This Hypercard stack was prepared by:
Dennis L. Bricker,
Dept. of Industrial Engineering,
University of Iowa,
Iowa City, Iowa 52242
e-mail: dbricker@icaen.uiowa.edu
An airline operates 7 flights daily between New York & Chicago.

Crews are based in either city, and fly to-&-from the other city each day.
<table>
<thead>
<tr>
<th>Flight #</th>
<th>Leave Chgo.</th>
<th>Arrive N.Y.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 am</td>
<td>10 am</td>
</tr>
<tr>
<td>2</td>
<td>9 am</td>
<td>1 pm</td>
</tr>
<tr>
<td>3</td>
<td>noon</td>
<td>4 pm</td>
</tr>
<tr>
<td>4</td>
<td>3 pm</td>
<td>7 pm</td>
</tr>
<tr>
<td>5</td>
<td>5 pm</td>
<td>9 pm</td>
</tr>
<tr>
<td>6</td>
<td>7 pm</td>
<td>11 pm</td>
</tr>
<tr>
<td>7</td>
<td>8 pm</td>
<td>midnight</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flight #</th>
<th>Leave N.Y.</th>
<th>Arrive Chgo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 am</td>
<td>9 am</td>
</tr>
<tr>
<td>2</td>
<td>8 am</td>
<td>10 am</td>
</tr>
<tr>
<td>3</td>
<td>10 am</td>
<td>noon</td>
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<td>4</td>
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<td>6 pm</td>
</tr>
<tr>
<td>7</td>
<td>6 pm</td>
<td>8 pm</td>
</tr>
</tbody>
</table>
- Each of airline's crews live either in Chicago or in New York
- Each day, a crew must fly one NY-Chgo and one Chgo-NY flight
- There must be at least 1 hour layover time between flights
- At end of the day, a crew must be in its home city.
● The airline wants to assign the flights to the crews to minimize total layover time

● How many crews should be based in each city?
\[ X_{ij} = \begin{cases} 
1 & \text{if a crew is assigned flight \#i from Chgo-NY and flight \#j from NY-Chgo.} \\
0 & \text{otherwise} \end{cases} \]

For example, \( X_{14} = 1 \) indicates that a crew is assigned Chgo-NY Flight \#1, departing at 6 AM & arriving at 10 AM. After a 2-hour layover, they return to Chicago on NY-Chgo Flight \#4, departing at noon and arriving in Chicago at 2 PM.
<table>
<thead>
<tr>
<th></th>
<th>j=1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>6</td>
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<td>∞</td>
</tr>
</tbody>
</table>

- These pairs flown by a Chicago-based crew.
- These pairs flown by a NY-based crew.
- These pairs are not feasible.
Row Reduction

\[
\begin{array}{cccc}
\infty & \infty & \infty & 2 & 4 & 6 & 8 \\
\infty & \infty & \infty & \infty & 1 & 3 & 5 \\
3 & 2 & \infty & \infty & \infty & \infty & 2 \\
6 & 5 & 3 & 1 & \infty & \infty & \infty \\
8 & 7 & 5 & 3 & 1 & \infty & \infty \\
10 & 9 & 7 & 5 & 3 & 1 & \infty \\
11 & 10 & 8 & 6 & 4 & 2 & \infty
\end{array}
\]

\[
\begin{array}{cccc}
\infty & \infty & \infty & 0 & 2 & 4 & 6 \\
\infty & \infty & \infty & \infty & 0 & 2 & 4 \\
1 & 0 & \infty & \infty & \infty & \infty & 0 \\
5 & 4 & 2 & 0 & \infty & \infty & \infty \\
7 & 6 & 4 & 2 & 0 & \infty & \infty \\
9 & 8 & 6 & 4 & 2 & 0 & \infty \\
9 & 8 & 6 & 4 & 2 & 0 & \infty
\end{array}
\]
Column Reduction

\[
\begin{array}{cccc}
\infty & \infty & \infty & 0 \\
\infty & \infty & \infty & 0 \\
1 & 0 & \infty & \infty \\
5 & 4 & 2 & 0 \\
7 & 6 & 4 & 2 \\
9 & 8 & 6 & 4 \\
9 & 8 & 6 & 4 \\
\end{array}
\quad
\begin{array}{cccc}
\infty & \infty & \infty & 0 \\
\infty & \infty & \infty & 0 \\
0 & 0 & \infty & \infty \\
4 & 4 & 0 & 0 \\
6 & 6 & 2 & 2 \\
8 & 8 & 4 & 4 \\
8 & 8 & 4 & 4 \\
\end{array}
\]

\[
\begin{array}{cccc}
\infty & \infty & \infty & 0 \\
\infty & \infty & \infty & 0 \\
0 & 0 & \infty & \infty \\
4 & 4 & 0 & 0 \\
6 & 6 & 2 & 2 \\
8 & 8 & 4 & 4 \\
8 & 8 & 4 & 4 \\
\end{array}
\quad
\begin{array}{cccc}
\infty & \infty & \infty & 0 \\
\infty & \infty & \infty & 0 \\
0 & 0 & \infty & \infty \\
4 & 4 & 0 & 0 \\
6 & 6 & 2 & 2 \\
8 & 8 & 4 & 4 \\
8 & 8 & 4 & 4 \\
\end{array}
\]
<table>
<thead>
<tr>
<th></th>
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<th>2</th>
<th>4</th>
<th>6</th>
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</thead>
<tbody>
<tr>
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<td>6</td>
<td>2</td>
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<td>0</td>
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<td>4</td>
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<td>2</td>
<td>0</td>
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</tr>
</tbody>
</table>

The zeroes can be covered by only five lines!

Therefore, the matrix reductions are not complete.
### Mixed Reduction

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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Rightarrow

<p>| | | | | | |</p>
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<thead>
<tr>
<th></th>
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<tbody>
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</tbody>
</table>

Only six lines are required to cover all the zeroes!

<table>
<thead>
<tr>
<th>2</th>
<th>2</th>
<th>2</th>
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<td>2</td>
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</tbody>
</table>

Therefore, the matrix reductions are not complete.
```
\begin{array}{cccc}
\infty & \infty & \infty & 0 \\
\infty & 2 & 4 & 2 \\
\infty & 0 & 2 & 0 \\
\hline
0 & \infty & \infty & \infty \\
0 & \infty & \infty & \infty \\
2 & 2 & 2 & 2 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
\end{array}
\quad \Rightarrow
\begin{array}{cccc}
\infty & \infty & \infty & 0 \\
\infty & 2 & 4 & 2 \\
\infty & 0 & 2 & 0 \\
\hline
0 & 0 & \infty & \infty \\
0 & 0 & \infty & \infty \\
0 & 0 & 2 & \infty \\
2 & 2 & 2 & 4 \\
2 & 2 & 2 & 4 \\
\end{array}
```
Only six lines are required to cover all the zeroes!

Therefore, the matrix reductions are not complete.
### Mixed Reduction

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tbody>
</table>

\[ \begin{array}{cccccc}
\infty & \infty & \infty & 0 & 2 & 6 & 2 \\
\infty & \infty & \infty & \infty & 0 & 4 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 2 \\
0 & 0 & 0 & 2 & \infty & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty \\
2 & 2 & 2 & 4 & 2 & 0 & \infty \\
2 & 2 & 2 & 4 & 2 & 0 & \infty \\
\end{array} \]
The zeroes cannot be covered by fewer than seven lines!

Therefore, we should be able to find a zero-cost assignment.
We look for a row or a column with a single zero...

...this indicates a necessary assignment
Total cost = 25 hours
(= total layover time)

(there are several optimal solutions)
Optimal Solution

2 Chicago-based crews, flying
   Flight #1 to NY, then Flight #4 to Chgo.
   Flight #2 to NY, then Flight #7 to Chgo.

5 New-York-based crews, flying
   Flight #1 to Chgo, then Flight #3 to NY
   Flight #2 to Chgo, then Flight #4 to NY
   Flight #3 to Chgo, then Flight #5 to NY
   Flight #5 to Chgo, then Flight #6 to NY
   Flight #6 to Chgo, then Flight #7 to NY

|←|