

# Chicken Feed



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A firm manufactures **chicken feed** by mixing three different ingredients.

Each ingredient contains **four key nutrients**:

protein, fat, vitamin A, and vitamin B.

The amount of each nutrient contained in 1 kilogram of the **three basic ingredients** is:

| Ingredient | Protein<br>(grams) | Fat<br>(grams) | Vitamin A<br>(units) | Vitamin B<br>(units) |
|------------|--------------------|----------------|----------------------|----------------------|
| 1          | 25                 | 11             | 235                  | 12                   |
| 2          | 45                 | 10             | 160                  | 6                    |
| 3          | 32                 | 7              | 190                  | 10                   |

The costs per kg of Ingredients 1, 2, and 3 are  
\$0.55, \$0.42, and \$0.38, respectively.

Each kilogram of the feed must contain

- *at least* 35 grams of protein,
- a *minimum* of 8 grams (and a *maximum* of 10 grams) of fat,
- *at least* 180 units of vitamin A and
- *at least* 9 units of vitamin B.

*Formulate an LP model for finding the feed mix that has the minimum cost per kg.*



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## Decision variables

X1 = kg. of Ingredient 1 included in mixture

X2 = kg. of Ingredient 2 included in mixture

X3 = kg. of Ingredient 3 included in mixture

# Chicken Feed

## Complete LP Formulation (LINDO)

```
MIN      0.55 X1 + 0.42 X2 + 0.38 X3
st
    25 X1 + 45 X2 + 32 X3 >= 35  ! Protein constraint
    11 X1 + 10 X2 + 7 X3 >= 8    ! Min Fat constraint
    11 X1 + 10 X2 + 7 X3 <= 10  ! Max Fat constraint
    235 X1 + 160 X2 + 190 X3 >= 180 ! Vitamin A constraint
    12 X1 + 6 X2 + 10 X3 >= 9    ! Vitamin B constraint
          X1 + X2 + X3 = 1      ! total wt of mixture

END
```

OBJECTIVE FUNCTION VALUE

1) 0.3986364

| VARIABLE | VALUE    | REDUCED COST |
|----------|----------|--------------|
| X1       | 0.045455 | 0.000000     |
| X2       | 0.272727 | 0.000000     |
| X3       | 0.681818 | 0.000000     |

| ROW | SLACK OR SURPLUS | DUAL PRICES |
|-----|------------------|-------------|
| 2)  | 0.227273         | 0.000000    |
| 3)  | 0.000000         | -0.034545   |
| 4)  | 2.000000         | 0.000000    |
| 5)  | 3.863636         | 0.000000    |
| 6)  | 0.000000         | -0.015909   |
| 7)  | 0.000000         | 0.020909    |

The minimum cost mixture costs \$0.398/kg and consists of  
 0.045 kg of Ingredient 1,  
 0.273 kg of Ingredient 2 and  
 0.682 kg of Ingredient 3

*Which constraints are “tight” or “binding”?*  
*Which are “loose”?*

RANGES IN WHICH THE BASIS IS UNCHANGED:

| VARIABLE | CURRENT<br>COEF | OBJ COEFFICIENT RANGES |                       |
|----------|-----------------|------------------------|-----------------------|
|          |                 | ALLOWABLE<br>INCREASE  | ALLOWABLE<br>DECREASE |
| X1       | 0.550000        | INFINITY               | 0.116667              |
| X2       | 0.420000        | 0.087500               | 0.380000              |
| X3       | 0.380000        | 0.126667               | 0.350000              |

| ROW | CURRENT<br>RHS | RIGHTHAND SIDE RANGES |                       |
|-----|----------------|-----------------------|-----------------------|
|     |                | ALLOWABLE<br>INCREASE | ALLOWABLE<br>DECREASE |
| 2   | 35.000000      | 0.227273              | INFINITY              |
| 3   | 8.000000       | 2.000000              | 0.250000              |
| 4   | 10.000000      | INFINITY              | 2.000000              |
| 5   | 180.000000     | 3.863636              | INFINITY              |
| 6   | 9.000000       | 0.068493              | 0.333333              |
| 7   | 1.000000       | 0.017241              | 0.003453              |

***The optimal tableau:***

| ROW | (BASIS) | X1    | X2    | X3    | SLK 2 | SLK 3  | SLK 4 |
|-----|---------|-------|-------|-------|-------|--------|-------|
| 1   | ART     | 0.000 | 0.000 | 0.000 | 0.000 | 0.035  | 0.000 |
| 2   | SLK 5   | 0.000 | 0.000 | 0.000 | 0.000 | -5.455 | 0.000 |
| 3   | X1      | 1.000 | 0.000 | 0.000 | 0.000 | -0.182 | 0.000 |
| 4   | SLK 4   | 0.000 | 0.000 | 0.000 | 0.000 | 1.000  | 1.000 |
| 5   | X2      | 0.000 | 1.000 | 0.000 | 0.000 | -0.091 | 0.000 |
| 6   | X3      | 0.000 | 0.000 | 1.000 | 0.000 | 0.273  | 0.000 |
| 7   | SLK 2   | 0.000 | 0.000 | 0.000 | 1.000 | 0.091  | 0.000 |

| ROW | SLK 5 | SLK 6   |        |
|-----|-------|---------|--------|
| 1   | 0.000 | 0.016   | -0.399 |
| 2   | 1.000 | -11.591 | 3.864  |
| 3   | 0.000 | -0.136  | 0.045  |
| 4   | 0.000 | 0.000   | 2.000  |
| 5   | 0.000 | 0.182   | 0.273  |
| 6   | 0.000 | -0.045  | 0.682  |
| 7   | 0.000 | 3.318   | 0.227  |

## ***Sensitivity Analysis***

- Suppose that the cost of ingredient #2 varies considerably from day to day. If all other ingredient costs remain the same, by how much can the cost of ingredient increase or decrease without changing the optimal formula for chicken feed? \_\_\_\_\_
- If experts change their recommendation of vitamin A content from 180 units/kg to 181 units/kg,  
will this change the optimal cost? \_\_\_\_\_  
will it change the optimal formula? \_\_\_\_\_ (*If so, how?*)
- If experts change their recommendation of vitamin B content from 9 units/kg to 10 units/kg,  
will this change the optimal cost? \_\_\_\_\_  
will it change the optimal formula? \_\_\_\_\_ (*If so, how?*)

## LINGO model:

```
SETS:
    INGREDIENT /1..3/: PROTEIN, FAT, VITA, VITB, COST, X;
ENDSETS

DATA:
    PROTEIN = 25 45 32;
    FAT = 11 10 7;
    VITA = 235 160 190;
    VITB = 12 6 10;
    COST = 0.55 0.42 0.38;
ENDDATA

MIN = @SUM(INGREDIENT: COST*X);
@SUM(INGREDIENT: PROTEIN * X) >= 35; ! minimum protein ;
@SUM(INGREDIENT: FAT * X) >= 8; ! minimum fat ;
@SUM(INGREDIENT: FAT * X) <= 10; ! maximum fat ;
@SUM(INGREDIENT: VITA * X) >= 180; ! minimum vitamin A ;
@SUM(INGREDIENT: VITB * X) >= 9; ! minimum vitamin B ;
@SUM(INGREDIENT: X) = 1 ; ! total weight = 1 kg;

END
```

**Chicken  
Feed**

Suppose that a new ingredient has become available with the following characteristics:

|                      |        |
|----------------------|--------|
| Protein              | 2.2%   |
| Fat                  | 0.9%   |
| Vitamin A (units/kg) | 200    |
| Vitamin B (units/kg) | 5      |
| Cost (\$/kg)         | \$0.36 |

- Modify the LINGO model in order to consider this ingredient.
- Is the solution changed?