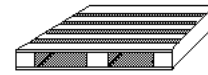


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An analyst for Labatt's Ontario Breweries was faced with a question posed by the warehouse manager concerning the repair and replacement policy for pallets used in the shipment of beer.



Should the pallets be repaired when damaged, and if so, how often? This question always arose in early summer when the supply of pallets became tighter as a result of increased summer shipments.

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Labatt's belongs to a pallet pool with the other breweries in Ontario and Quebec and their common retail outlets.

The pallets were used for shipments to the retail outlets from the breweries and for the return of empties. Each brewery shipped its products to many locations throughout both provinces. The use of a common bottle and pallet by breweries in Canada allowed the return of empties to any brewery.

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Damaged pallets were repaired by individual breweries if feasible. If a center block had been damaged, the pallet was not repaired.

When questioned, the foreman at Labatt's in charge of pallet repair said: "Pallets are damaged through abuse by the handlers." Often, in repairing a pallet, the new nails would split a center block which would necessitate scrapping of the pallet.

Approximately 10 percent of the damaged pallets were unreparable.

New bottles, when shipped to the breweries from the glass manufacturers, were shipped on new pallets. The bill for the new pallets was included with the bill for the new bottles.

From time to time, additional pallets could be ordered by the individual breweries, if required. The number of new pallets purchased by each brewery was recorded, and at the end of each fiscal year, breweries that had purchased more than their share were compensated by those breweries that had purchased less than their share. (A brewery's share of new pallets to be purchased was determined by its share of the market.)

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Average cost of a new pallet is \$4.47
Average cost of repairing a pallet was \$2.07

The foreman in charge of repairs considered those over 2 years old not worth repairing, and they were scrapped.

Labatt's current repair policy is to repair a damaged pallet only if the age is no more than two years.

\$0.75 was the most ever received for damaged pallets, but the warehouse manager thought that as much as \$1.50 might be obtained if the scrapped pallets were in a better condition.

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Average Damage Rates

description	% damaged in year #			
	one	two	three	four
New pallets	22	45	33	--
Pallets repaired in year #1	--	47	48	5
Pallets repaired in year #2	--	--	83	17

Using the damage rates, we will compute the transition probabilities

Note that each row sums to 100%!

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Current Policy

pallets over two years old are scrapped when damaged

Consider the pallet to age 1 year on the last day of the year. Assume that the state of the system is observed on the last day of the year.

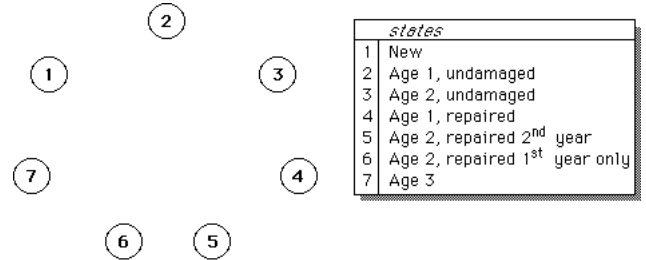
Possible states:

- 1 New (less than one year old)
- 2 Age 1, never damaged
- 3 Age 2, never damaged
- 4 Age 1, repaired
- 5 Age 2, repaired second year (& possibly first)
- 6 Age 2, repaired first year only
- 7 Age 3

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Rather than model a "typical" pallet, which has a limited lifetime (no more than 4 years), we will model a pallet AND its future replacements, so that when a pallet is scrapped, the state of the pallet returns to "New", i.e., the state of the replacement pallet.

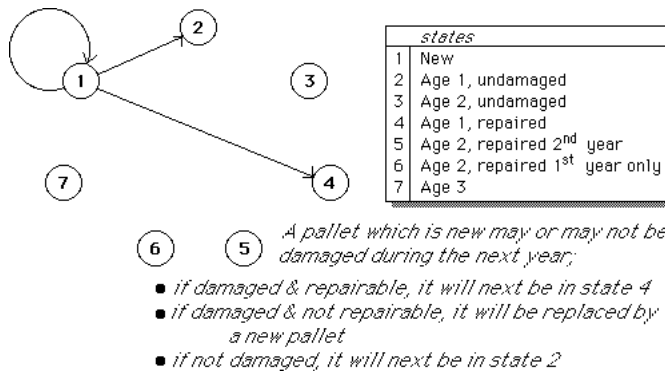
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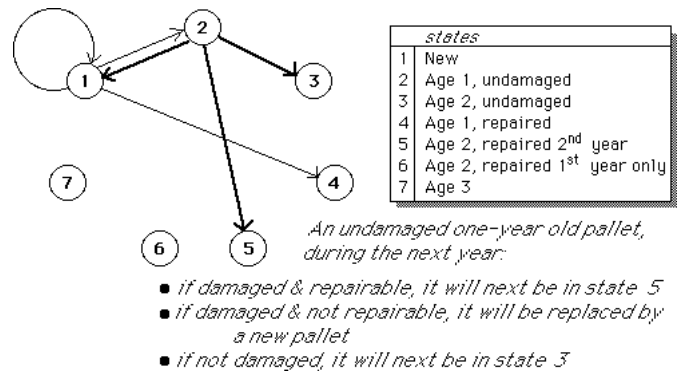
states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

What transitions are possible?

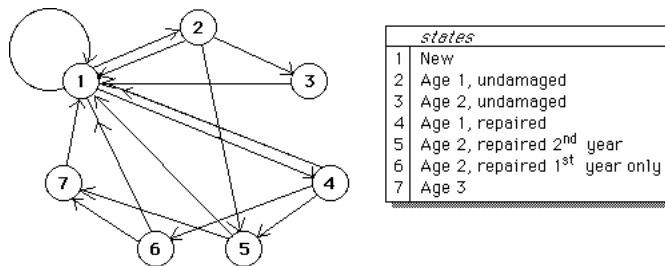
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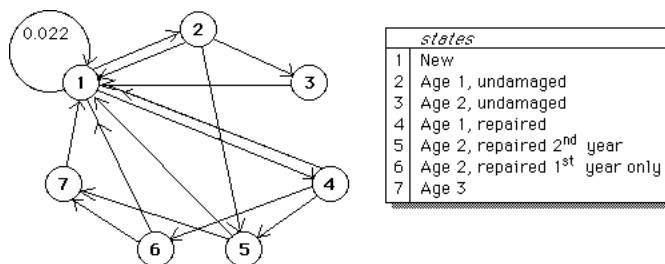
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Average Damage Rates

description	% damaged in year #			
	one	two	three	four
New pallets	22	45	33	--
Pallets repaired in year #1	--	47	48	5
Pallets repaired in year #2	--	--	83	17

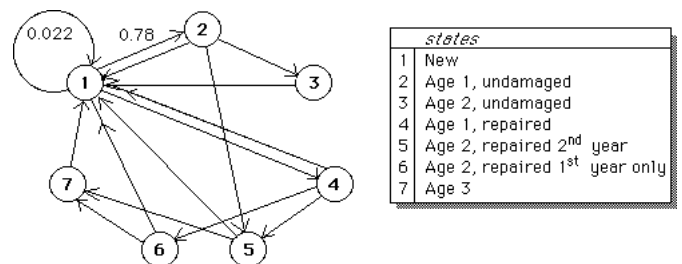
Using the damage rates, we will compute the transition probabilities

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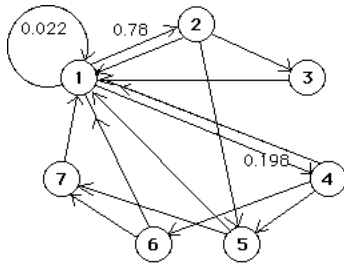
p_{11} = probability that new pallet is damaged in first year, but is not repairable
 = $0.22 \times 0.10 = 0.022$

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p_{12} = probability that new pallet is not damaged in its first year
 = $1 - 0.22 = 0.78$

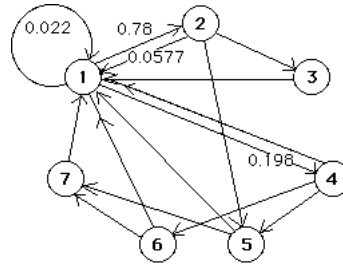
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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

p_{14} = probability that new pallet is damaged in its first year, and is repairable
 = $0.22 \times 0.90 = 0.198$

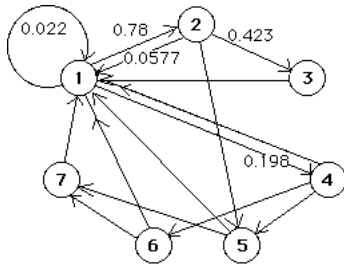
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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

p_{21} = probability that undamaged 1-yr-old pallet is damaged next year & is unrepairable
 = $\frac{0.45}{1-0.22} \times 0.10 = 0.0577$

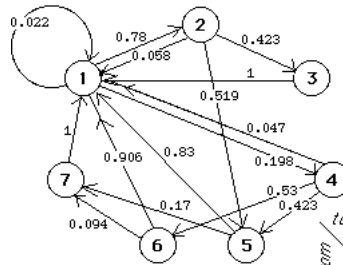
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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

p_{23} = probability that undamaged 1-yr-old pallet is not damaged in its second year
 = $\frac{0.33}{0.33+0.45} = 0.423$

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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transition Probability Matrix								
	from	1	2	3	4	5	6	7
1	1	0.022	0.78	0	0.198	0	0	0
2	1	0.0577	0.058	1	0.519	0	0	0
3	1	0	0	0	0	0	0	0
4	1	0.047	0	0	0	0.423	0.53	0
5	1	0.83	0	0	0	0	0	0.17
6	1	0.906	0	0	0	0	0	0.094
7	1	0	0	0	0	0	0	0

etc.

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Steady State Distribution

i		P(i)
1	New	0.3339594
2	Age 1, undamaged	0.26048833
3	Age 2, undamaged	0.11018657
4	Age 1, repaired	0.066123962
5	Age 2, Repair yr 2	0.16316388
6	Age 2, repair yr 1	0.0350457
7	Age 3	0.031032156

Current Policy

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System costs/revenues by state

i		--COST--
1	New	3.72
2	Age 1, undamaged	0
3	Age 2, undamaged	0
4	Age 1, repaired	2.07
5	Age 2, Repair yr 2	2.07
6	Age 2, repair yr 1	0
7	Age 3	0

Current Policy

assuming 75¢ scrap value

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i	state	Pi	C	Pi×C
1	New	0.3339594	3.72	1.242329
2	Age 1, undamaged	0.26048833	0	0
3	Age 2, undamaged	0.11018657	0	0
4	Age 1, repaired	0.066123962	2.07	0.1368766
5	Age 2, Repair yr 2	0.16316388	2.07	0.33774923
6	Age 2, repair yr 1	0.0350457	0	0
7	Age 3	0.031032156	0	0

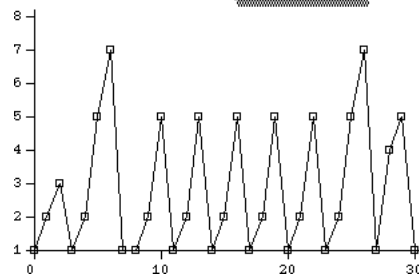
Current Policy

The average cost/period in steady state is 1.7169548

Annual cost of a pallet, under the current policy, is \$1.7169548

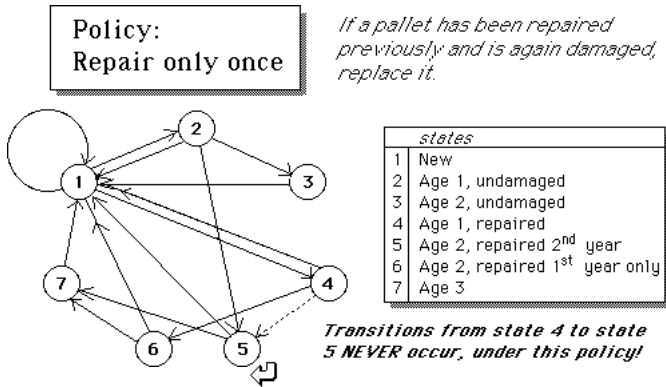
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Repetition #1

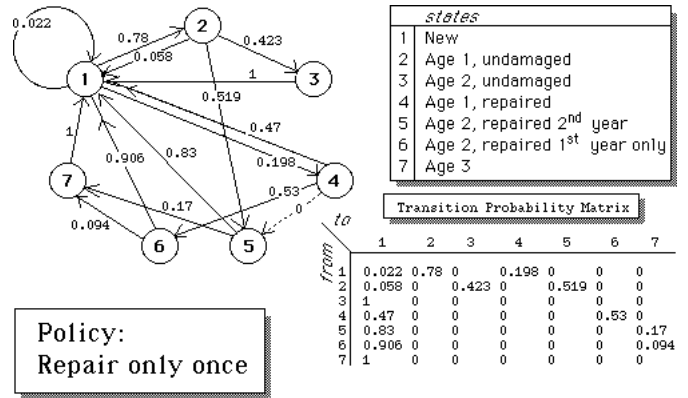


Simulation of a pallet (current policy)

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Policy:
Repair only once

Steady State Distribution

i	P(i)	
1	New	0.34525812
2	Age 1, undamaged	0.26930133
3	Age 2, undamaged	0.11391446
4	Age 1, repaired	0.068361107
5	Age 2, Repair yr 2	0.13976739
6	Age 2, repair yr 1	0.036231387
7	Age 3	0.027166207

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Assuming 75¢ scrap value:

i	state	Pi	C	Pi×C
1	New	0.34525812	3.72	1.2843602
2	Age 1, undamaged	0.26930133	0	0
3	Age 2, undamaged	0.11391446	0	0
4	Age 1, repaired	0.068361107	2.07	0.14150749
5	Age 2, Repair yr 2	0.13976739	2.07	0.2893185
6	Age 2, repair yr 1	0.036231387	0	0
7	Age 3	0.027166207	0	0

The average cost/period in steady state is 1.7151862
Annual cost per pallet, under this policy, is
\$1.7151862
which is slightly less than that of the current policy,
\$1.7169548

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Policy:
Repair only once

Assuming 75¢ scrap value:

Present value of all future costs, for each initial state:

(using interest rate 20%, i.e. discount factor 0.83333333)

i	PV
1	11.40129
2	8.7882537
3	9.5010753
4	10.666072
5	11.301878
6	9.3522251
7	9.5010753

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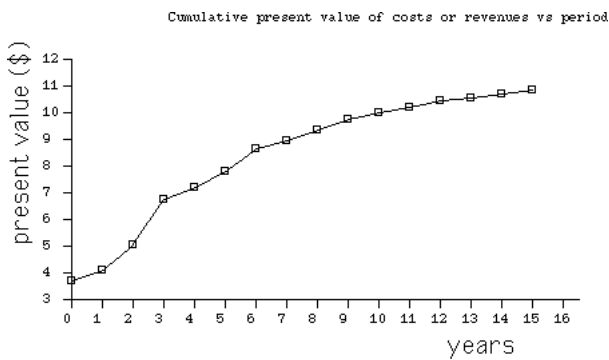
Policy:
Repair only once

Present Values by Period

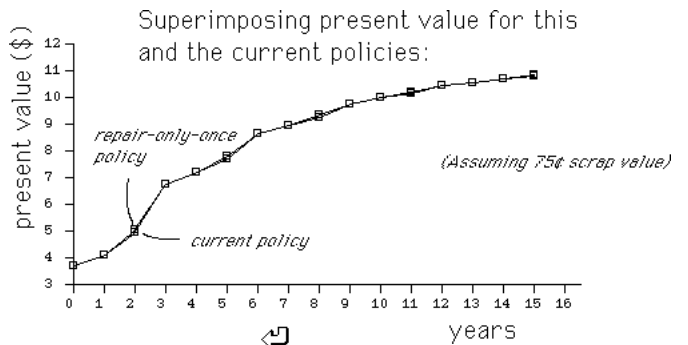
n	Cn	PVCn	Vn
0	3.72	3.72	3.72
1	0.4917	0.40975	4.12975
2	1.3632708	0.94671583	5.0764658
3	2.9289745	1.6950084	6.7714742
4	0.91987306	0.44361162	7.2150858
5	1.5014744	0.60340889	7.8184947
6	2.4965153	0.83607792	8.6545726
7	1.1930783	0.33296626	8.9875389
8	1.5865413	0.36897879	9.3565177
9	2.217937	0.42985104	9.7863687
10	1.3728005	0.22168264	10.008051
11	1.6381974	0.22048169	10.228533
12	2.0385921	0.22864167	10.457175
13	1.4904986	0.13930778	10.596483
14	1.6694245	0.13002574	10.726508
15	1.9231645	0.1248239	10.851332

r=20%
initial state: 1

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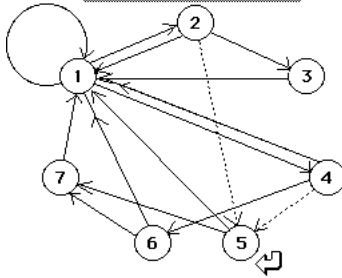
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Policy:
Repair in first year only

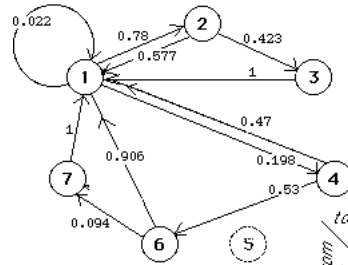
If a pallet is age 1 or greater, and is damaged, replace it



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transitions into state 5 will never occur!

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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Transition Probability Matrix		to						
from		1	2	3	4	5	6	7
1	1	0.022	0.78	0	0.198	0	0	0
1	2	0.577	0	0.423	0	0	0	0
1	3	1	0	0	0	0	0	0
1	4	0.47	0	0	0	0	0.53	0
1	5	0	0	0	0	1	0	0
1	6	0.906	0	0	0	0	0	0.094
1	7	1	0	0	0	0	0	0

Policy:
Repair in first year only

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Transition Probability Matrix

from		to						
		1	2	3	4	5	6	7
1	1	0.022	0.78	0	0.198	0	0	0
1	2	0.577	0	0.423	0	0	0	0
1	3	1	0	0	0	0	0	0
1	4	0.47	0	0	0	0.53	0	0
1	5	0.906	0	0	0	0	0.094	0
1	6	1	0	0	0	0	0	0

- 1 New
- 2 Age 1, undamaged
- 3 Age 2, undamaged
- 4 Age 1, repaired
- 5 Age 2, repaired
- 6 Age 3

Policy:
Repair in first year only

Let's eliminate the state Age 2, repaired 2nd year and renumber the states

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Assuming 75¢ scrap value:

i	State	Pi	C	PixC
1	New	0.41275506	3.72	1.5354488
2	Age 1, undamaged	0.32194895	0	0
3	Age 2, undamaged	0.13618441	0	0
4	Age 1, repaired	0.081725502	2.07	0.16917179
5	Age 2, repaired	0.043314516	0	0
6	Age 3	0.0040715645	0	0

The average cost/period in steady state is 1.7046206

The annual cost per pallet will be \$1.7046206

Policy:
Repair in first year only

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Policy:
Repair in first year only

Present value of all future costs, for each initial state:
(using interest rate 20%, i.e. discount factor 0.83333333)

i	State	FV
1	New	11.480285
2	Age 1, undamaged	8.8924378
3	Age 2, undamaged	9.5669045
4	Age 1, repaired	10.72563
5	Age 2, repaired	9.417023
6	Age 3	9.5669045

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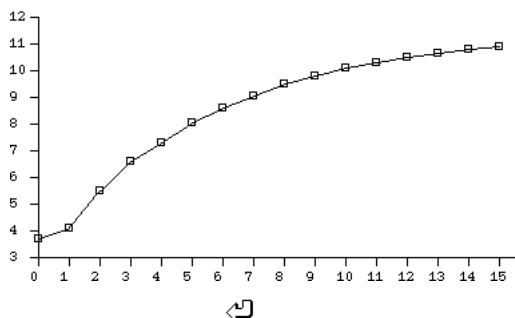
Present Values by Period

n	Cn	PVCn	Vn
0	3.72	3.72	3.72
1	0.4917	0.40975	4.12975
2	2.0312238	1.4105721	5.5403221
3	1.8927972	1.0953688	6.6356908
4	1.3905154	0.67058035	7.3062712
5	1.9267595	0.77432145	8.0805926
6	1.6221106	0.54324155	8.6238342
7	1.6918101	0.47215315	9.0959873
8	1.75084	0.40718943	9.5031768
9	1.665803	0.32284378	9.8260206
10	1.7226107	0.27821125	10.104232
11	1.7034514	0.22926409	10.333496
12	1.6983235	0.19047523	10.523974
13	1.7111102	0.159927	10.683901
14	1.7010238	0.1324869	10.816388
15	1.7053782	0.11068838	10.927076

Policy:
Repair in first year only

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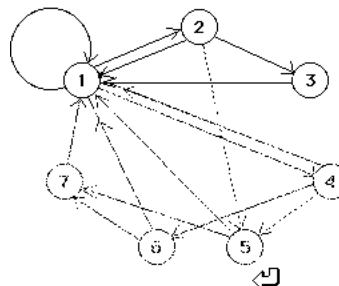
Cumulative present value of costs or revenues vs period



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Policy:
No repairs

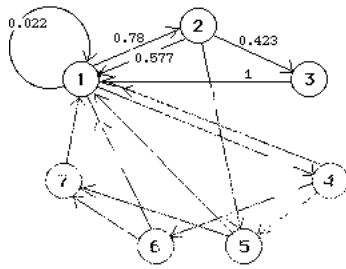
Always replace a damaged pallet (thereby eliminating the need for a repair shop.)



states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged
4	Age 1, repaired
5	Age 2, repaired 2 nd year
6	Age 2, repaired 1 st year only
7	Age 3

Only states 1, 2, & 3 are possible!

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states	
1	New
2	Age 1, undamaged
3	Age 2, undamaged

Transition Probability Matrix

from \ to	1	2	3
1	0.22	0.78	0
2	0.577	0	0.423
3	1	0	0

**Policy:
No repairs**

Assuming a 75¢ scrap value:

**Policy:
No repairs**

i	Pi	C	Pi×C
1	0.47394713	3.72	1.7630833
2	0.36967876	0	0
3	0.15637411	0	0

The average cost/period in steady state is 1.7630833

The annual cost per pallet will be \$1.7630833

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Assuming a 75¢ scrap value:

**Policy:
No repairs**

Present value of all future costs, for each initial state:
(using interest rate 20%, i.e. discount factor 0.8333333)

i	State	FV
1	New	11.877869
2	Age 1, undamaged	9.2003991
3	Age 2, undamaged	9.8982239

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Initial state: 1, interest rate= 20%

**Policy:
No repairs**

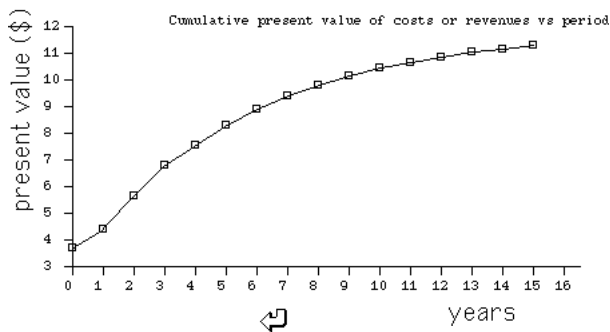
Present Values by Period

n	Cn	PVCn	Vn
0	3.72	3.72	3.72
1	0.8194	0.682	4.402
2	1.8542712	1.2876883	5.6896883
3	2.0036456	1.1595171	6.8492054
4	1.5453582	0.74525377	7.5944592
5	1.8535378	0.74459526	8.3393545
6	1.764365	0.59085228	8.9302368
7	1.732239	0.48343512	9.4136729
8	1.786719	0.41553373	9.8292066
9	1.7548243	0.3400967	10.169303
10	1.761727	0.28452875	10.453832
11	1.7668662	0.23779896	10.691631
12	1.7605801	0.19746078	10.889092
13	1.7637876	0.16485044	11.053942
14	1.7633598	0.13734204	11.191284
15	1.7626352	0.11440467	11.305689

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Assuming a 75¢ scrap value:

Summary



Policy	Annual Cost per Pallet	
	75¢ scrap value	\$1.50 scrap value
Current policy	\$ 1.7169548	
Repair only once	\$ 1.7151862	
Repair First Year Only	\$ 1.7046206	
Never Repair	\$ 1.7630833	