Linear Programming Models

Definitions

linear function $f(x_1, x_2, \cdots x_n) = \sum_{j=1}^{n} c_j x_j$
$= c_1 x_1 + c_2 x_2 + \cdots c_n x_n$

linear inequality $\sum_{j=1}^{n} a_j x_j \leq b$

Linear Programming

--an optimization problem for which:

- we maximize or minimize a linear function of the *decision variables* (the *objective* function)
- the values of the decision variables must satisfy a set of *constraints*, each consisting of a linear equation or a linear inequality
- a sign restriction (nonnegative, i.e.≥ 0, nonpositive, i.e., ≤ 0) may be associated with each decision variable.

Assumptions of LP Model

- **Proportionality**: the contribution of each variable to the objective function or constraint function is proportional to the value of that variable

E.g., doubling a decision variable will double the contribution to the cost function.

This implies that there are no economies of scale!

Assumptions of LP Model

- **Additivity:** The contribution of any variable to the objective or a constraint function is independent of the values of the other decision variables.
- **Certainty:** All parameters (cost coefficients, right-hand-sides of constraints, etc.) are known with certainty, i.e., are not random.
- **Continuity:** Each decision variable can assume real values, i.e., are not restricted to a discrete set of values, e.g., integers

Product Mix

Consider a manufacturer that produces 2 products: Widgets and Frisbees. Each product is made from the two raw materials, polyester and polypropylene. The following table gives the amounts of raw material required per unit of the two products:

Examples

- Product Mix
- Blending
- Investment Planning
- Feedem-Speedem Airlines
- Fine-Webb Paper Company