easily be shown that the best solutions to the problem are such that:

a lot arrives only when we run out of inventory at the end of the previous period.

That is, at the beginning of a period,

- **either** a lot arrives
- **or** there is inventory remaining from the previous period,

but ***never both!***

The Fixed EOQ method just presented does <u>*not*</u> *satisfy this property…*

Period Order Quantity (POQ)

The EOQ is expressed as a “time supply”, that is, an integer number of periods’ supply.

Example

Divide the **EOQ** just computed by the average demand to obtain the average number of periods per order, and round to the nearest integer (but greater than zero!)

Example: The EOQ computed earlier is **164**. Since the average demand is 100/month, this quantity satisfies demand for an average of **1.64 months**, which is then rounded to **2 months**.

A **two-months supply** is then always ordered. (The quantity varies as the demand varies.)

Period	Demand	Order receipt	Inventory	Holding Cost $.40	Setup Cost $54.00
Initial Inventory			0.		
January	10.	72.	62.	24.8	54.
February	62.		0.		
March	12.	142.	130.	52.	54.
April	130.		0.		
May	154.	283.	129.	51.6	54.
June	129.		0.		
July	88.	140.	52.	20.8	54.
August	52.		0.		
September	124.	284.	160.	64.	54.
October	160.		0.		
November	238.	279.	41.	16.4	54.
December	41.		0.		
Totals	1,200.	1,200.		229.6	324.
Average demand	100.	EOQ =	164.		
Total cost =	553.6	POQ =	2.		

POQ

The Total annual cost is ***$553.60****, compared to* ***$858*** *using the fixed EOQ method, a reduction of over* ***$304****,* or **35.5%**!

Part Period Balancing (PPB)

This method is based upon a property of the classical EOQ:

For the optimal Q*,

the annual holding cost = annual ordering cost

The number of periods covered by the replenishment is therefore made so that these two costs are as close as possible (subject to the restriction that we order an integer number of periods' supply.)

Example

PPB

- Average demand = **D** = 100 / month.
- Ordering cost = **A** = $54
- holding cost = **h** = $0.40 / month

Consider the order to be placed in **January**:

If we order a **1-month supply** (10), there will be **$0** holding cost, since there is no end-of-month inventory.

If we order a **2-month supply** (10+62), we will have 62 in inventory at the end of the first month, for which we pay $1\times\$0.40\times 62$ = **$24.80** holding cost. (**<** $54 = A)

If we order a **3-month supply** (10+62+12), we will have to carry an additional 12 units for 2 months, at a cost of $2\times\$0.40\times 12=\$9.60$, a total of $0 + $24.80 + $9.60 = **$34.40** holding cost (**<** $54=A)

If we order a **4-month supply** (10+62+12+130), we will have to carry an additional 130 units for 3 months, at a cost of $3\times\$0.40\times 130= \$156$, a total of $0 + $24.80 + $9.60 + $156 = **$190.40** holding cost **(> $54=A)**

PPB

Period	Demand	Carrying cost	Cumulative Carrying cost
January	10	0	0
February	62	1×\$0.40×62=\$24.80	\$24.80
March	12	2×\$0.40×12=\$9.60	\$34.40
April	130	3×\$0.40×130=\$156.00	\$190.40

Since \$54 is nearer the **3-months supply**, we order

10+62+12 = **84** in January.

PPB

Now consider the order to be placed in **April**, when inventory again reaches zero:

Period	**Demand**	**Carrying cost**	**Cumulative Carrying cost**
April	130	0	0
May	154	1×\$0.40×154=\$61.60	\$61.60
June	129	2×\$0.40×129=\$103.20	\$164.80

The order in **April** should be a **2-month supply, *i.e.*,** 130+154 = **284**, for April & May.

Etc.

PPB

Period	Demand	Order receipt	Inventory	Holding Cost $.40	Setup Cost $54.00
Initial Inventory			0.		
January	10.	84.	74.	29.6	54.
February	62.		12.	4.8	
March	12.		0.		
April	130.	284.	154.	61.6	54.
May	154.		0.		
June	129.	217.	88.	35.2	54.
July	88.		0.		
August	52.	176.	124.	49.6	54.
September	124.		0.		
October	160.	398.	238.	95.2	54.
November	238.		0.		
December	41.	41.	0.		54.
Totals	1,200.	1,200.		276.	324.
Average demand	100.				
Total cost =	600.				

In this example, the total annual cost is **$600**, more than the **POQ** cost ($553.60) but less than the **EOQ** cost ($800).

Wagner-Whitin Algorithm

W-W

The previous algorithms are ***heuristic*** in nature, and will not guarantee the minimum-cost production schedule.

The Wagner-Whitin (W-W) algorithm, on the other hand, does guarantee the optimal (minimum-cost) solution.

W-W belongs to a class of methods referred to as ***"dynamic programming"***, which can be computationally expensive, compared to the heuristic methods.

Period	Demand	Order receipt	Inventory	Holding Cost $.40	Setup Cost $54.00
Initial Inventory			0.		
January	10.	84.	74.	29.6	54.
February	62.		12.	4.8	
March	12.		0.		
April	130.	130.	0.		54.
May	154.	283.	129.	51.6	54.
June	129.		0.		
July	88.	140.	52.	20.8	54.
August	52.		0.		
September	124.	124.	0.		54.
October	160.	160.	0.		54.
November	238.	279.	41.	16.4	54.
December	41.		0.		
Totals	1,200.	1,200.		123.2	378.
Average demand	100.				
Total cost =	501.2				

*Other heuristic methods have been suggested, for example the **Silver-Meal** algorithm.*

Comparison of results:

Method	Cost of solution found
EOQ (economic order quantity)	\$800.00
POQ (period order quantity)	\$553.60
PPB (part period balancing)	\$600.00
W-W (Wagner-Whitin)	\$501.20