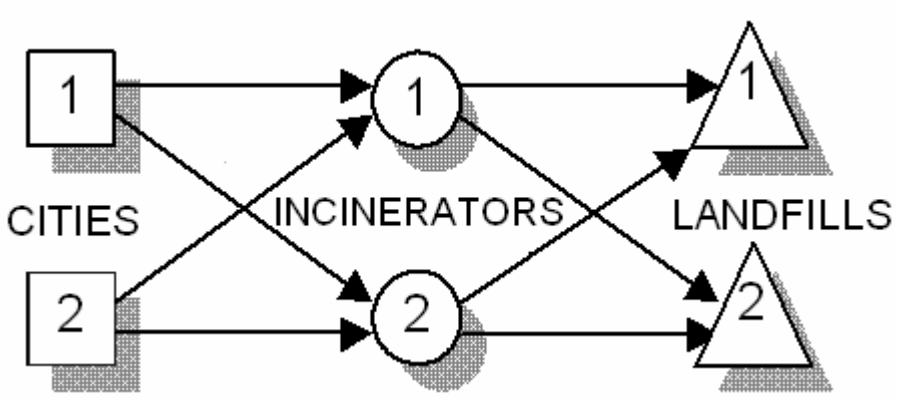
Two cities use incinerators to reduce waste before placing it in landfills.



- **City 1** produces **500** tons of waste per day, and
- **City 2** produces **400** tons of waste per day.
- Waste must be burned at **incinerator 1** or **incinerator 2.**
- Each incinerator can process up to **500** tons of waste per day.
- The cost to incinerate waste is $**40**/ton at incinerator 1 and $**30**/ton at incinerator 2.
- Incineration reduces each ton of waste to **0.2** tons of debris, which must be dumped at one of **two landfills**.
- Each landfill can receive at most **150** tons of debris per day.
- It costs $3 per mile to transport a ton of material (either debris or waste).
- Distances (in miles) between locations are:

	Incinerator #1	Incinerator #2
City# 1	**30**	**5**
City #2	**36**	**42**

	Landfill #1	Landfill #2
Incin #1	**5**	**8**
Incin #2	**9**	**6**

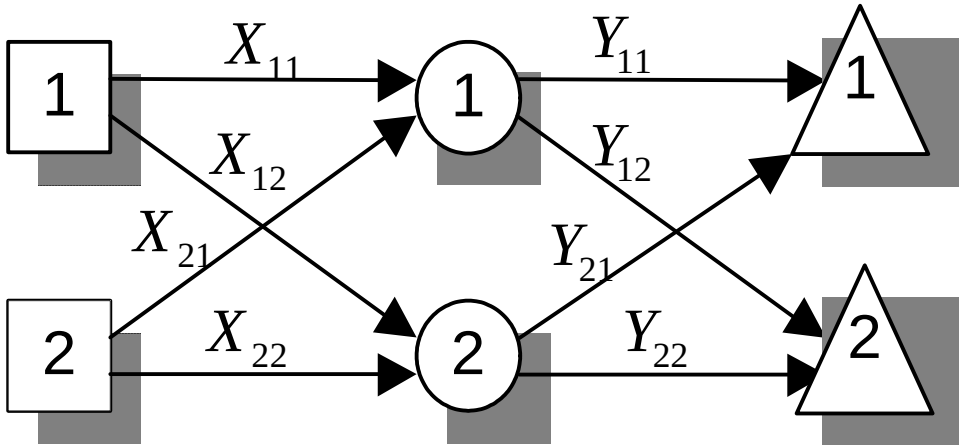
Formulate an LP that can be used to minimize the total cost of disposing of the waste of both cities.

Definition of variables

X_{ij} = tons/day of City #i waste that is sent to Incinerator #j, i=1,2; j=1,2

Y_{jk} = tons/day of debris sent from Incinerator #j to Landfill #k, j=1,2; k=1,2

(There are a total of 8 variables.)



Complete the table entries which would make this a feasible *(not necessarily optimal!)* solution:

Variable	Value		Variable	Value
X_{11}	100		Y_{11}	70
X_{12}			Y_{12}	
X_{21}	400		Y_{21}	80
X_{22}	0		Y_{22}	

Match the written restriction with the mathematical constraint below.

___ 1. City #1 must dispose of 500 tons of waste each day.

___ 2. Landfill #1 can receive at most 200 tons/day of waste.

___ 3. Incinerator #2 reduces one ton of trash to 0.2 tons of debris, e.g., ashes & other waste.

___ 4. Incinerator #2 can process at most 500 tons of waste per day.

a. $Y_{11} + Y_{21} - X_{11} - X_{12} \le 500$

b. $Y_{11} + Y_{21} \le 200$

c. $X_{12} + X_{22} \le 500$

d. $X_{12} + X_{22} = 0.2\left(Y_{21} + Y_{22}\right)$

e. $Y_{11} + Y_{22} = 0.2\left(X_{11} + X_{22}\right)$

f. $X_{11} + X_{12} \ge 500$

g. $Y_{21} + Y_{22} = 0.2\left(X_{12} + X_{22}\right)$

h. $\left(X_{11} + Y_{11}\right) = 0.2\left(X_{12} + Y_{12}\right)$

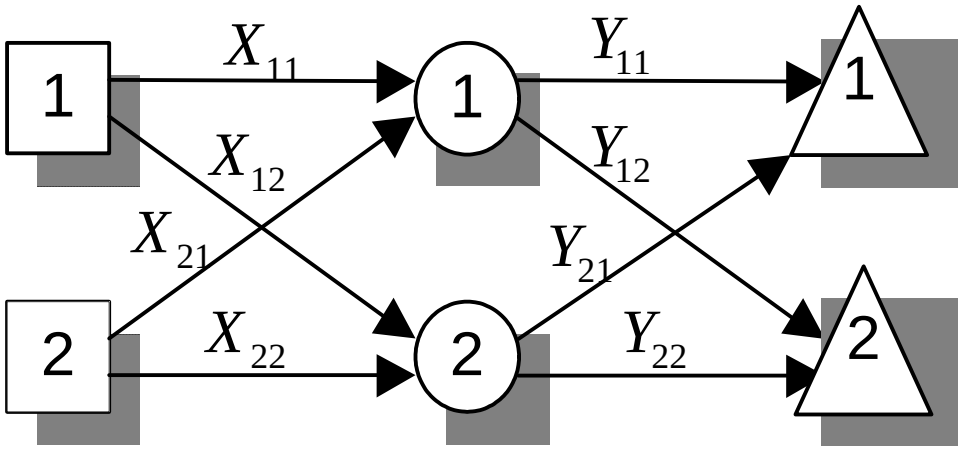
i. $X_{12} + X_{22} \ge 500$

j. $X_{12} - X_{22} = 0.2\left(Y_{12} - Y_{22}\right)$

k. $Y_{11} + Y_{21} \ge 200$

l. $Y_{11} + Y_{12} \ge 500$

m. *None of the above*



LINGO model (with sets):

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MODEL: ! WASTE INCINERATION & DISPOSAL IN LANDFILLS;
SETS:  ! DECISION VARIABLES ARE X, Y, & BURN
         X = TRANSPORT FROM CITIES TO INCINERATORS
         BURN = WASTE BURNED IN INCINERATOR
         Y = TRANSPORT FROM INCINERATORS TO LANDFILLS;
     CITIES/1..2/: OUTPUT;
     INCINERATORS/1..2/: ICOST, ICAP, BURN;
     LANDFILLS/1..2/: LCAP;
     PICKUP(CITIES,INCINERATORS): DX,X;
     DISPOSE(INCINERATORS,LANDFILLS): DY,Y;
ENDSETS
DATA:
     OUTPUT= 500 400;
     ICOST =   40 30;
     ICAP  = 500 500;
     LCAP  = 200 200;
     DX    =  30   5
              36  42;
     DY    =   5   8
              9   6;
     COSTPERMI = 3.0 ;   ! TRANSPORTATION COST PER MILE;
     REDUCTION = 0.2 ;   ! REDUCTION FACTOR FOR WASTE TO DEBRIS;
ENDDATA
```

```
     MIN = TRANSPORTCOST + BURNCOST;

     BURNCOST = @SUM(INCINERATORS:ICOST*BURN);
     TRANSPORTCOST = COSTPERMI*(@SUM(PICKUP: DX*X)
          + @SUM(DISPOSE: DY*Y) );

     @FOR(INCINERATORS(J):
          BURN(J)=@SUM(CITIES(I): X(I,J) );
          BURN(J) <= ICAP(J);
          REDUCTION* BURN(J) = @SUM(LANDFILLS(K):  Y(J,K) );
     );

     @FOR(CITIES(I):
          @SUM(INCINERATORS(J): X(I,J)) = OUTPUT(I);
     );

     @FOR(LANDFILLS(K):
          @SUM(INCINERATORS(J): Y(J,K) ) <= LCAP(K)
     );
END
```

The optimal solution:

Variable	Value	Reduced Cost
X(1, 2)	500.0000	0.0000000
X(2, 1)	400.0000	0.0000000
Y(1, 1)	80.00000	0.0000000
Y(2, 2)	100.0000	0.0000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
TRANSPORTCOST	1.000000	INFINITY	0.4623656
BURNCOST	1.000000	0.8600000	8.440000

Righthand Side Ranges

Row	Current RHS	Allowable Increase	Allowable Decrease
2	0.0	INFINITY	31000.00
3	0.0	INFINITY	53700.00
4	0.0	100.0000	400.0000
5	500.0000	INFINITY	100.0000
6	0.0	80.00000	120.0000
7	0.0	0.0	500.0000
8	500.0000	INFINITY	0.0
9	0.0	100.0000	100.0000
10	500.0000	0.0	500.0000
11	400.0000	100.0000	400.0000
12	200.0000	INFINITY	120.0000
13	200.0000	INFINITY	100.0000

LINDO model

```
MIN     TRANCOST + BURNCOST
SUBJECT TO
       2)   BURNCOST - 40 BURN_1 - 30 BURN_2 =    0
       3)   TRANCOST - 15 Y_1_1 - 24 Y_1_2 - 27 Y_2_1
            - 18 Y_2_2 - 90 X_1_1 - 15 X_1_2
            - 108 X_2_1 - 126 X_2_2 =    0
       4)   BURN_1 - X_1_1 - X_2_1 =    0
       5)   BURN_1 <=   500
       6)   0.2 BURN_1 - Y_1_1 - Y_1_2 =    0
       7)   BURN_2 - X_1_2 - X_2_2 =    0
       8)   BURN_2 <=   500
       9)   0.2 BURN_2 - Y_2_1 - Y_2_2 =    0
      10)   X_1_1 + X_1_2 =    500
      11)   X_2_1 + X_2_2 =    400
      12)   Y_1_1 + Y_2_1 <=   200
      13)   Y_1_2 + Y_2_2 <=   200
END
```

ROW	(BASIS)	TRANCOST	BURNCOST	BURN_1	BURN_2	Y_1_1	Y_1_2
1	ART	0.000	0.000	0.000	0.000	0.000	9.000
2	BURNCOST	0.000	1.000	0.000	0.000	0.000	0.000
3	TRANCOST	1.000	0.000	0.000	0.000	0.000	-9.000
4	BURN_1	0.000	0.000	1.000	0.000	0.000	0.000
5	SLK 5	0.000	0.000	0.000	0.000	0.000	0.000
6	Y_1_1	0.000	0.000	0.000	0.000	1.000	1.000
7	X_1_2	0.000	0.000	0.000	0.000	0.000	0.000
8	SLK 8	0.000	0.000	0.000	0.000	0.000	0.000
9	BURN_2	0.000	0.000	0.000	1.000	0.000	0.000
10	Y_2_2	0.000	0.000	0.000	0.000	0.000	0.000
11	X_2_1	0.000	0.000	0.000	0.000	0.000	0.000
12	SLK 12	0.000	0.000	0.000	0.000	0.000	-1.000
13	SLK 13	0.000	0.000	0.000	0.000	0.000	1.000

ROW	Y_2_1	Y_2_2	X_1_1	X_1_2	X_2_1	X_2_2	SLK 5
1	9.000	0.000	84.400	0.000	0.000	8.600	0.000
2	0.000	0.000	-10.000	0.000	0.000	10.000	0.000
3	-9.000	0.000	-74.400	0.000	0.000	-18.600	0.000
4	0.000	0.000	-1.000	0.000	0.000	1.000	0.000
5	0.000	0.000	1.000	0.000	0.000	-1.000	1.000
6	0.000	0.000	-0.200	0.000	0.000	0.200	0.000
7	0.000	0.000	1.000	1.000	0.000	0.000	0.000
8	0.000	0.000	-1.000	0.000	0.000	1.000	0.000
9	0.000	0.000	1.000	0.000	0.000	-1.000	0.000
10	1.000	1.000	0.200	0.000	0.000	-0.200	0.000
11	0.000	0.000	0.000	0.000	1.000	1.000	0.000
12	1.000	0.000	0.200	0.000	0.000	-0.200	0.000
13	-1.000	0.000	-0.200	0.000	0.000	0.200	0.000

ROW	SLK 8	SLK 12	SLK 13	
1	0.00E+00	0.00E+00	0.00E+00	-0.85E+05
2	0.000	0.000	0.000	31000.000
3	0.000	0.000	0.000	53700.000
4	0.000	0.000	0.000	400.000
5	0.000	0.000	0.000	100.000
6	0.000	0.000	0.000	80.000
7	0.000	0.000	0.000	500.000
8	1.000	0.000	0.000	0.000
9	0.000	0.000	0.000	500.000
10	0.000	0.000	0.000	100.000
11	0.000	0.000	0.000	400.000
12	0.000	1.000	0.000	120.000
13	0.000	0.000	1.000	100.000