Deterministic Production Planning

Dynamic Programming Model

author

Acme Computer must plan its production for the next 8 weeks, based upon scheduled deliveries of computer systems.

period #	1	2	3	4	5	6	7	8
Demand =	3	5	3	4	3	5	1	2

Maximum regular-time production of this computer system is 3, with 2 additional units possible when overtime is used.

Setup cost is $10K each week that production is scheduled.

Unit production costs are $2K during regular time, and $3K during O.T.

Computers can be produced in advance of demand and stored, at a cost of $1K each, with a maximum storage capacity of 6 computers.

Any computers remaining in stock at the end of the 8-week planning period are valued at $2K each.

Inventory Cost

i	1	2	3	4	5	6
h[i]	1	2	3	4	5	6
Δ	1	1	1	1	1	1

based upon beginning-of-week stock on hand

Production Cost

x	1	2	3	4	5
C[x]	12	14	16	19	22
Δ	12	2	2	3	3

producing 4 or 5 units requires use of overtime

Salvage values

i	1	2	3	4	5	6
S[i]	2	4	6	8	10	12
Δ	2	2	2	2	2	2

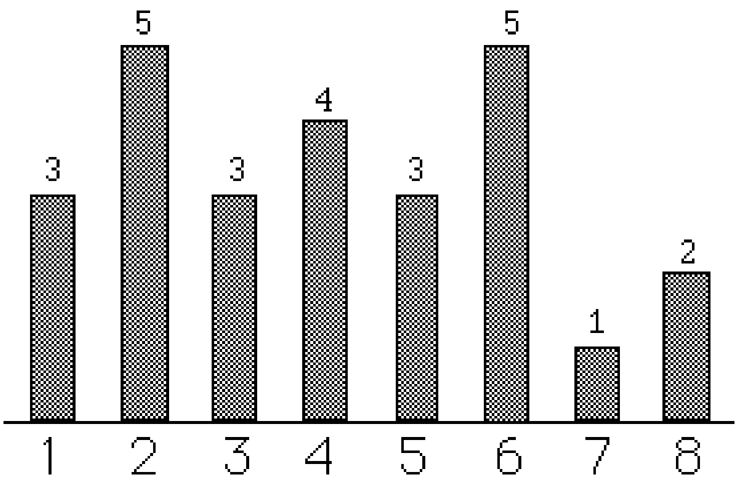
value of stock on hand at end of 8th week

(Δ indicates marginal costs & values.)

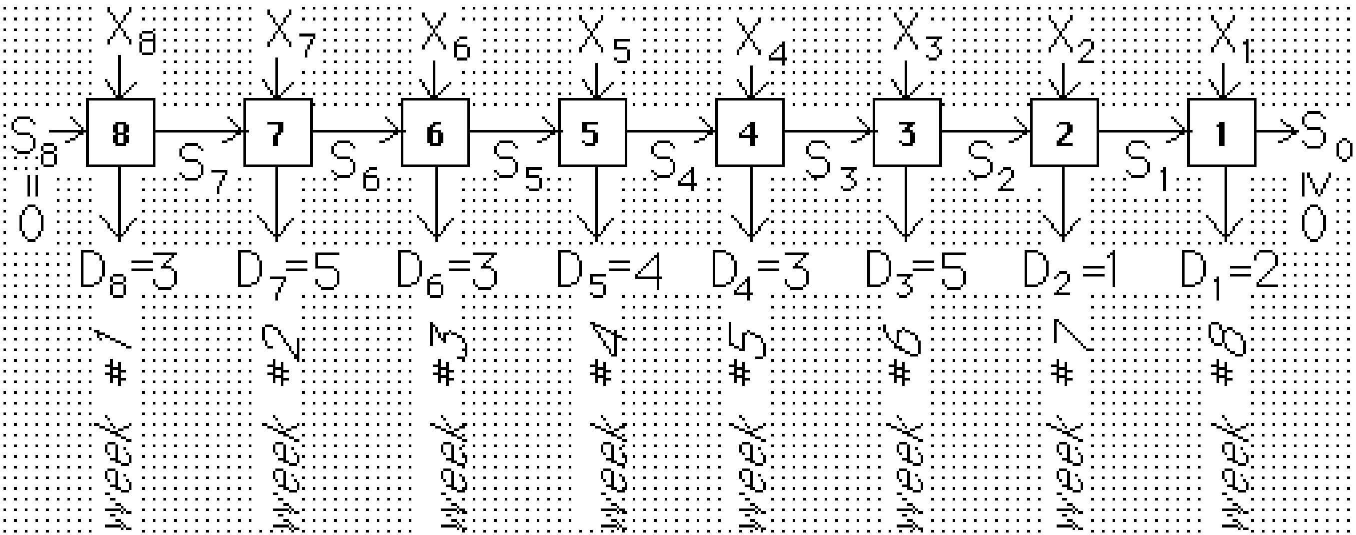
Demands

stage #	8	7	6	5	4	3	2	1
period #	1	2	3	4	5	6	7	8
Demand =	3	5	3	4	3	5	1	2

Demand is "lumpy". Known with certainty.



Stage n → n weeks remaining to be planned



"state" = inventory at beginning of the week

$$S_{n-1} = S_n + X_n - D_n$$

$f_n(S_n)$ = minimum cost of satisfying demand during the last n weeks of the planning period, if initial stock-on-hand is S_n

What is the minimum cost of satisfying the demand during the 8 weeks, if initial stock-on-hand is zero?

i.e., what is $f_8(0) = ????$

$f_0(S_0) = -2S_0$ *(salvage value of final inventory)*

Stage 1

week #8 (final stage)

demand is 2

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	14.00	14.00	15.00	16.00
1	9999.99	13.00	13.00	13.00	14.00	15.00
2	2.00	12.00	12.00	12.00	13.00	14.00
3	1.00	11.00	11.00	11.00	12.00	13.00
4	0.00	10.00	10.00	10.00	11.00	9999.99
5	¯1.00	9.00	9.00	9.00	9999.99	9999.99
6	¯2.00	8.00	8.00	9999.99	9999.99	9999.99

Example:

If stock-on-hand is 1
and production qty is 4,

storage cost:	1	← *charged for beginning stock*
setup cost:	10	
prod'n cost:	9	
salvage:	-6	← *received for final stock (1+4-2=3)*
net:	14	

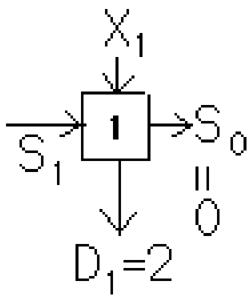
Stage 1

week #8 (final stage)

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	14.00	14.00	15.00	16.00
1	9999.99	13.00	13.00	13.00	14.00	15.00
2	2.00	12.00	12.00	12.00	13.00	14.00
3	1.00	11.00	11.00	11.00	12.00	13.00
4	0.00	10.00	10.00	10.00	11.00	9999.99
5	-1.00	9.00	9.00	9.00	9999.99	9999.99
6	-2.00	8.00	8.00	99[illegible]	[illegible]	[illegible]

State	Optimal Values	Optimal Decisions	Resulting State
0	14.00	2	0
		3	1
1	13.00	1	0
		2	1
		3	2
2	2.00	0	0
3	1.00	0	1
4	0.00	0	2
5	-1.00	0	3
6	-2.00	0	4

$f_1(S_1)$



Stage 2 *week #7* *demand is 1*

s \ x:	0	1	2	3	4	5
0	9999.99	26.00	27.00	18.00	20.00	22.00
1	15.00	26.00	17.00	18.00	20.00	22.00
2	15.00	16.00	17.00	18.00	20.00	22.00
3	5.00	16.00	17.00	18.00	20.00	9999.99
4	5.00	16.00	17.00	18.00	9999.99	9999.99
5	5.00	16.00	17.00	9999.99	9999.99	9999.99
6	5.00	16.00	9999.99	9999.99	9999.99	9999.99

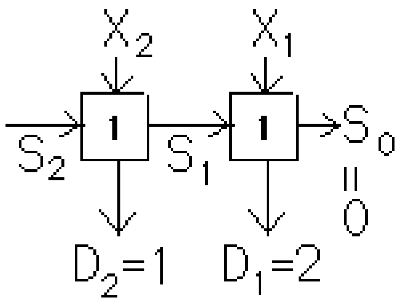
State	$f_1(S)$
0	14.00
1	13.00
2	2.00
3	1.00
4	0.00
5	¯1.00
6	¯2.00

For example, if S=stock=1 and X=prod'n=3,

storage cost:	1
setup cost:	10
prod'n cost:	6
cost of stage 1, with S = 3 $f_1(3)$:	1
net	18

Stage 2 *week #7*

s \ x:	0	1	2	3	4	5
0	9999.99	26.00	27.00	18.00	20.00	22.00
1	15.00	26.00	17.00	18.00	20.00	22.00
2	15.00	16.00	17.00	18.00	20.00	22.00
3	5.00	16.00	17.00	18.00	20.00	9999.99
4	5.00	16.00	17.00	18.00	9999.99	9999.99
5	5.00	16.00	17.00	9999.99	9999.99	9999.99
6	5.00	16.00	9999.99	9999.99	9999.99	9999.99



State	Optimal Values	Optimal Decisions	Resulting State
0	18.00	3	2
1	15.00	0	0
2	15.00	0	1
3	5.00	0	2
4	5.00	0	3
5	5.00	0	4
6	5.00	0	5

$f_2(S_2)$ S_1

Stage 3

demand is 5

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	9999.99	9999.99	40.00
1	9999.99	9999.99	9999.99	9999.99	38.00	38.00
2	9999.99	9999.99	9999.99	36.00	36.00	39.00
3	9999.99	9999.99	35.00	34.00	37.00	30.00
4	9999.99	34.00	33.00	35.00	28.00	31.00
5	23.00	32.00	34.00	26.00	29.00	32.00
6	21.00	33.00	25.00	27.00	30.00	33.00

S	$f_2(S)$
0	18.00
1	15.00
2	15.00
3	5.00
4	5.00
5	5.00
6	5.00

Stage 4

demand is 3

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	56.00	57.00	58.00
1	9999.99	9999.99	55.00	55.00	56.00	53.00
2	9999.99	54.00	54.00	54.00	51.00	52.00
3	43.00	53.00	53.00	49.00	50.00	48.00
4	42.00	52.00	48.00	48.00	46.00	47.00
5	41.00	47.00	47.00	44.00	45.00	9999.99
6	36.00	46.00	43.00	43.00	9999.99	9999.99

S_3	$f_3(S_3)$
0	40.00
1	38.00
2	36.00
3	30.00
4	28.00
5	23.00
6	21.00

Stage 5

demand is 4

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	9999.99	75.00	75.00
1	9999.99	9999.99	9999.99	73.00	73.00	74.00
2	9999.99	9999.99	72.00	71.00	72.00	67.00
3	9999.99	71.00	70.00	70.00	65.00	67.00
4	60.00	69.00	69.00	63.00	65.00	67.00
5	58.00	68.00	62.00	63.00	65.00	63.00
6	57.00	61.00	62.00	63.00	61.00	9999.99

S	$f_4(S)$
0	56.00
1	53.00
2	51.00
3	43.00
4	42.00
5	41.00
6	36.00

Stage 6

demand is 3

S \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	91.00	92.00	89.00
1	9999.99	9999.99	90.00	90.00	87.00	88.00
2	9999.99	89.00	89.00	85.00	86.00	84.00
3	78.00	88.00	84.00	84.00	82.00	83.00
4	77.00	83.00	83.00	80.00	81.00	83.00
5	72.00	82.00	79.00	79.00	81.00	9999.99
6	71.00	78.00	78.00	79.00	9999.99	9999.99

S	$f_5(S)$
0	75.00
1	73.00
2	67.00
3	65.00
4	60.00
5	58.00
6	57.00

Stage 7

demand is 5

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	9999.99	9999.99	111.00
1	9999.99	9999.99	9999.99	9999.99	109.00	110.00
2	9999.99	9999.99	9999.99	107.00	108.00	108.00
3	9999.99	9999.99	106.00	106.00	106.00	103.00
4	9999.99	105.00	105.00	104.00	101.00	103.00
5	94.00	104.00	103.00	99.00	101.00	99.00
6	93.00	102.00	98.00	99.00	97.00	99.00

S	$f_6(S)$
0	89.00
1	87.00
2	84.00
3	78.00
4	77.00
5	72.00
6	71.00

Stage 8

demand is 3

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	127.00	128.00	129.00
1	9999.99	9999.99	126.00	126.00	127.00	126.00
2	9999.99	125.00	125.00	125.00	124.00	125.00
3	114.00	124.00	124.00	122.00	123.00	119.00
4	113.00	123.00	121.00	121.00	117.00	119.00
5	112.00	120.00	120.00	115.00	117.00	9999.99
6	109.00	119.00	114.00	115.00	9999.99	9999.99

S	$f_7(S)$
0	111.00
1	109.00
2	107.00
3	103.00
4	101.00
5	94.00
6	93.00

s \ x:	0	1	2	3	4	5
0	9999.99	9999.99	9999.99	127.00	128.00	129.00
1	9999.99	9999.99	126.00	126.00	127.00	126.00
2	9999.99	125.00	125.00	125.00	124.00	125.00
3	114.00	124.00	124.00	122.00	123.00	119.00
4	113.00	123.00	121.00	121.00	117.00	119.00
5	112.00	120.00	120.00	115.00	117.00	9999.99
6	109.00	119.00	114.00	115.00	9999.99	9999.99

Stage 8

State	Optimal Values	Optimal Decisions	Resulting State
0	127.00	3	0
1	126.00	2	0
		3	1
		5	3
2	124.00	4	3
3	114.00	0	0
4	113.00	0	1
5	112.00	0	2
6	109.00	0	3

Optimal Returns & Decisions

Stage 8:

State	Optimal Values	Optimal Decisions	Resulting State
0	127.00	3	0
1	126.00	2	0
		3	1
		5	3
2	124.00	4	3
3	114.00	0	0
4	113.00	0	1
5	112.00	0	2
6	109.00	0	3

Initially, stock-on-hand, S_8 , is zero, so production lot size X_8 is 3, and (since the demand $D_8 = 3$), the end-of-week stock-on-hand is zero.

Stage 7:

State	Optimal Values	Optimal Decisions	Resulting State
0	111.00	5	0
1	109.00	4	0
2	107.00	3	0
3	103.00	5	3
4	101.00	4	3
5	94.00	0	0
6	93.00	0	1

The stock-on-hand entering the second week (stage 7) will be zero, and the optimal lot size, X_5, will be 5. Because the demand D is 5, the end-of-week inventory will be zero.

Stage 6:

State	Optimal Values	Optimal Decisions	Resulting State
0	89.00	5	2
1	87.00	4	2
2	84.00	5	4
3	78.00	0	0
4	77.00	0	1
5	72.00	0	2
6	71.00	0	3

Stock-on-hand at beginning of third week (stage 6) will be zero, and so the production lot size, X_6, will be 5. Since demand $D_6 = 3$, the stock-on-hand at end of week will be 2.

Stage 5:

State	Optimal Values	Optimal Decisions	Resulting State
0	75.00	4	0
		5	1
1	73.00	3	0
		4	1
2	67.00	5	3
3	65.00	4	3
4	60.00	0	0
5	58.00	0	1
6	57.00	0	2

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
0	56.00	3	0
1	53.00	5	3
2	51.00	4	3
3	43.00	0	0
4	42.00	0	1
5	41.00	0	2
6	36.00	0	3

Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
0	40.00	5	0
1	38.00	4	0
		5	1
2	36.00	3	0
		4	1
3	30.00	5	3
4	28.00	4	3
5	23.00	0	0
6	21.00	0	1

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
0	18.00	3	2
1	15.00	0	0
2	15.00	0	1
3	5.00	0	2
4	5.00	0	3
5	5.00	0	4
6	5.00	0	5

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
0	14.00	2	0
		3	1
1	13.00	1	0
		2	1
		3	2
2	2.00	0	0
3	1.00	0	1
4	0.00	0	2
5	¯1.00	0	3
6	¯2.00	0	4

Acme Computer

*** Optimal value is 127 ***

STAGE	STATE	DECISION
8	0	3
7	0	5
6	0	5
5	2	5
4	3	0
3	0	5
2	0	3
1	2	0
0	0	