Decision Analysis

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The decision process:

1) the decision-maker selects a decision from among the alternatives d_k, k=1,...n
2) after the decision is selected, one of the possible "states of nature", s_j, occurs
3) the decision-maker receives a "payoff" r_{kj} determined from a payoff table.

Three Classes of Decision Problems

- Decisions under *certainty*
  i.e., a single state of nature is possible
- Decisions under *risk*, in which the probability distribution of the state of nature is known
- Decisions under *uncertainty*, in which the state of nature has an unknown probability distribution

Criteria for decision-making under...

- risk
  - maximize expected return
  - maximize expected utility
  - minimize expected regret
- uncertainty
  - maximize minimum return
  - maximize maximum return
  - minimize maximum regret

NEWSBOY EXAMPLE

MAKE or BUY EXAMPLE

NEWSBOY PROBLEM

- The newsboy buys newspapers from the delivery truck at the beginning of the day, at a cost of 10¢ per paper
- During the day, he sells papers for 25¢ each
- Demand is a random variable, but with a known probability distribution:
  $P_0 = 0.1$, $P_1 = 0.3$, $P_2 = 0.4$, $P_3 = 0.2$
- At the end of the day, any leftover papers are without any value

NEWSBOY PROBLEM

Let d = # of papers ordered at beginning of the day (the "decision")
s = demand for papers ("state of nature")

Min(s,d) = # of papers sold

Payoff r_{ds} = 25(# of papers sold) − 10(# of papers ordered)
= $25\min\{s,d\} - 10\times d$

How many newspapers should the newsboy order from the delivery truck at the beginning of the day?

Because the probability distribution of the demand (state of "nature") is known, this is ***decision-making under risk***.

Payoff Table

State of Nature (demand)

Decision	0	1	2	3
0	0	0	0	0
1	-10	15	15	15
2	-20	5	30	30
3	-30	-5	20	45

Calculation of Expected Payoff

$$\sum_{j=1}^{4} r_{kj} \times P_j \quad \text{for } k=0,1,2,3$$

Decision		
0	0(0.1) + 0(0.3) + 0(0.4) + 0(0.2)	= 0
1	-10(0.1) +15(0.3) +15(0.4) +15(0.2)	= 12.5
2	-20(0.1) + 5(0.3) + 30(0.4) + 30(0.2)	= 17.5
3	-30(0.1) - 5(0.3) + 20(0.4) + 45(0.2)	= 12.5

To maximize the expected payoff, the newsboy should order 2 papers.

NEWSBOY PROBLEM

Suppose that nothing is known about the probability distribution of the demand

(although we still assume that possible demands are 0, 1, 2, & 3)

This is now an example of

decision-making under uncertainty

decision-making under uncertainty

Three commonly-used criteria:

- **maximin**, i.e., maximize the minimum payoff
- **maximax**, i.e., maximize the maximum payoff
- **minimax regret**, where "regret" is the opportunity cost of not making the best decision for a given state of nature.

MAXIMIN Criterion

$$\underset{k}{\text{Maximum}} \left\{ \underset{j}{\text{minimum}}\ r_{kj} \right\}$$

- a very conservative or pessimistic approach
- each decision is evaluated by calculating the worst payoff that can be received if you make that decision

MAXIMIN Criterion

$$\underset{k}{\text{Maximum}} \left\{ \underset{j}{\text{minimum}}\ r_{kj} \right\}$$

State of Nature (demand)

Decision	0	1	2	3	minimum payoff
0	0	0	0	0	0 ☜
1	-10	15	15	15	-10
2	-20	5	30	30	-20
3	-30	-5	20	45	-30

The newsboy should order no papers from the delivery truck!

MAXIMAX Criterion

$$\underset{k}{\text{Maximum}} \left\{ \underset{j}{\text{maximum}}\ r_{kj} \right\}$$

- a very optimistic approach
- each decision is evaluated by the best payoff that can be received if you make that decision

MAXIMAX Criterion

$$\underset{k}{\text{Maximum}} \left\{ \underset{j}{\text{maximum}}\ r_{kj} \right\}$$

State of Nature (demand)

Decision	0	1	2	3	payoff
0	0	0	0	0	0
1	-10	15	15	15	15
2	-20	5	30	30	30
3	-30	-5	20	45	45 ☜

The newsboy should order 3 papers from the delivery truck!

MINIMAX REGRET

$$\underset{k}{\text{Minimum}} \left\{ \underset{j}{\text{maximum}} \left[\max_{i} r_{ij} \right] - r_{kj} \right\}$$

"Regret" is the opportunity cost of not making the best decision for a given state of nature

For example, if the state of nature (i.e. demand) will be 2, the best decision that could have been made is of course 2, which earns a payoff of 30¢

If we instead had ordered 3, our payoff will be 20¢, and our regret is 10¢

Payoff

decision \ demand	0	1	2	3
0	[0]	0	0	0
1	-10	[15]	15	15
2	-20	5	[30]	30
3	-30	-5	20	[45]

Regret

decision \ demand	0	1	2	3
0	0	15	30	45
1	10	0	15	30
2	20	10	0	15
3	30	20	10	0

regret = 15 - (-5)

regret = 30 - 20

Each payoff is subtracted from the maximum payoff in its column:

$$\text{Regret}_{ij} = \left[\underset{k}{\text{Maximum}}\, r_{kj} \right] - r_{ij}$$

MINIMAX REGRET

$$\underset{k}{\text{Minimum}} \left\{ \max_{j} \left[\max_{i} r_{ij} \right] - r_{kj} \right\}$$

State of Nature (demand)

Decision	*0*	*1*	*2*	*3*	max regret
0	0	15	30	45	45
1	10	0	15	30	30
2	20	10	0	15	20
3	30	20	10	0	30

("regret")

The newsboy should order 2 newspapers from the delivery truck.

EVPI (Expected Value of Perfect Information)

Imagine the current sequence of events:

- Mother Nature, using the probability distribution, generates a random demand
- The newsboy, not knowing what demand had been determined by Nature, orders his newspapers
- The demand is then revealed to the newsboy, and he then receives a payoff

EVPI (Expected Value of Perfect Information)

Consider a new scenario:

- The newsboy pays Mother Nature a fee
- Mother Nature determines the demand as before
- Mother Nature then tells the newsboy what the demand will be
- The newsboy orders his newspapers
- The newsboy receives his payoff

What is the largest fee which the newsboy should be willing to pay?

EVPI

EVPI = {expected return with new scenario}
− {expected return with current scenario}

Assuming that, after learning what the demand will be, the newsboy orders enough to exactly satisfy the demand,

Expected return with new scenario is $\sum_{i=0}^{3} r_{ii} P_i$

$$= 0(0.1) + (15¢)(0.3) + (30¢)(0.4) + (45¢)(0.2$$
$$= 25.5¢$$

EVPI

Since the newsboy's expected return is currently 17.5¢

then

EVPI = 25.5¢ − 17.5¢ = 8¢

That is, possessing knowledge of the demand before he orders the newspapers will increase his expected return by 8¢.

Relationship between EVPI and "regret"

Regret

decision \ demand	0	1	2	3	Expected regret
0	0	15	30	45	**25.5¢**
1	10	0	15	30	**13 ¢**
2	20	10	0	15	**8 ¢**
3	30	20	10	0	**13 ¢**
P_j	0.1	0.3	0.4	0.2	

EVPI = Minimum Expected Regret

EXAMPLE

A manufacturer has a choice of either

- buying 9000 of a certain part at $20 each,

or

- making them at a setup cost of $50,000 plus $12 each

Average cost = $\frac{\$50{,}000 + 9000 \times \$12}{9000}$ = $17.56 per unit

Unfortunately, while the bought product is ***always*** satisfactory,
the product he makes is often ***defective***,
having a distribution of the percent defective (p) as:

p	0%	10%	20%	30%	40%
P{p}	.1	.2	.3	.25	.15

If a defective part is installed and discovered on final test of the product, it must be corrected at a cost of $10 each.

Construct a Payoff table,

with the 5 "states of nature" being the % defective, and the decisions being "make" and "buy".

Decision	percent defective				
	0%	10%	20%	30%	40%
Make	-158000	-167000	-176000	-185000	-194000
Buy	-180000	-180000	-180000	-180000	-180000

(A cost is interpreted as a ***negative*** *payoff, in order to be consistent with the criteria discussed earlier.)*

Payoff Table

Decision	p= 0%	10%	20%	30%	40%
Make	-158000	-167000	-176000	-185000	-194000
Buy	-180000	-180000	-180000	-180000	-180000

Regret table:

Decision	percent defective				
	0%	10%	20%	30%	40%
Make	0	0	0	5000	14000
Buy	22000	13000	4000	0	0

What is the decision, using criterion...

- **MAXIMIN** ?
- **MAXIMAX** ?
- **MINIMAX REGRET** ?
- **MAXIMUM EXPECTED PAYOFF** ?

What is...

- **EVPI** ? *Expected Value of Perfect Information*

MAXIMIN

Decision	p= 0%	10%	20%	30%	40%
Make	-158000	-167000	-176000	-185000	-194000
Buy	-180000	-180000	-180000	-180000	-180000

Minimum payoff for the decision "Make" is -194000,

Minimum payoff for the decision "Buy" is -180000.

Therefore, the decision selected by the maximin criterion will be "Buy",
since -180000>-194000.

MAXIMAX

Decision	p= 0%	10%	20%	30%	40%
Make	-158000	-167000	-176000	-185000	-194000
Buy	-180000	-180000	-180000	-180000	-180000

Maximum payoff for decision "Make" is -158000,

Maximum payoff for decision "Buy" is -180000.

Therefore, the decision selected by the maximax criterion is "Make".
since -158000>-180000.

MINIMAX REGRET *regret table*

Decision	p= 0%	10%	20%	30%	40%
Make	0	0	0	5000	14000
Buy	22000	13000	4000	0	0

The maximum regret for decision "Make" is 14000, and for "Buy" is 22000.

Therefore, the decision selected by the "minimax regret" criterion is "Make".

MAXIMUM EXPECTED PAYOFF

Decision	p= 0%	10%	20%	30%	40%
Make	-158000	-167000	-176000	-185000	-194000
Buy	-180000	-180000	-180000	-180000	-180000
probability:	*0.10*	*0.20*	*0.30*	*0.25*	*0.15*

The expected payoff for decision "Make" is
0.1x(-158000) + 0.2x(167000) + 0.3x(-176000) + 0.25x(-185000) + 0.15x(-194000) = -177350,
while for the decision "Buy" it is -180000.
Therefore, the decision selected by this criterion is "Make".

EVPI

Expected Value of Perfect Information:

If the manufacturer had a prediction of the defective rate in advance ***(possessed perfect information)***, he would choose

"Make" if p= 0, 10, or 20%,
and
"Buy" if p=30 or 40%:

Decision	p= 0%	10%	20%	30%	40%
Make	**-158000**	**-167000**	**-176000**	-185000	-194000
Buy	-180000	-180000	-180000	**-180000**	**-180000**
probability:	*0.10*	*0.20*	*0.30*	*0.25*	*0.15*

EVPI

defect rate:	0	10%	20%	30%	40%	
Payoff:	-158	-167	-176	-180	-180	$\times 10^3$
probability:	0.1	0.2	0.3	0.25	0.15	

EVWPI = Expected Value With Perfect Information
= 0.1x(-158000) + 0.2x(167000) + 0.3x(-176000) + 0.25x(-180000) + 0.15x(-180000)
= -174000.

EVWOI = Expected Value Without Information
= -177350

EVPI

EVWPI = - $174000

EVWOI = -$177350

EVPI = EVWPI - EVWOI = $3350

i.e., with perfect information,
the manufacturer's payoff is 3350 more than without.

Regret Table:

Decision	p= 0%	10%	20%	30%	40%	Expected regret
Make	0	0	0	5000	14000	$3350 ⟸
Buy	22000	13000	4000	0	0	$6000
probability:	*0.10*	*0.20*	*0.30*	*0.25*	*0.15*	

The decision which ***maximizes expected payoff*** is "Make"

The expected regret of this decision is
0 + 0 + 0 + 0.25x$5000 + 0.15x$14000 = $3350

EVPI = Minimum Expected Regret