

Assume that, in the ordinary simplex tableau, previous pivots have increased the density such that we cannot make good use of sparse matrix techniques.

Let's count the number of multiplications & divisions per pivot.

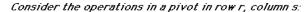


- "ordinary" simplex method
- "revised" simplex method

requires the least computational effort?

Computational effort per pivot depends upon the problem parameters

- n = # columns of A
- m = # constraints
- d = density of A (% nonzero elements)



[c ¹	$\widehat{\mathbb{C}}^2$	 Ĉŝ	 Ĉ	ĝ
Â1	\widehat{A}_1^2	 \widehat{A}_1^s	 \widehat{A}_1^n	ĥ₁
\widehat{A}_2^1	\widehat{A}_2^2	 \widehat{A}_2^{s}	 \widehat{A}_2^n	6₁ 6₂ 6r
:	÷	:	:	:
\widehat{A}_r^1	\widehat{A}_r^2	 $\widehat{(\widehat{A}_r^s)}$	 Ân Ar	ĥ₁
:	÷	$\widetilde{}$:	:
\widehat{A}_{m}^{1}	\widehat{A}_m^{2}	 \widehat{A}_m^s	 \widehat{A}_{m}^{n}	b̂ _m _

→ Ordinary Simplex Method

Pivoting in full tableau, with 100% density

→ Revised Simplex Method

Explicit basis inverse maintained, and density less than 100%

→ Comparison of Algorithms



Operation Count (x and ÷) per iteration

☐ Minimum Ratio Test (pivot row selection)

m divisions

Pivot:

 $oldsymbol{\square}$ Divide row r by $\widehat{\mathsf{A}}_{\mathsf{r}}^{\mathsf{s}}$ (need not divide in basic columns).

n-m divisions



□ For i=1,2,...m+1, i≠r, $\text{add} \quad -\widehat{A}_i^s \text{ times row r to row i}$

(only necessary to compute elements in nonbasic columns)

(n-m) multiplications per each of m rows



Total number of multiplications & divisions:

$$N_S = m + (n-m) + m(n-m)$$

= m + n + mn - m²

per iteration.



Revised Simplex Method

 $fill \mbox{Pricing each of (n-m) nonbasic columns} \qquad \widehat{C}^j = \pi \ \mbox{$\mathbb{A}j (selecting pivot column)

(dm) multiplications per each of (n-m) columns

4

- $\mbox{\ensuremath{\square}}$ Pivot (update of basis inverse matrix, rhs, & π)
 - divide row r of $(A^B)^{-1} \& \hat{b}$ by pivot element (m+1) divisions
 - For i= 0,1, 2, ...m (i = r):
 Add multiple of row r to row i

(m+1) multiplications per each of m rows

\square Computing substitution rates $\widehat{A}^s = (A^B)^{-1} A^j$ (computing pivot column)

dm multiplications per each of m rows

☐ Minimum ratio test (pivot row selection)

m divisions

Revised Simplex Method

Total number of multiplications & divisions:

$$N_R = dm(n-m) + dm^2 + m + (m+1) + (m+1)n$$

= $dmn + m^2 + 3m + 1$
per iteration.



Comparison of Algorithms

Multiplicatons & Divisions per iteration:

Ordinary Simplex
$$N_S = m + n + mn - m^2$$

Revised Simplex
$$N_R = dm n + m^2 + 3m + 1$$

Under what conditions is the revised simplex method more efficient that the ordinary simplex method?

That is, when is
$$N_R < N_S$$
 ?

So the revised simplex method is more efficient than the ordinary simplex method when the density of the coefficient matrix A satisfies:

$$d < 1 - 2\frac{m}{n}$$

For example:

m	n	1-2 m
10	50	60%
100	1000	80%
100	10000	98%

If m=10 & n=50, then the revised simplex method is more efficient if the density is less than about 60%

$$N_S = m + n + mn - m^2$$

 $N_R = dmn + m^2 + 3m + 1$

For large LP problems in the "real world", the density is typically no more than 5%.

If m=100 and n=1000, $N_S = 91100$

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	d=1%	d=5%		
N_R	11301	15301		
N_R/N_s	0.124	0.168		

