

🕼 Example Four

Max z = subject to	$2x_1$	- X2	+ X3		
	·				(0
	$\mathfrak{I}\mathfrak{I}\mathfrak{X}_1$	+ X ₂	+ X3	2	60
1	$\mathbf{x_1}$	- X2	+ X ₃ + 2X ₃ - X ₃	≤	10
l	X ₁	+ X2	- X3	≤	20
	X1	ι, Χ ₂ ,	X ₃ ≥	0	

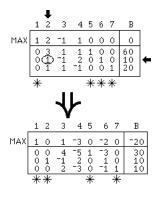
Max z =	$2x_1 - x_2 + x_3$	
subject to		
	$\begin{cases} 3x_1 + x_2 + x_3 + x_4 &= 6\\ x_1 - x_2 + 2x_3 &+ x_5 &= 1\\ x_1 + x_2 - x_3 &+ x_6 &= 2 \end{cases}$	0
-	$x_1 - x_2 + 2x_3 + x_5 = 1$	0
	$x_1 + x_2 - x_3 + x_6 = 2$	0
	$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$	

Add slack variables to convert the inequalities to equations

-	Z	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	
	1	2	3	4	5	6	7	В
MAX	1	2	-1	1	0	0	0	0
	0 0 0	3 1 1	1	1 -2 -1	1	0 1 0	0 0 1	60 10 20

EXAMPLE ONE

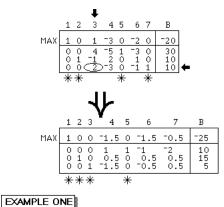
EXAMPLE ONE



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EXAMPLE ONE

MAX	-Z X ₁ X ₂ <u>1 2 3</u> <u>1 0 0 -1</u> 0 0 0 1 0 1 0 0 0 0 1 -1 ***	456 .50 1.5	7 -0.5	B -25 10 15 5	optimal tableau!
-z = -25	⇒ z = 2 <u>5</u>	5		<i>basic</i> z = 2 X ₁ = 1 X ₂ = 5 X ₄ = 1	
EXAMPLE ONE		K⊅			



Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$ subject to $\begin{array}{rl} x_1 + 2 x_2 + 3 x_3 & = 15 \\ 2 x_1 & + x_2 & - 5 x_3 & = 20 \end{array}$ $x_1 + 2x_2 - x_3 + x_4 = 10$ x_1 , x_2 , x_3 , $x_4 \ge 0$ $-\mathbf{z} \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_3 \mathbf{x}_4$ 1 2 3 4 5 В MAX 1 1 3 -1 0 2 0 1 0 2 0 1 2 1 2 -5 -5 -1 0 0 1 15 20 10 *

EXAMPLE TWO

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-z and X_4 can serve as the basic variables in the top and bottom rows, respectively.

We need basic variables for rows 2&3 also!

Minimize $\mathbf{w} = \mathbf{x}_5 + \mathbf{x}_6$ $z + x_1 + 2x_2 + 3x_3 - x_4$ = 0 subject to $x_1 + 2x_2 + 3x_3$ = 15+ x₅ $2x_1 + x_2 - 5x_3$ $+ x_6 = 20$ $x_1 + 2x_2 - x_3 + x_4$ = 10 x_1 , x_2 , x_3 , $x_4 \geq 0$, $\ x_5 \geq 0, x_6 \geq 0$

Artificial variables X₅ & X₆ are added to the first two constraints to serve as initial basic variables.

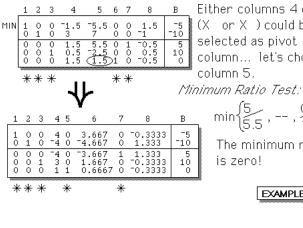
EXAMPLE TWO

	1	2	3	4	5	6	7	8	В	
MIN	1 0	0 1	-3 2	-3 4	2 2	0 0	0 0	0 0	-35 10	
	0 0 0	0 0 0	1 2 1	2 1 2	-5 -5	0 0 1	1 0 0	0 1 0	15 20 10	
	*	*				*	*	*		

We are minimizing the Phase-One objective, and select a pivot column having a negative reduced cost in the objective row.

Two columns have a negative reduced cost. Pivoting in either column should reduce the value of the objective.

EXAMPLE TWO



Either columns 4 or 5 (X or X) could be selected as pivot column... let's choose column 5.

 $\min \left\{ \frac{5}{5.5}, --, \frac{0}{1.5} \right\}$

The minimum ratio is zero!

EXAMPLE TWO

	1	2	3	4	5	6	7	8	В
MIN	1	0 1	0	3 0	0	2.75	0.25	0	-3.75 -15
	0000	0 0 0	0 1 0	-3 2 0	001	-2.75 0.75 -0.25	0.75 0.25 0.25	1 0 0	3.75 11.25 1.25
	*	*	*	÷	*			*	

In this tableau, one of the artificial variables remains basic (and positive).

This indicates that the original LP had no feasible solution, since a feasible solution (with artificial variables zero) would be optimal for Phase One, if such a solution exists!

EXAMPLE TWO

MIN

 $^{1}_{0}$

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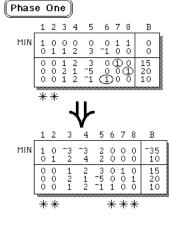
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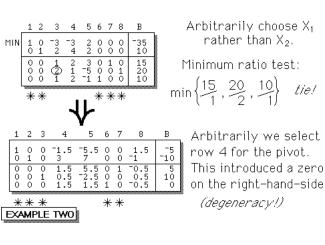
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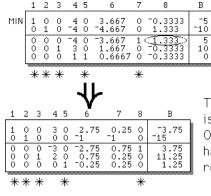


First, pivot so as to eliminate X₅ & X₆ from the top row and X₄ from the second row.

We now have a basic "pseudo-feasible" solution with which to begin the Simplex method.

EXAMPLE TWO





Choose column #8 for the next pivot. There is only one candidate row for the pivot.

The resulting tableau is optimal (for Phase One), since no column has a negative reduced cost.

EXAMPLE TWO

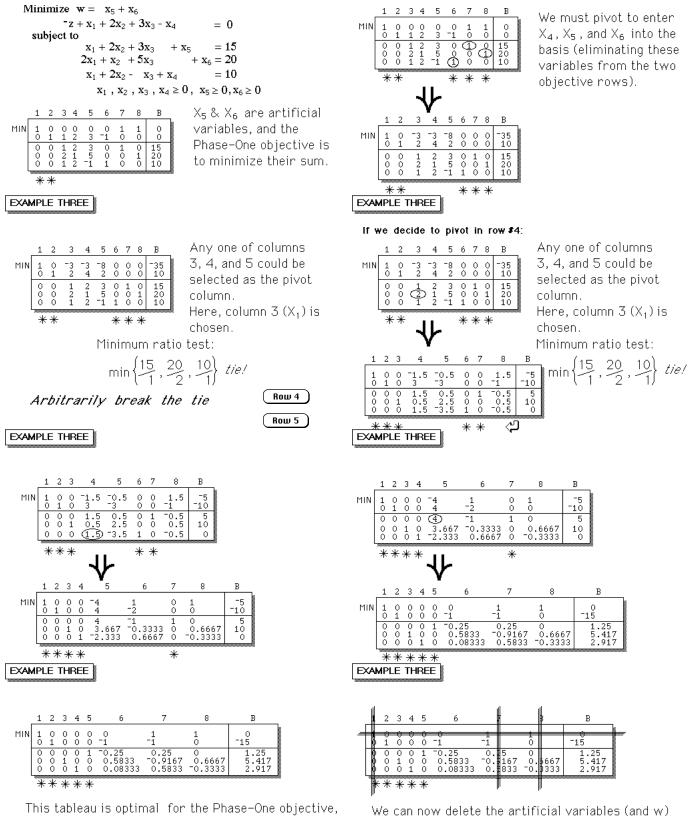
Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$ subject to

 $x_1 + 2x_2 + 3x_3$ = 15 $2x_1 + x_2 + 5x_3$ = 20 $x_1 + 2x_2 - x_3 + x_4 = 10$ x_1 , x_2 , x_3 , $x_4 \ge 0$

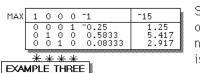
-z & X₄ can serve as basic variables in the first and last rows.

							-			
	1	2	3	4	5	В				
MAX	1	1	2	3	-1	0	1			
	0	1 2 1	2	3 5	0	15 20	1			
	ŏ	1	2	-1	1	10	ł			
	*				*					
EXAMPLE THREE										

The second and third rows will require artificial variables to serve as basic variables.



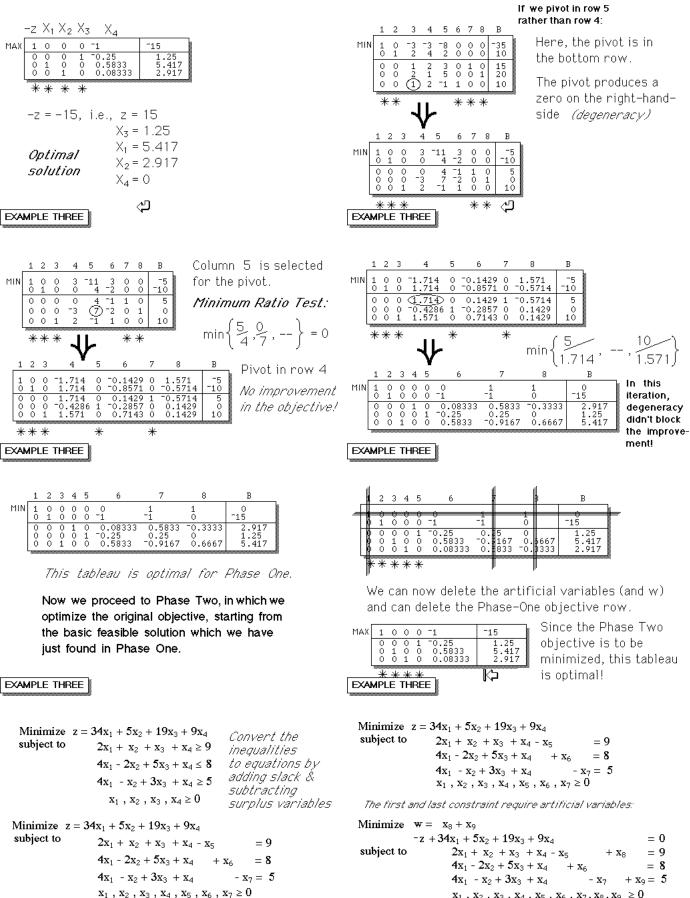
and provides us with a basic feasible solution with which to begin Phase Two.



and can delete the Phase-One objective row.

Since the Phase Two objective is to be minimized, this tableau is optimal!

EXAMPLE THREE

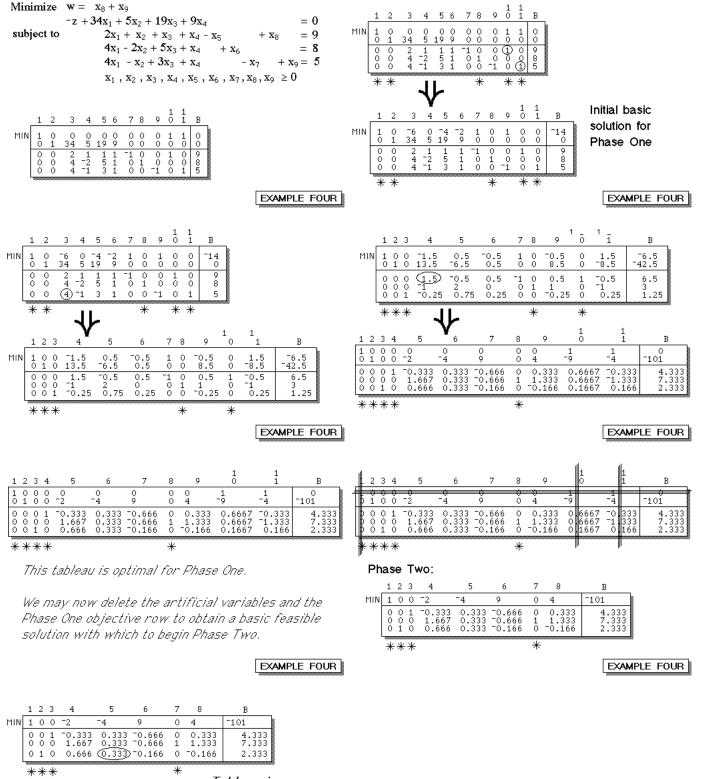


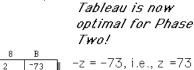
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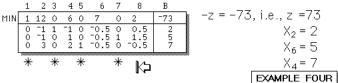
EXAMPLE FOUR

 x_1 , x_2 , x_3 , x_4 , x_5 , x_6 , x_7 , x_8 , $x_9 \ge 0$

EXAMPLE FOUR







0 0 0