Production Planning with 2 Items & Random Demand

Consider a production facility which can be devoted in each period to one of two products. For simplicity, we assume that the production rate is deterministic and that production is always at full capacity. Demand for the two products is random.

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- In our previous stochastic DP model for production planning, there was only a **single** item being produced.
- Suppose that there is a single facility which produces **two** items, each experiencing **random** demand.
- Production rate is constant—if the facility is devoted to item #i in a period, it produces *only* that item at *full* capacity.

Example

Daily Demand distribution:

- Item #1

D_1	0	1	2	3	4
$P\{D_1\}$	0.1	0.2	0.4	0.2	0.1

Item #2

D_2	0	1	2	3	4	5
$P\{D_2\}$	0.05	0.1	0.2	0.3	0.2	0.15

Costs & Production Rates:

Item	Production Rate	Production Cost	Storage Cost/day	Shortage Cost/day
1	4/day	\$5/day	\$0.50	\$20
2	3/day	\$10/day	\$1.00	\$30

Salvage value:

Item	Value
1	\$1.00
2	\$1.50

- The demand distributions are stationary, i.e., remain the same each day.
- Production is to be planned for a six-day planning horizon.
- Backorders are limited to 1 unit; storage is limited to 3 units.
- State: (I_1, I_2) where I_i is the inventory position for item #i: $I_i \in \{-1, 0, 1, 2, 3\}$
- Decision: $x \in \{0, 1, 2\}$ where x is the item to be produced.
- Demand = $[d_1, d_2]$ where d_i is demand for item #i

If there had been a cost for changing machine setup from one item to the other, then a state variable would need to be added, specifying the current machine setup (the decision x in the previous stage.)

Dynamic Programming Model:

State: (s_1, s_2) where s_i = inventory position for item *i*,

$s_i = s_i^+ - s_i^-$, $s_i^+ = \max\{0, s_i\}$ is the stock on hand and

$s_i^- = \max\{0, -s_i\}$ is the number of backorders

Decision: $x \in \{0,1,2\}$ ***is the item to be produced***

$R^x = (R_1^x, R_2^x)$ is the vector of production rates for the products corresponding to decision x, namely

$R^0 = (0,0), R^1 = (3,0), R^2 = (0,4)$

Demand: ***(d_1,d_2) with*** *probability* $p_1 \times p_2$

Costs:

h = holding cost,
k = shortage cost,
g = production cost,
a = salvage value,
b = penalty for any final backorders at stage 0

Optimal value function:

$f_n(s_1, s_2)$ = minimum expected cost for the final n stages if, at the beginning of that interval, the state of the system is (s_1, s_2)

Recursion:

For n=1, 2, ….

$$f_n(s_1,s_2) = \min_x \left\{ \sum_{i=1}^{2} \left(h_i s_i^+ + k_i s_i^-\right) + g_x + \sum_{d_1}\sum_{d_2} p_1 p_2 f_{n-1}\left(s_1 + R_1^x - d_1, s_2 + R_2^x - d_2\right) \right\}$$

where

$$f_0(s_1,s_2) = \sum_{i=1}^{2} \left([h_i - a_i] s_i^+ + [k_i + b_i] s_i^- \right)$$

APL Implementation of Model

State is two-dimensional (s1,s2) where si = inventory position of item i

```
s← ,s1 ∘.,s2
```

where si is list of inventory positions for product i

Demand for product i is random vector di, so that demand set is

```
d ← ,d1 ∘., d2
```

and (assuming independence of demands)

```
P ← ,p1 ∘.× p2
```

Decision set is {0,1,2} where **0** indicates machine is idle, and i=**1** or **2** specifies the item to be produced.

It is assumed that machine produces at full capacity (**R**i) if not idle.

Costs and demands are ***stationary***:

H is holding cost matrix,

```
H←H1 ∘.+ H2

H
    0     1     2     3
    0.5   1.5   2.5   3.5
    1     2     3     4
    1.5   2.5   3.5   4.5
```

K is shortage cost matrix,

```
K← K1 ∘.+ K2

K
    0   30    60    90
   20   50    80   110
   40   70   100   130
   60   90   120   150
```

G is production cost vector of length 3,

```
G

      0  5  10
```

A is salvage value matrix,

```
A← A1 ∘.+ A2
A
    0  1.5  3  4.5
    1  2.5  4  5.5
    2  3.5  5  6.5
    3  4.5  6  7.5
```


B is cost matrix for filling any final backorders

```
B← B1 ∘.+ B2
B
  0 10 20 30
  5 15 25 35
 10 20 30 40
 15 25 35 45
```

R is (nested) vector of production rates

```
R←(0,R1,0),¨(0,0,R2)
R
  0 0     4 0      0 3
```

where Ri is production rate for product i

The following arrays are the same for all stages, and may be defined for use at each stage:

```
    ∇ Define_Arrays;⎕io
      ⍝
      ⍝  Define arrays for DP model for planning production
      ⍝        of two products
      ⎕io←0
      H←H1 ∘.+ H2
      A←A1 ∘.+ A2
      B←B1 ∘.+ B2
      K←K1 ∘.+ K2
      R←(0,R1,0),¨(0,0,R2)
      Current_C←(((H[0⌈s])+K[0⌈-s])∘.+G)∘.+(⍴d)⍴0
      Next_S←(⌈/s)⌊(⌊/s)⌈s∘.+R∘.-d
      PAUSE
    ∇
```


APL Definition of Optimal Value Function

```
    ∇ z←F N;t
      ⍝
      ⍝ Optimal Value Function
      ⍝    for DP model of production planning problem with
      ⍝    two  products and constant production rate
      ⍝
      :if N>NN
         z←((H-A)[1+0⌈s]+(K+B)[1+0⌈-s]),BIG
      :else
         ⍝ Recursive definition of optimal value function
         z←P Minimize_E Current_C+(F N+1)[TRANSITION Next_S ]
      :endif
    ∇
```

Recursion type: forward

s		x: 0	1	2	Minimum
¯1	¯1	120.0000	105.2250	101.8000	101.8000
¯1	0	86.0000	71.2250	63.4500	63.4500
¯1	1	78.9500	64.1750	60.0500	60.0500
¯1	2	63.8000	49.0250	60.7000	49.0250
¯1	3	56.4500	41.6750	61.5500	41.6750
0	¯1	98.5000	78.9250	80.3000	78.9250
0	0	64.5000	44.9250	41.9500	41.9500
0	1	57.4500	37.8750	38.5500	37.8750
0	2	42.3000	22.7250	39.2000	22.7250
0	3	34.9500	15.3750	40.0500	15.3750
1	¯1	95.9750	74.5750	77.7750	74.5750
1	0	61.9750	40.5750	39.4250	39.4250
1	1	54.9250	33.5250	36.0250	33.5250
1	2	39.7750	18.3750	36.6750	18.3750
1	3	32.4250	11.0250	37.5250	11.0250
2	¯1	90.4000	74.7500	72.2000	72.2000
2	0	56.4000	40.7500	33.8500	33.8500
2	1	49.3500	33.7000	30.4500	30.4500
2	2	34.2000	18.5500	31.1000	18.5500
2	3	26.8500	11.2000	31.9500	11.2000
3	¯1	81.7250	75.0750	63.5250	63.5250
3	0	47.7250	41.0750	25.1750	25.1750
3	1	40.6750	34.0250	21.7750	21.7750
3	2	25.5250	18.8750	22.4250	18.8750
3	3	18.1750	11.5250	23.2750	11.5250

Stage 6

Sample Calculation

Consider state s=(1,2) and decision x=1, that is, there are currently one unit of item 1 and 2 of item 2 in stock, and the decision is made to produce item 1.

The costs in stage **6** are

- **storage** cost H[1,2] = 0.5 + 2.0 = 2.5
- **production** cost = 5

Total: **7.5**

To this we must add the expected value of the **terminal costs**, i.e., $f_7(s_1, s_2)$.

The terminal costs (i.e., $f_7(s_1, s_2)$) are determined by the salvage values, cost of filling final backorders, etc., and are shown in the following table:

State #	(s_1, s_2)	Cost $F_7(s)$
1	-1 -1	65
2	-1 0	25
3	-1 1	24.5
4	-1 2	24
5	-1 3	23.5
6	0 -1	40
7	0 0	0
8	0 1	-0.5
9	0 2	-1
10	0 3	-1.5
11	1 -1	39.5
12	1 0	-0.5
13	1 1	-1
14	1 2	-1.5
15	1 3	-2
16	2 -1	39
17	2 0	-1
18	2 1	-1.5
19	2 2	-2
20	2 3	-2.5
21	3 -1	38.5
22	3 0	-1.5
23	3 1	-2
24	3 2	-2.5
25	3 3	-3

For example, if the terminal state is $(1, -1)$ then the terminal costs are:

- Item #1: storage (0.5) + salvage (-1) = -0.5
- Item #2: shortage (30) + cost of filling final backorders (10) = 40

Total: 39.5

Table of computations of expected terminal cost for s = (1,2) and x = 1:

Demand #	Demand D	Probability P{D}	Resulting state	Terminal Costs	P{D}xCost
1	0 0	0.005	3 2	-2.5	-0.0125
2	0 1	0.01	3 1	-2	-0.02
3	0 2	0.02	3 0	-1.5	-0.03
4	0 3	0.01	3 -1	38.5	0.385
5	0 4	0.005	3 -1	38.5	0.1925
6	1 0	0.01	3 2	-2.5	-0.025
7	1 1	0.02	3 1	-2	-0.04
8	1 2	0.04	3 0	-1.5	-0.06
9	1 3	0.02	3 -1	38.5	0.77
10	1 4	0.01	3 -1	38.5	0.385
11	2 0	0.02	3 2	-2.5	-0.05
12	2 1	0.04	3 1	-2	-0.08
13	2 2	0.08	3 0	-1.5	-0.12
14	2 3	0.04	3 -1	38.5	1.54
15	2 4	0.02	3 -1	38.5	0.77

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Table of computations, continued….

Demand #	Demand D	Probability P{D}	Resulting state	Terminal Costs	P{D}xCost
15	2 4	0.02	3 -1	38.5	0.77
16	3 0	0.03	2 2	-2	-0.06
17	3 1	0.06	2 1	-1.5	-0.09
18	3 2	0.12	2 0	-1	-0.12
19	3 3	0.06	2 -1	39	2.34
20	3 4	0.03	2 -1	39	1.17
21	4 0	0.02	1 2	-1.5	-0.03
22	4 1	0.04	1 1	-1	-0.04
23	4 2	0.08	1 0	-0.5	-0.04
24	4 3	0.04	1 -1	39.5	1.58
25	4 4	0.02	1 -1	39.5	0.79
26	5 0	0.015	0 2	-1	-0.015
27	5 1	0.03	0 1	-0.5	-0.015
28	5 2	0.06	0 0	0	0
29	5 3	0.03	0 -1	40	1.2
30	5 4	0.015	0 -1	40	0.6

For example, after producing 4 units of item 1 there will be 5 units of item 1 and 2 of item 2 available to satisfy the demand.

If the demand happens to be (3,2), then the resulting state is (5,2) – (3,2) = (2,0), i.e., 2 units of item 1 and 0 units of item 2 remain.

In this case, the terminal cost (from earlier table) is $f_7(2,0) = -1$.

The sum of the products $p_d \times f_7(s+R-d)$ in the last column is 10.875.

Adding this to the costs in stage 6 (namely, 7.5) yields 18.375.

Stage 5

s		\ x: 0	1	2	Minimum
¯1	¯1	151.8000	139.6187	132.8325	132.8325
¯1	0	117.9650	105.8890	90.8625	90.8625
¯1	1	110.9550	99.0193	81.4675	81.4675
¯1	2	94.8325	83.0029	76.9825	76.9825
¯1	3	83.8625	71.6136	75.4100	71.6136
0	¯1	130.6562	112.3962	111.7061	111.7061
0	0	96.8281	78.6847	69.6951	69.6951
0	1	89.8285	71.8333	60.2178	60.2178
0	2	73.7061	55.9259	55.6881	55.6881
0	3	62.6951	44.6451	54.0950	44.6451
1	¯1	128.6512	106.1475	109.7724	106.1475
1	0	94.8460	72.3971	67.6611	67.6611
1	1	87.8762	65.4962	58.0099	58.0099
1	2	71.7724	49.7058	53.3819	49.7058
1	3	60.6611	38.8920	51.7475	38.8920
2	¯1	124.0225	102.9175	105.2279	102.9175
2	0	90.2470	69.1305	62.9666	62.9666
2	1	83.3175	62.1790	53.0416	53.0416
2	2	67.2279	46.6222	48.2619	46.6222
2	3	55.9666	36.4958	46.5613	36.4958
3	¯1	116.1187	101.3262	97.5029	97.5029
3	0	82.3890	67.4912	55.1136	55.1136
3	1	75.5193	60.4813	44.9859	44.9859
3	2	59.5029	45.0363	40.0819	40.0819
3	3	48.1136	35.4213	38.3350	35.4213

Stage 4

s		\ x: 0	1	2	Minimum
¯1	¯1	182.8325	171.3996	160.1865	160.1865
¯1	0	148.6355	137.2891	116.6006	116.6006
¯1	1	140.3020	129.1122	108.6568	108.6568
¯1	2	122.1865	111.2105	105.6728	105.6728
¯1	3	109.6006	98.2100	105.1505	98.2100
0	¯1	161.7762	144.3193	139.1273	139.1273
0	0	127.5790	110.2601	95.5105	95.5105
0	1	119.2447	102.1712	87.5081	87.5081
0	2	101.1273	84.3845	84.4097	84.3845
0	3	88.5105	71.2683	83.8305	71.2683
1	¯1	159.8856	138.4867	137.3328	137.3328
1	0	125.7056	104.4046	93.6495	93.6495
1	1	117.4040	96.2531	85.4724	85.4724
1	2	99.3328	78.3519	82.1113	78.3519
1	3	86.6495	65.1929	81.4015	65.1929
2	¯1	155.4430	135.8825	133.0364	133.0364
2	0	121.2901	101.7509	89.2221	89.2221
2	1	113.0380	93.4861	80.7302	80.7302
2	2	95.0364	75.3848	76.8749	75.3848
2	3	82.2221	62.3042	75.9205	62.3042
3	¯1	147.8996	134.8151	125.7105	125.7105
3	0	113.7891	100.6127	81.7100	81.7100
3	1	105.6122	92.1983	72.7767	72.7767
3	2	87.7105	73.8563	68.2851	68.2851
3	3	74.7100	60.8383	67.0174	60.8383

Stage 3

s		\ x: 0	1	2	Minimum
¯1	¯1	210.1865	199.8592	186.9948	186.9948
¯1	0	175.8279	165.4934	143.7571	143.7571
¯1	1	167.3164	156.9505	136.1235	136.1235
¯1	2	148.9948	138.4483	132.7472	132.7472
¯1	3	136.7571	125.5529	131.9563	125.5529
0	¯1	189.1335	173.5171	165.9392	165.9392
0	0	154.7748	139.1442	122.6703	122.6703
0	1	146.2626	130.5700	114.9767	114.9767
0	2	127.9392	111.9265	111.4857	111.4857
0	3	115.6703	98.7347	110.6375	98.7347
1	¯1	187.4379	167.7453	164.2133	164.2133
1	0	153.0785	133.3629	120.8378	120.8378
1	1	144.5636	124.7472	112.9416	112.9416
1	2	126.2133	106.0089	109.1793	106.0089
1	3	113.8378	92.7135	108.1963	92.7135
2	¯1	183.3318	164.9191	160.0463	160.0463
2	0	148.9705	130.5275	116.4680	116.4680
2	1	140.4472	121.8712	108.1992	108.1992
2	2	122.0463	103.0529	103.9122	103.0529
2	3	109.4680	89.8228	102.6690	89.8228
3	¯1	176.3592	163.3094	152.9483	152.9483
3	0	141.9934	128.9122	109.0529	109.0529
3	1	133.4505	120.2309	100.2269	100.2269
3	2	114.9483	101.4065	95.2391	95.2391
3	3	102.0529	88.3120	93.6508	88.3120

Stage 2

s		\ x: 0	1	2	Minimum
¯1	¯1	236.9948	226.7247	214.1007	214.1007
¯1	0	202.6711	192.3905	171.0050	171.0050
¯1	1	194.2602	183.9395	163.3652	163.3652
¯1	2	176.1007	165.5994	160.0488	160.0488
¯1	3	164.0050	152.8571	159.2723	152.8571
0	¯1	215.9421	200.4526	193.0454	193.0454
0	0	181.6181	166.1074	149.9193	149.9193
0	1	173.2066	157.6139	142.2208	142.2208
0	2	155.0454	139.1208	138.7918	138.7918
0	3	142.9193	126.0756	137.9592	126.0756
1	¯1	214.2502	194.7793	191.3244	191.3244
1	0	179.9254	160.4191	148.0952	148.0952
1	1	171.5106	151.8683	140.2026	140.2026
1	2	153.3244	133.2468	136.5022	133.2468
1	3	141.0952	120.0687	135.5346	120.0687
2	¯1	210.1581	192.0576	187.1704	187.1704
2	0	175.8307	157.6822	143.7435	143.7435
2	1	167.4053	149.0739	135.4945	135.4945
2	2	149.1704	130.3262	131.2684	130.3262
2	3	136.7435	117.1688	130.0405	117.1688
3	¯1	203.2247	190.5130	180.0994	180.0994
3	0	168.8905	156.1282	136.3571	136.3571
3	1	160.4395	147.4844	127.5713	127.5713
3	2	142.0994	128.6956	122.6440	122.6440
3	3	129.3571	115.6395	121.0717	115.6395

Stage 1

s		\ x: 0	1	2	Minimum
¯1	¯1	264.1007	253.8357	241.3101	241.3101
¯1	0	229.7911	219.5160	198.2526	198.2526
¯1	1	221.4080	211.0940	190.6502	190.6502
¯1	2	203.3101	192.8156	187.3463	187.3463
¯1	3	191.2526	180.1136	186.5763	180.1136
0	¯1	243.0479	227.5705	220.2549	220.2549
0	0	208.7382	193.2400	177.1671	177.1671
0	1	200.3545	184.7769	169.5065	169.5065
0	2	182.2549	166.3463	166.0906	166.0906
0	3	170.1671	153.3426	165.2648	153.3426
1	¯1	241.3564	221.9076	218.5341	218.5341
1	0	207.0458	187.5623	175.3434	175.3434
1	1	198.6589	179.0435	167.4887	167.4887
1	2	180.5341	160.4878	163.8035	160.4878
1	3	168.3434	147.3538	162.8439	147.3538
2	¯1	237.2655	219.1974	214.3816	214.3816
2	0	202.9523	184.8371	170.9939	170.9939
2	1	194.5553	176.2622	162.7824	162.7824
2	2	176.3816	157.5845	158.5752	157.5845
2	3	163.9939	144.4742	157.3569	144.4742
3	¯1	230.3357	217.6601	207.3156	207.3156
3	0	196.0160	183.2906	163.6136	163.6136
3	1	187.5940	174.6810	154.8645	154.8645
3	2	169.3156	155.9656	149.9597	149.9597
3	3	156.6136	142.9591	148.3986	142.9591

Stage 6

State	Decision	Optimal Value
¯1 ¯1	Produce 2	96.8000
¯1 0	Produce 2	58.4500
¯1 1	Produce 2	55.0500
¯1 2	Produce 1	47.2750
¯1 3	Produce 1	39.9250
0 ¯1	Produce 2	75.5500
0 0	Produce 2	37.2000
0 1	Produce 2	33.8000
0 2	Produce 1	21.9750
0 3	Produce 1	14.6250
1 ¯1	Produce 2	73.5250
1 0	Produce 2	35.1750
1 1	Produce 2	31.7750
1 2	Produce 1	18.3750
1 3	Produce 1	11.0250
2 ¯1	Produce 2	68.9500
2 0	Produce 2	30.6000
2 1	Produce 2	27.2000
2 2	Produce 1	18.5500
2 3	Produce 1	11.2000
3 ¯1	Produce 2	61.7750
3 0	Produce 2	23.4250
3 1	Produce 2	20.0250
3 2	Produce 1	18.8750
3 3	Produce 1	11.5250

Stage 5

State	Decision	Optimal Value
¯1 ¯1	Produce 2	128.1575
¯1 0	Produce 2	86.8375
¯1 1	Produce 2	78.7425
¯1 2	Produce 2	74.9075
¯1 3	Produce 1	68.6424
0 ¯1	Produce 2	107.0748
0 0	Produce 2	65.7143
0 1	Produce 2	57.5383
0 2	Produce 1	52.9334
0 3	Produce 1	42.2776
1 ¯1	Produce 1	103.8438
1 0	Produce 2	63.8429
1 1	Produce 2	55.4734
1 2	Produce 1	47.2336
1 3	Produce 1	36.9784
2 ¯1	Produce 1	100.9725
2 0	Produce 2	59.4426
2 1	Produce 2	50.7811
2 2	Produce 1	44.6588
2 3	Produce 1	34.9963
3 ¯1	Produce 2	93.8816
3 0	Produce 2	52.1424
3 1	Produce 2	43.2089
3 2	Produce 2	38.9546
3 3	Produce 1	34.0388

Stage 4

State	Decision	Optimal Value
¯1 ¯1	Produce 2	156.4215
¯1 0	Produce 2	113.5260
¯1 1	Produce 2	105.9880
¯1 2	Produce 2	102.9054
¯1 3	Produce 1	95.3232
0 ¯1	Produce 2	135.3609
0 0	Produce 2	92.4337
0 1	Produce 2	84.8354
0 2	Produce 1	81.2295
0 3	Produce 1	68.6011
1 ¯1	Produce 2	133.6046
1 0	Produce 2	90.5894
1 1	Produce 2	82.8066
1 2	Produce 1	75.4358
1 3	Produce 1	62.7715
2 ¯1	Produce 2	129.4012
2 0	Produce 2	86.2131
2 1	Produce 2	78.0991
2 2	Produce 1	72.6276
2 3	Produce 1	60.1107
3 ¯1	Produce 2	122.2589
3 0	Produce 2	78.8232
3 1	Produce 2	70.2484
3 2	Produce 2	65.9208
3 3	Produce 1	58.7602

Stage 3

State	Decision	Optimal Value
¯1 ¯1	Produce 2	183.8250
¯1 0	Produce 2	140.8559
¯1 1	Produce 2	133.3093
¯1 2	Produce 2	129.9061
¯1 3	Produce 1	122.6649
0 ¯1	Produce 2	162.7673
0 0	Produce 2	119.7663
0 1	Produce 2	112.1580
0 2	Produce 2	108.6483
0 3	Produce 1	95.8949
1 ¯1	Produce 2	161.0395
1 0	Produce 2	117.9321
1 1	Produce 2	110.1225
1 2	Produce 1	102.9387
1 3	Produce 1	89.9427
2 ¯1	Produce 2	156.8722
2 0	Produce 2	113.5634
2 1	Produce 2	105.3864
2 2	Produce 1	100.0526
2 3	Produce 1	87.1341
3 ¯1	Produce 2	149.7859
3 0	Produce 2	106.1649
3 1	Produce 2	97.4472
3 2	Produce 2	92.5621
3 3	Produce 1	85.6714

State	Decision	Optimal Value
¯1 ¯1	Produce 2	211.1423
¯1 0	Produce 2	168.1251
¯1 1	Produce 2	160.5094
¯1 2	Produce 2	157.1776
¯1 3	Produce 1	149.9731
0 ¯1	Produce 2	190.0869
0 0	Produce 2	147.0395
0 1	Produce 2	139.3657
0 2	Produce 2	135.9223
0 3	Produce 1	123.1938
1 ¯1	Produce 2	188.3645
1 0	Produce 2	145.2137
1 1	Produce 2	137.3454
1 2	Produce 1	130.2881
1 3	Produce 1	117.2042
2 ¯1	Produce 2	184.2079
2 0	Produce 2	140.8599
2 1	Produce 2	132.6356
2 2	Produce 1	127.3802
2 3	Produce 1	114.3294
3 ¯1	Produce 2	177.1347
3 0	Produce 2	133.4731
3 1	Produce 2	124.7154
3 2	Produce 2	119.8114
3 3	Produce 1	112.8213

Stage 2

State	Decision	Optimal Value
¯1 ¯1	Produce 2	238.4124
¯1 0	Produce 2	195.3758
¯1 1	Produce 2	187.7774
¯1 2	Produce 2	184.4676
¯1 3	Produce 1	177.2332
0 ¯1	Produce 2	217.3571
0 0	Produce 2	174.2904
0 1	Produce 2	166.6337
0 2	Produce 2	163.2121
0 3	Produce 1	150.4603
1 ¯1	Produce 2	215.6359
1 0	Produce 2	172.4659
1 1	Produce 2	164.6145
1 2	Produce 1	157.5849
1 3	Produce 1	144.4744
2 ¯1	Produce 2	211.4823
2 0	Produce 2	168.1150
2 1	Produce 2	159.9060
2 2	Produce 1	154.6832
2 3	Produce 1	141.6006
3 ¯1	Produce 2	204.4149
3 0	Produce 2	160.7332
3 1	Produce 2	151.9862
3 2	Produce 2	147.0852
3 3	Produce 1	140.0913

Stage 1

Suppose that the **initial inventory position** (state at stage 6) is **(0,0)**, that is, we initially have no stock of either item.

Consulting the table, we find that

- the minimum expected cost for the six-day planning period is **$174.29**, and
- the optimal decision is to produce item **#2**.

Table showing **optimal policies** for each stage:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
6	3	3	3	2	2	3	3	3	2	2	3	3	3	2	2	3	3	3	2	2	3	3	3	2	2
5	3	3	3	3	2	3	3	3	2	2	2	3	3	2	2	2	3	3	2	2	3	3	3	3	2
4	3	3	3	3	2	3	3	3	2	2	3	3	3	2	2	3	3	3	2	2	3	3	3	3	2
3	3	3	3	3	2	3	3	3	3	2	3	3	3	2	2	3	3	3	2	2	3	3	3	3	2
2	3	3	3	3	2	3	3	3	3	2	3	3	3	2	2	3	3	3	2	2	3	3	3	3	2
1	3	3	3	3	2	3	3	3	3	2	3	3	3	2	2	3	3	3	2	2	3	3	3	3	2

The optimal policies have converged....