A flowshop is modeled as a Markov queueing network of four machine types, with $\lambda=5$ jobs/hr entering at machine $A$, routed to either machine $B_1$ or $B_2$ (whose service rates $\mu$ differ), then to machine $C_1$ or $C_2$ (which also have different service rates $\mu$) and finally to machine $D$ and then exits.

How should the jobs be routed so that the average number of jobs in the flowshop is minimized?

The average total number of jobs in the flowshop is

$$L_{\text{total}} = L_A + L_{B_1} + L_{B_2} + L_{C_1} + L_{C_2} + L_D$$

$$= 5 + \frac{\lambda_1}{3-\lambda_1} + \frac{\lambda_2}{4-\lambda_2} + \frac{\lambda_3}{3-\lambda_3} + \frac{\lambda_4}{4-\lambda_2-\lambda_4} + \frac{\lambda_5}{8-\lambda_1-\lambda_5}$$

where the $\lambda$'s are chosen to satisfy conservation of flow, i.e., rate of flow out of machine center = rate of flow into machine center.
Routing in Queueing Network 3/26/2006 page 5 of 9

\[ \lambda = 5 \]

\[ \mu_A = 6 \]

\[ \lambda_1 \]

\[ \mu_{B1} = 3 \]

\[ \lambda_3 \]

\[ \mu_{C1} = 3 \]

\[ \lambda_5 \]

\[ \mu_D = 8 \]

\[ \lambda_6 \]

\[ A \]

\[ B1 \]

\[ B2 \]

\[ C1 \]

\[ C2 \]

\[ D \]

\[ L^*_{total} = 5 + \text{Minimum} \]

\[ \frac{\lambda_1}{3-\lambda_1} + \frac{\lambda_2}{4-\lambda_2} + \frac{\lambda_3}{3-\lambda_3} + \frac{\lambda_4}{4-\lambda_2-\lambda_4} + \frac{\lambda_5}{8-\lambda_3-\lambda_5} \]

s.t.

\[ \lambda_1 + \lambda_2 = 5 \]

\[ \lambda_3 + \lambda_4 = \lambda_1 \]

\[ \lambda_2 + \lambda_4 = \lambda_5 \]

\[ \lambda_3 + \lambda_5 = \lambda_4 \]

\[ \lambda_i \geq 0, \ i=1,2,...6 \]

\( \text{That is, } \frac{2.072}{5} = 41.44\% \text{ of jobs when leaving machine } A \text{ should be routed to machines } B1\&C1, \text{ and the remainder to machine } B2\&C2. \)

\( \text{This should result in an average of approximately } 16.59 \text{ jobs in the flowshop at any time.} \)
According to Little's Law for queues,

\[ L = \lambda W, \quad \text{where} \]

\( L \) = average number in the queueing system,
\( W \) = average time in the queueing system,
\( \lambda \) = average arrival rate,

and so the **average time** spent by a job in this flowshop will be

\[ W = \frac{L}{\lambda} = \frac{16.59}{5/hr} = 3.318 \text{ hours} \]