Covering Problems

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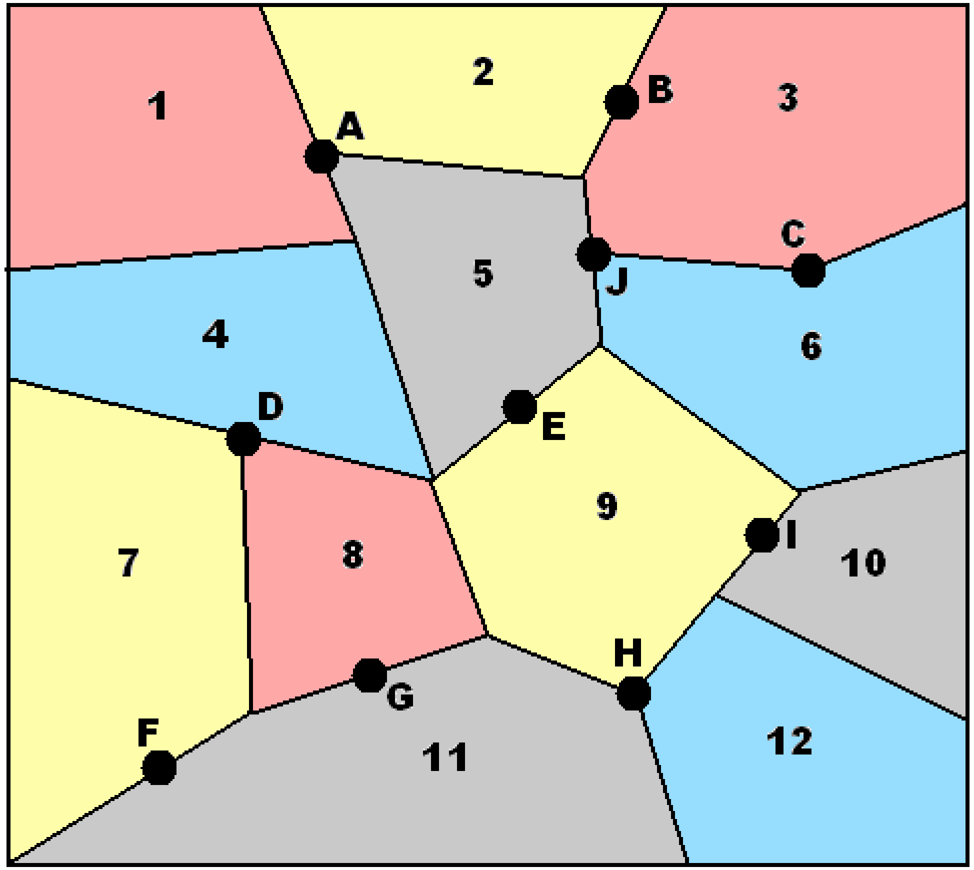


A mobile phone operator wants to provide service to a currently uncovered geographical region. Seven locations are being considered for installation for towers for this purpose.

Because of the distances and obstacles such as mountains and tall buildings, each tower can serve only a small number of the twelve communities in the region. Furthermore, the costs of building towers depends upon the site.

The following table gives the following information: an “X” indicating that a tower covers a community; the population (in thousands) of each community; the cost (in US$ millions) of purchasing the land and building the tower.

site→ community↓	A	B	C	D	E	F	G	H	I	J	Pop. (K)
1	X			X							2
2	X	X								X	4
3		X	X							X	3
4				X			X				3
5	X			X	X					X	5
6			X		X				X	X	6
7				X		X	X				2
8				X	X		X				7
9					X		X	X	X		6
10								X	X		5
11						X	X	X			4
12							X	X	X		3
Cost (US$M)	0.8	0.6	0.9	1.4	1.4	1.2	1.6	0.9	0.8	0.8	

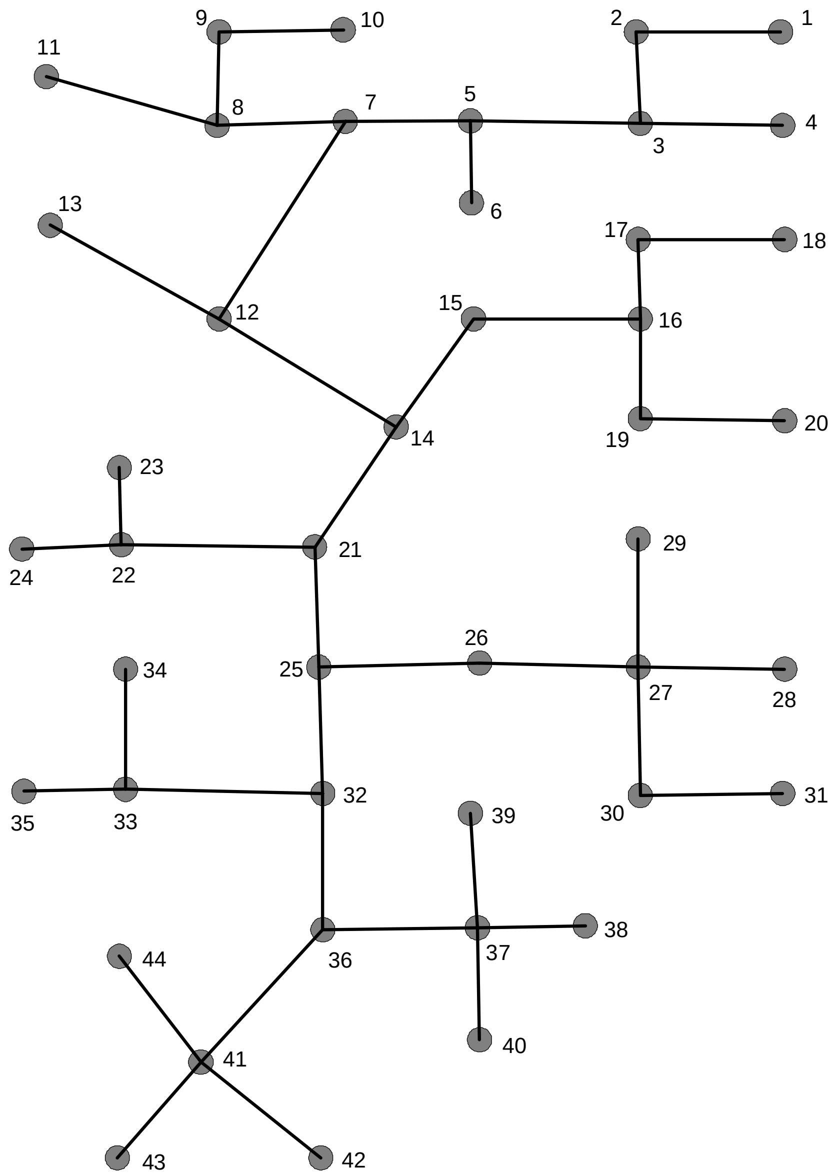
I. The mobile telephone company wishes to cover all the communities at the lowest cost of constructing the towers. Where should be towers be placed?

II. **Suppose that the telephone company has a budget of US$5 million and wants to provide service to as many customers as possible. Where should the towers be placed?**

Surveillance Camera Placement

During the last few months, the industrial zone of *Billston* has suffered from a series of break-ins and thefts during the night. The zone is watched by security officers, but there are too few of them.

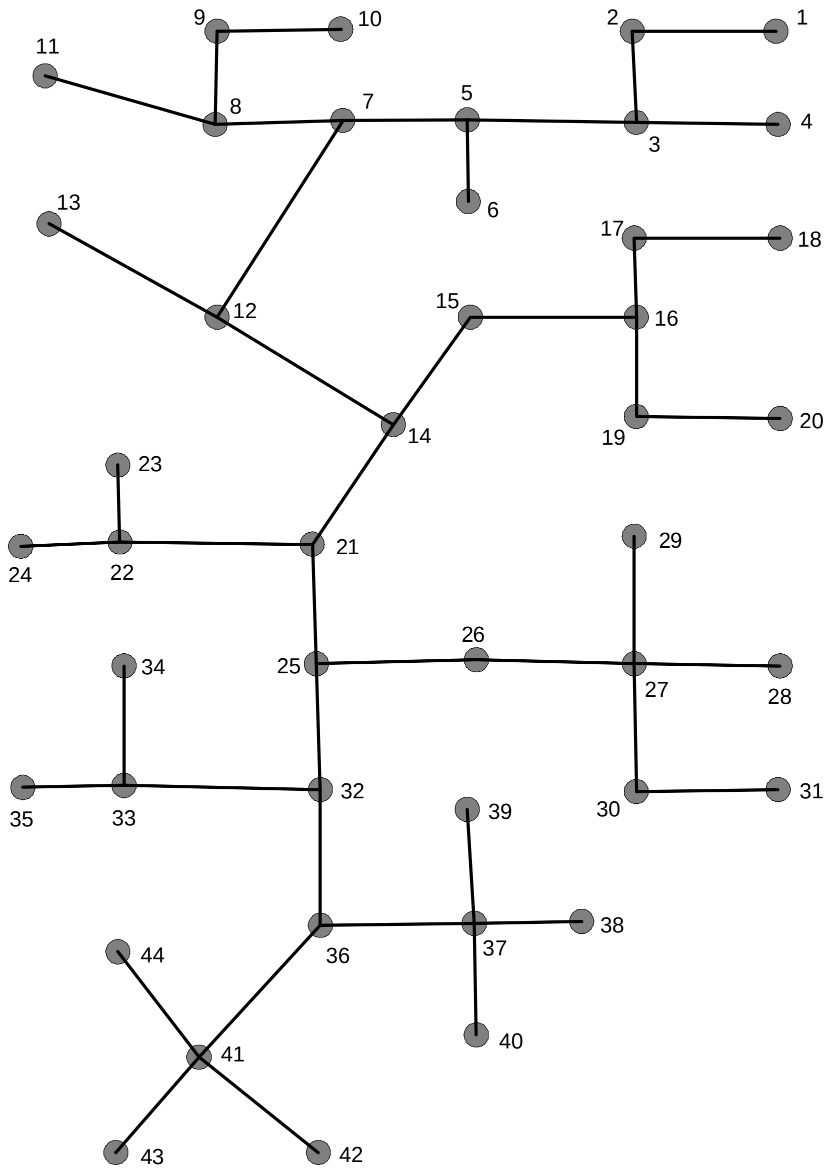
The town council in charge of security in this zone decides to install **surveillance cameras** to aid the security force with their task.



These cameras can be directed and pivot through 360º.

By installing a camera at the intersection of several streets, it is possible to survey all adjoining streets.

The map shows the industrial zone with the limits of the zone to be covered by closed circuit TV (CCTV) surveillance and the **44** possible intersections where the cameras might be installed.



What is the minimum number of cameras that have to be installed to survey all the streets of this zone, and where should they be placed?

(Adapted from *Applications of Optimization with Xpress^{MP}*, section 15.2)

Information Retrieval

A database is distributed over a network of ten nodes and stored at several locations. The database has received an inquiry for 6 pieces of data. The table below indicates at which node each piece of data has been stored.

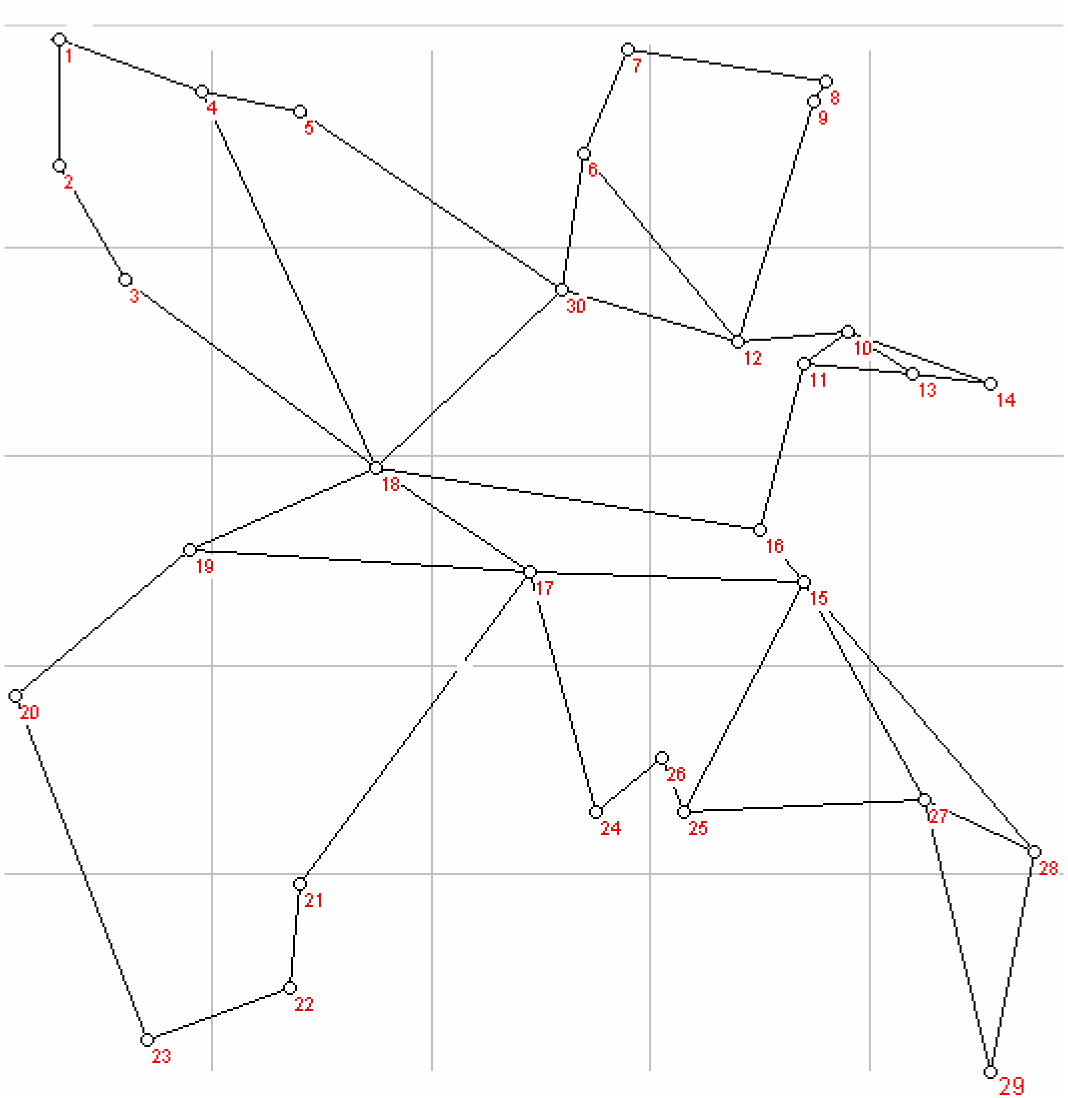
nodes→ requests↓	1	2	3	4	5	6	7	8	9	10
1	x				x	x		x		
2	x	x		x		x			x	x
3		x	x	x			x	x		
4	x		x		x	x	x		x	
5		x	x	x				x	x	x
6					x		x	x		x

What is the smallest number of nodes which together can fill the request?

A map of the main streets in an urban area are shown on the right.

The city currently has fire stations at nodes #4, 6, 9, 10, 21, and 26.

The city government has decided to place ambulances at the fire stations so that each intersection is no more than **75 seconds** of travel time away from the nearest station.



What is the smallest number which can satisfy this requirement, and where should they be placed?

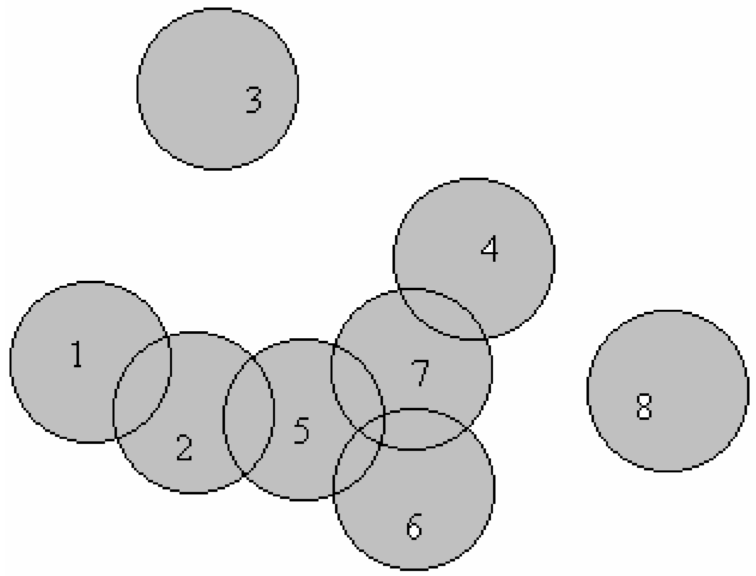
The following shortest path lengths (in seconds) between every pair of nodes in the network were computed (using Floyd's algorithm) and, for those in the list of six available sites, shown below:

sites→ nodes↓	4	6	9	10	21	26
1	14	65	93	79	107	102
2	26	77	105	91	96	91
3	39	66	94	80	83	78
4	0	51	79	65	93	88
5	9	42	70	56	102	97
6	51	0	31	33	91	86
7	62	11	20	44	102	97
8	80	29	2	36	120	93
9	79	31	0	34	119	91
10	65	33	34	0	89	57
11	70	38	39	5	84	52
12	55	23	24	10	95	67
13	72	40	41	7	94	62
14	79	47	48	14	101	69
15	81	60	61	27	62	30

sites→ nodes↓	4	6	9	10	21	26
16	75	54	55	21	68	36
17	56	54	82	52	37	32
18	39	37	65	51	54	49
19	58	56	84	70	68	63
20	79	77	105	91	59	84
21	93	91	119	89	0	69
22	103	101	129	99	10	79
23	114	112	140	113	24	93
24	80	78	99	65	61	8
25	93	85	86	52	74	5
26	88	86	91	57	69	0
27	105	84	85	51	86	27
28	114	93	94	60	95	38
29	132	111	112	78	113	54
30	38	13	41	27	78	73

Radio Station Licenses

The map that follows shows the locations of 8 applicants for low-power radio station licenses and the approximate range of their signals.



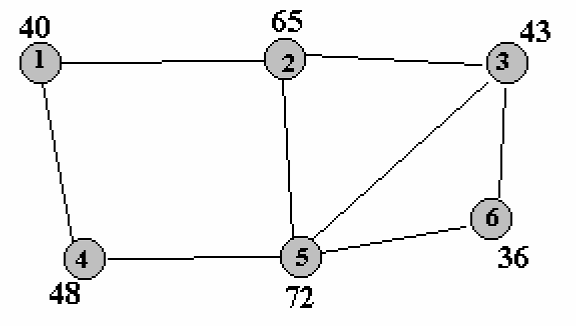
Regulators have scored the quality of applications on a scale of 0 to 100:

Applicant	1	2	3	4	5	6	7	8
Score	45	30	84	73	80	70	61	91

They wish to select the highest-quality combination of applications that has no overlap in signal ranges.

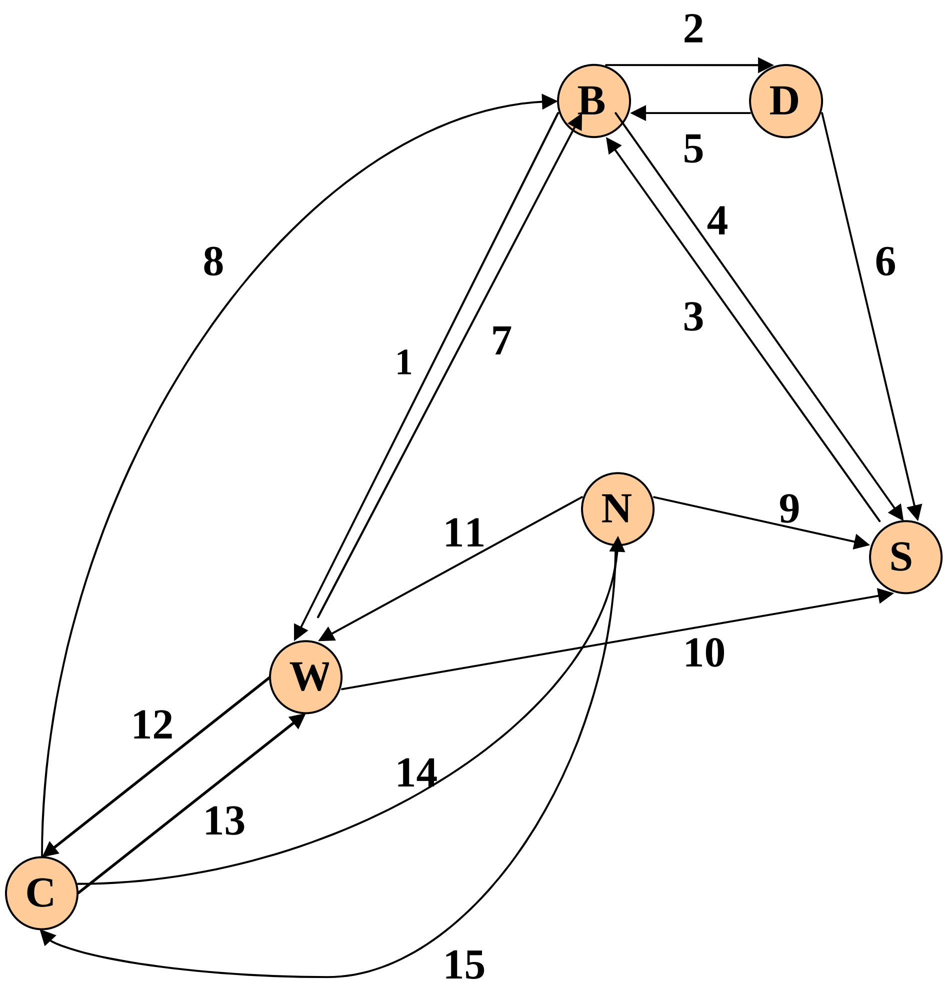
Traffic Monitoring Stations

The following map shows the 8 intersections at which automatic traffic monitoring devices might be installed:



A station at any particular node can monitor all the road links meeting that intersection. Numbers next to nodes reflect the monthly cost (in thousands of dollars) of operating a station at that location.

At which nodes should stations be installed to provide full coverage at minimum total cost?



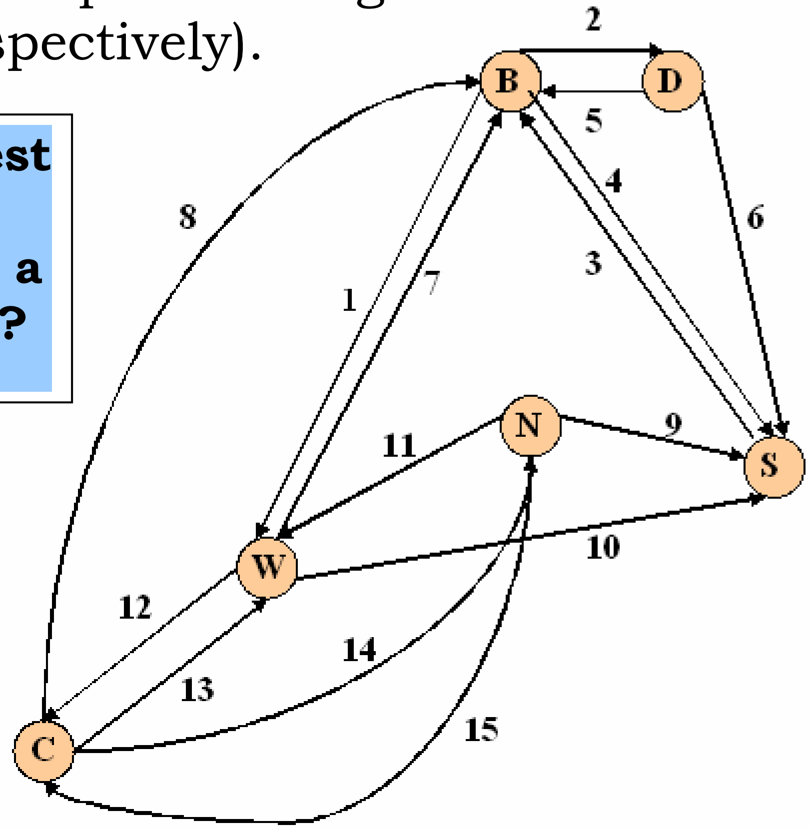
A domestic airline provides commuter service between six cities (**B**eijing, **D**alien, **S**hanghai, **N**anjing, **W**uhan, and **C**hungqing). The flight numbers are shown on the map.

The airline wants to assign flight crews to trips consisting of two or three flights which form a circuit, so that the crew could be based in one of the cities.

For simplicity, assume that there are three possible flight times: Morning, Afternoon, Night (M,A, & N, respectively).

Flight #	from /to	time
1	B→W	N
2	B→D	A
3	S→B	N
4	B→S	M
5	D→B	M
6	D→S	N
7	W→B	A
8	C→B	N
9	N→S	A
10	W→S	A
11	N→W	N
12	W→C	M
13	C→W	N
14	C→N	A
15	N→C	M

What is the smallest number of trips which will provide a crew for all flights?



Assume that a trip can include at most one layover and three flights, and that deadheading (traveling as passengers) on a flight is allowed.

THE SET COVERING MODEL

GIVEN DATA:

m nodes to be covered, n candidate sets

$$a_{ij} = \begin{cases} 1 & \text{if set } i \text{ covers node } j \\ 0 & \text{otherwise} \end{cases}$$

$$C_j = \text{cost of set } j$$

DECISION VARIABLES:

$$X_j = \begin{cases} 1 & \text{if set } j \text{ is included in the cover} \\ 0 & \text{otherwise} \end{cases}$$

Set Covering Problem

Objective:

$$Minimize \sum_{j=1}^{n} C_j X_j \quad or \quad Minimize \sum_{j=1}^{n} X_j$$

Constraints

for each node *i*, *i=1,2,...m*, $\sum_{j=1}^{n} a_{ij} X_j \geq 1$

$$X_j \in \{0,1\}, \quad j = 1, 2, \ldots n$$