

**classifying**

# **Simplex Tableaus**

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**Note:** We assume that the tableau is set up so that the basic variable in the objective row is  $-Z$  rather than  $Z$ .

a. The number in the objective row of a **nonbasic** variable indicates the change in the objective function per unit increase in the variable.

- if maximizing, we call it **relative profit**
- if minimizing, we call it **reduced cost**

b. **Any** nonbasic variable with

- positive relative profit (if maximizing) or
- negative reduced cost (if minimizing)

can be chosen to enter the basis.

*(That is, we need not choose the “most positive” relative profit or “most negative” reduced cost.)*

**Example:** either  $X_1$  or  $X_3$  could be selected as the pivot column:

$-Z$ (max)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	1	0	3	0	0	-27
0	5	0	1	1	0	11
0	0	1	6	0	0	52
0	-2	0	2	0	1	33

*For convenience, suppose we are minimizing— similar conclusions will follow the maximizing case:*

c. If no nonbasic variable has a negative reduced cost, the basic solution is **optimal**.

Example:

$-Z$ (min)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	1	0	3	0	0	-27
0	5	0	1	1	0	11
0	0	1	6	0	0	52
0	-2	0	2	0	1	33

- d. If a nonbasic variable has zero reduced cost, there may be another basic solution with the **same objective value**.

**Example:** a pivot in  $X_1$  column does not change the objective value

$-Z$ (min)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	0	0	-3	0	0	-27
0	5	0	1	1	0	10
0	0	1	6	0	0	52
0	-2	0	2	0	1	32

(If reduced costs are  $\geq 0$ , and one nonbasic reduced cost is zero, then there are **multiple optimal solutions**.)

**Example:** A pivot in the  $X_1$  column gives another basic solution with the same objective value:

$-Z$ (min)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	0	0	-3	0	0	-27
0	1	0	0.2	0.2	0	2
0	0	1	6	0	0	52
0	0	0	0.4	0.4	1	36

e. If a nonbasic variable with negative reduced cost has no positive substitution rate, i.e., no candidate for the pivot, the LP has an **unbounded solution** (even if some other nonbasic variable does have an eligible pivot).

**Example:** Although  $X_3$  could enter the basis, the LP has an unbounded solution because  $X_1$  can increase without limit, sending the objective function to  $-\infty$ :

$-Z$ (min)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	-1	0	-3	0	0	-27
0	-5	0	1	1	0	11
0	0	1	6	0	0	52
0	-2	0	2	0	1	33

- f. If a basic variable has the value zero, we call the basic solution ***degenerate***.

***Example:*** the basic variable  $X_3$  is zero:

$-Z$ (min)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	-1	0	-3	0	0	-27
0	5	0	1	1	0	0
0	0	1	6	0	0	52
0	2	0	2	0	1	33

- g. If the pivot column has a *positive substitution rate* in a row with zero on the right-hand-side (i.e., solution is *degenerate*), then
- the minimum ratio (RHS/positive substitution rate) is zero
  - the simplex method is forced to pivot in that row
  - the variable entering the basis will remain zero in the new tableau, that is, the new basic solution will also be degenerate
  - there will be *no changes in the values of any variable* or the objective function

*(Degenerate pivots will continue until a pivot column has only zero or negative substitution rates in rows with zero right-hand-sides.)*

*Example:* Whether  $X_1$  or  $X_3$  is selected for the pivot row,  $X_4$  will leave the basis but the entering variable will remain zero.

$-Z$ (min)	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	RHS
1	-1	0	-3	0	0	-27
0	5	0	1	1	0	0
0	0	1	6	0	0	52
0	2	0	2	0	1	33

- h. *It is possible but very rare that the simplex method will get “stuck” in a **cycle** of degenerate basic solutions and unable to converge to the optimal solution.*  
*(Special rules for choosing the pivot column can guarantee this doesn't happen, but are seldom implemented in computer software.)*