LP

Model Formulation

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author

- Sunco Oil Refinery
- Shoemakers of America
- Paper Recycling
- Boilen Oil Company
- Product Mix

SunCo processes oil into aviation fuel and heating oil.

It costs $40 to purchase each barrel of oil, which is then distilled and yields 0.5 barrel of aviation fuel and 0.5 barrel of heating oil.

Output from the distillation may be sold directly or processed in the catalytic cracker.

If sold after distillation without further processing, aviation fuel sells for $60/barrel and heating oil for $40/barrel

SUNCO

It takes 1 hour to process 1000 barrels of aviation fuel in the catalytic cracker, and these can be sold for $130/barrel.

It takes 45 minutes to process 1000 barrels of heating oil in the cracker, and these can be sold for $90/barrel.

Each day, at most 20,000 barrels of oil can be purchased, and 8 hours of catalytic cracker time are available.

Formulate an LP to maximize Sunco's profits.

SUNCO

Decision Variables

OIL = # of barrels of oil purchased

HSOLD = # of barrels of heating oil sold

HCRACK = # of barrels of heating oil processed in catalytic cracker

ASOLD = # of barrels of aviation fuel sold

ACRACK = # of barrels of aviation fuel processed in catalytic cracker

SUNCO

Maximize 40 HSOLD + 90 HCRACK + 60 ASOLD + 130 ACRACK − 40 OIL

subject to

$$OIL \le 20000$$

available supply

$$0.5\ OIL = ASOLD + ACRACK$$

$$0.5\ OIL = HSOLD + HCRACK$$

aviation fuel & heating oil each constitute 50% of the output of the distillation

$$0.001 ACRACK + 0.00075\ HCRACK \le 8$$

available time for cracker

$$OIL \ge 0,\ ASOLD \ge 0,\ ACRACK \ge 0,\ HSOLD \ge 0,\ HCRACK \ge 0$$

SUNCO

```
MAX     40 HSOLD + 90 HCRACK + 60 ASOLD
          + 130 ACRACK - 40 OIL
SUBJECT TO
      2)  OIL <=  20000
      3) - ASOLD - ACRACK + 0.5 OIL =   0
      4) - HSOLD - HCRACK + 0.5 OIL =   0
      5)  0.00075 HCRACK + 0.001 ACRACK <=  8
END
LP OPTIMUM FOUND

      OBJECTIVE FUNCTION VALUE

           1)   760000.000
```

SUNCO

VARIABLE	VALUE	REDUCED COST
HSOLD	10000.000000	.000000
HCRACK	.000000	2.500000
ASOLD	2000.000300	.000000
ACRACK	8000.000000	.000000
OIL	20000.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	10.000000
3)	.000000	-60.000000
4)	.000000	-40.000000
5)	.000000	70000.000000

SUNCO

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
HSOLD	40.000000	INFINITY	2.500000
HCRACK	90.000000	2.500000	INFINITY
ASOLD	60.000000	3.333333	20.000000
ACRACK	130.000000	INFINITY	3.333333
OIL	-40.000000	INFINITY	10.000000

SUNCO

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	20000.0000	INFINITY	4000.0010
3	.0000	2000.0003	INFINITY
4	.0000	10000.0000	INFINITY
5	8.0000	2.0000	8.0000

SUNCO

ROW	(BASIS)	HSOLD	HCRACK	ASOLD	ACRACK	OIL	SLK 2
1	ART	.000	2.500	.000	.000	.000	10.000
2	OIL	.000	.000	.000	.000	1.000	1.000
3	ASOLD	.000	-.750	1.000	.000	.000	.500
4	HSOLD	1.000	1.000	.000	.000	.000	.500
5	ACRACK	.000	.750	.000	1.000	.000	.000

ROW	SLK 5	
1	0.70E+05	0.76E+06
2	.000	20000.000
3	-1000.000	2000.000
4	.000	10000.000
5	1000.000	8000.000

TABLEAU

SUNCO

Shoemakers of America forecasts the following demand for each of the next six months:

Month t	Demand D_t (pairs)
1	5000
2	6000
3	5000
4	9000
5	6000
6	5000

(Problem #18 of Chapter 4 Review Problems, page 184 of text by Winston.)

It takes a shoemaker 15 minutes to produce a pair of shoes.

Each shoemaker works 150 hours/month, plus up to 40 hours/month overtime.

A shoemaker is paid a regular salary of $2000/month, plus $50/hour for overtime.

At the beginning of each month, Shoemakers can either hire or fire workers.

It costs the company $1500 to hire a worker and $1900 to fire a worker.

The monthly holding cost per pair of shoes is 3% of the cost of producing a pair of shoes with regular-time labor.

(The raw materials in a pair of shoes cost $20.)

At the beginning of month 1, Shoemakers has 13 workers.

Formulate an LP that minimizes the cost of meeting (on time) the demands of the next six months.

Hints: Assume that, even though a shoemaker is paid for 150 hours of work per month, he may be idle part of the time if he is required in a later month, in order that the company avoid firing and hiring costs. You may find it useful to define the following decision variables for each month t, t=1,2,... 6:

Wt = # of shoemakers in the work force during month t
Ht= # of shoemakers hired at the beginning of month t
Ft= # of shoemakers fired at the beginning of month t
Rt= # of pairs of shoes produced during regular time in month t
Ot= # of pairs of shoes produced during overtime in month t
It= # of pairs of shoes in inventory at the end of month t

```
MIN  2000 W1 +2000 W2 +2000 W3 +2000 W4 +2000 W5 +2000 W6
   + 1500 H1 +1500 H2 +1500 H3 +1500 H4 +1500 H5 +1500 H6
   + 1900 F1 +1900 F2 +1900 F3 +1900 F4 +1900 F5 +1900 F6
   + 0.7 I1  +0.7 I2  +0.7 I3  +0.7 I4  +0.7 I5  +0.7 I6
   + 20 R1   +20 R2   +20 R3   +20 R4   +20 R5   +20 R6
   + 32.5 O1 +32.5 O2 +32.5 O3 +32.5 O4 +32.5 O5 +32.5 O6
```

```
 2)      - I1 + R1 + O1 =   5000
 3)   I1 - I2 + R2 + O2 =   6000
 4)   I2 - I3 + R3 + O3 =   5000
 5)   I3 - I4 + R4 + O4 =   9000
 6)   I4 - I5 + R5 + O5 =   6000
 7)   I5      + R6 + O6 =   5000
 8)        W1 - H1 + F1 =   13
 9) - W1 + W2 - H2 + F2 =   0
10) - W2 + W3 - H3 + F3 =   0
11) - W3 + W4 - H4 + F4 =   0
12) - W4 + W5 - H5 + F5 =   0
13) - W5 + W6 - H6 + F6 =   0
```

```
14) - 600 W1 + R1 <=    0
15) - 600 W2 + R2 <=    0
16) - 600 W3 + R3 <=    0
17) - 600 W4 + R4 <=    0
18) - 600 W5 + R5 <=    0
19) - 600 W6 + R6 <=    0
20) - 160 W1 + O1 <=    0
21) - 160 W2 + O2 <=    0
22) - 160 W3 + O3 <=    0
23) - 160 W4 + O4 <=    0
24) - 160 W5 + O5 <=    0
25) - 160 W6 + O6 <=    0
```

END

```
LP OPTIMUM FOUND AT STEP      29

        OBJECTIVE FUNCTION VALUE

        1)     852716.620
```

VARIABLE	VALUE	REDUCED COST
W1	10.416670	.000000
W2	10.416670	.000000
W3	10.416670	.000000
W4	10.416670	.000000
W5	10.000000	.000000
W6	8.333333	.000000
F1	2.583333	.000000
F5	.416667	.000000
F6	1.666667	.000000
I1	1250.000000	.000000
I2	1500.000000	.000000
I3	2750.000000	.000000
R1	6250.000000	.000000
R2	6250.000000	.000000
R3	6250.000000	.000000
R4	6250.000000	.000000
R5	6000.000000	.000000
R6	5000.000000	.000000

The LP solution, unfortunately, is not integer!

(The number of persons hired &/or fired each month is noninteger!)

If we add the integer restrictions, and re-solve, using LINDO, the result is:

```
OBJECTIVE FUNCTION VALUE

 1)     854080.000
```

Note: this requires branch-&-bound, with solutin of many LPs & much more computatinal effort!

VARIABLE	VALUE	REDUCED COST
F1	2.000000	-7579.997000
F3	1.000000	-3999.998000
F6	1.000000	-100.000000
W1	11.000000	.000000
W2	11.000000	.000000
W3	10.000000	.000000
W4	10.000000	.000000
W5	10.000000	.000000
W6	9.000000	.000000
I1	1400.000000	.000000
I2	2000.000000	.000000
I3	3000.000000	.000000
R1	6400.000000	.000000
R2	6600.000000	.000000
R3	6000.000000	.000000
R4	6000.000000	.000000
R5	6000.000000	.000000
R6	5000.000000	.000000

As a result of adding the integer restriction, the cost is increased by

(854080-852716.62) = $1363.38,

an increase of approximately 0.15%.

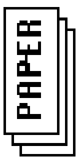
VARIABLE	LP VALUE	Integer VALUE
F1	2.583333	2
F3	0	1
F5	.416667	0
F6	1.666667	1
W1	10.416670	11
W2	10.416670	11
W3	10.416670	10
W4	10.416670	10
W5	10.000000	10
W6	8.333333	9
I1	1250.000000	1400
I2	1500.000000	2000
I3	2750.000000	3000
R1	6250.000000	6400
R2	6250.000000	6600
R3	6250.000000	6000
R4	6250.000000	6000
R5	6000.000000	6000
R6	5000.000000	5000

Compare the LP & Integer optimal values:

The integer solution isn't simply the result of rounding the non-integer solution!

(Exercise 30 of Review Problems, page 119 of text by Winston) "A paper recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are:

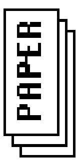
Input type	Cost $/ton	Pulp content
Box board	5	15%
Tissue paper	6	20%
Newsprint	8	30%
Book paper	10	40%



Two methods are available for processing the four inputs into pulp: de-inking and asphalt dispersion

It costs $20 to de-ink a ton of any input, a process which removes 10% of the pulp.

It costs #15 to apply asphalt dispersion to a ton of material, a processs which removes 20% of the pulp.



A total of 3000 tons, at most, can be run through the asphalt dispersion &/or de-inking process.

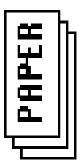
Grade 1 paper can be produced only with newsprint or book paper pulp

Grade 2 paper can be produced only with book paper, tissue paper, or box board pulp;

Grade 3 paper can be produced only with newsprint, tissue paper, or box board pulp.

To meet its demands, the company needs:

- 500 tons of pulp for grade 1 paper
- 500 tons of pulp for grade 2 paper
- 600 tons of pulp for grade 3 paper



Variables

BOX = tons of purchased boxboard

TISS = tons of purchased tissue

NEWS = tons of purchased newsprint

BOOK = tons of purchased book paper

PAPER

Variables, continued

BOX1 = tons of boxboard sent through de-inking
TISS1 = tons of tissue sent through de-inking
NEWS1 = tons of newsprint sent through de-inking
BOOK1 = tons of book paper sent through de-inking
BOX2 = tons of boxboard sent through asphalt dispersion
TISS2 = tons of tissue sent through asphalt dispersion
NEWS2 = tons of newsprint sent through asphalt dispersion
BOOK2 = tons of book paper sent through asphalt dispersion

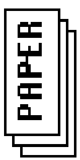
PAPER

Variables, continued

PBOX = tons of pulp recovered from boxboard
PTISS = tons of pulp recovered from tissue
PNEWS = tons of pulp recovered from newsprint
PBOOK = tons of pulp recovered from book paper
PBOX1 = tons of boxboard pulp used for grade 1 paper,
PBOX2 = tons of boxboard pulp used for grade 2 paper,

... etc.

PBOOK3 = tons of book paper pulp used for grade 3 paper.



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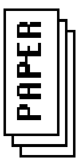
MIN  5 BOX    +6 TISS +    8 NEWS   +10 BOOK
   +20 BOX1 +20 TISS1 +20 NEWS1 +20 BOOK1
   +15 BOX2 +15 TISS2 +15 NEWS2 +15 BOOK2
SUBJECT TO
       2) - BOX + BOX1 + BOX2          <=   0
       3) - TISS + TISS1 + TISS2       <=   0
       4) - NEWS + NEWS1 + NEWS2       <=   0
       5) - BOOK + BOOK1 + BOOK2       <=   0
       6)   0.135 BOX1 + 0.12 BOX2 - PBOX      =   0
       7)   0.18 TISS1 + 0.16 TISS2 - PTISS    =   0
       8)   0.27 NEWS1 + 0.24 NEWS2 - PNEWS    =   0
       9)   0.36 BOOK1 + 0.32 BOOK2 - PBOOK    =   0
```



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10) - PBOX + PBOX2 + PBOX3          <=   0
11) - PTISS + PTISS2 + PTISS3       <=   0
12) - PNEWS + PNEWS1 + PNEWS3 <=   0
13) - PBOOK + PBOOK1 + PBOOK2  <=   0
14)   PNEWS1 + PBOOK1                  >=   500
15)   PBOX2 + PTISS2 + PBOOK2 >=   500
16)   PBOX3 + PTISS3 + PNEWS3 >=   600
17)   BOX1 + TISS1 + NEWS1 + BOOK1 <=   3000
18)   BOX2 + TISS2 + NEWS2 + BOOK2 <=   3000
END
```
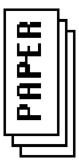
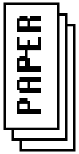



```
LP OPTIMUM FOUND AT STEP     23

     OBJECTIVE FUNCTION VALUE

     1)     140000.000
```

VARIABLE	VALUE	REDUCED COST
NEWS	2500.000000	.000000
BOOK	2833.333400	.000000
BOOK1	2333.333400	.000000
NEWS2	2500.000000	.000000
BOOK2	499.999930	.000000
PNEWS	600.000000	.000000
PBOOK	1000.000000	.000000
PNEWS3	600.000000	.000000
PBOOK1	500.000000	.000000
PBOOK2	500.000000	.000000



ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	8.000000
5)	.000000	10.000000
6)	.000000	-102.777800
7)	.000000	-102.777800
8)	.000000	-102.777800
9)	.000000	-83.333340
10)	.000000	102.777800
11)	.000000	102.777800
12)	.000000	102.777800
13)	.000000	83.333340
14)	.000000	-83.333340
15)	.000000	-83.333340
16)	.000000	-102.777800
17)	666.666500	.000000
18)	.000000	1.666666

The Boilen Oil Co. wishes to find the optimal mix of two possible blending processes.

Process	Input (barrels)		Output (gallons)	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	1	3	50	20
2	4	2	30	80

Available crude: 120 bbls of A, 180 of B

Required blends: 2800 gal. of X, 2200 of Y

Profits: $0.10/gallon of X, $0.12/gallon of Y

Product Mix

	Product			
	A	B	C	D
Machine X hr/unit	4	1	9	5
Machine Y hr/unit	1	2	6	8
Selling price $/unit	3550	2280	3775	3750
Direct costs $/unit	3150	2005	2825	2550

Available Machine Capacity: X: 150 hrs/week

Y: 260 hrs/we

What is the optimal production plan?

$$\text{Maximize } (3550-3150)A + (2280-2005)B + (3775-2825)C + (3750-2550)D$$

subject to

$$4A + B + 9C + 5D \le 150$$

$$A + 2B + 6C + 8D \le 260$$

$$A \ge 0,\ B \ge 0,\ C \ge 0,\ D \ge 0$$