PRODUCTION PLANNING WITH RANDOM DEMAND & BACKORDERING

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author

At the beginning of each period, the quantity to be produced of an item is to be decided.
The quantity demanded in each period is random, and independent of other period's demand.
In case of a shortage, demand may be backordered, but must be filled during the next period.

Planning Horizon = 4 periods
Maximum inventory = 6
Maximum backorder level = 3
Production Capacity = 3

DP Model

Stage n = # of periods remaining in planning period (n=4, 3, 2, 1, 0)

State S_n = inventory position (positive if stock on hand, negative if there are backorders)
$S_n \in \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

Decision X_n = quantity to be produced

Data

Inventory Cost
$1 each

i	1	2	3	4	5	6
h[i]	1	2	3	4	5	6
∆	1	1	1	1	1	1

Production Cost
setup: $25 plus $5 each

x	1	2	3
C[x]	30	35	40
∆	30	5	5

Shortage Cost
$3 each

b	1	2	3	4
S[b]	3	6	9	12
∆	3	3	3	3

marginal values

Salvage values
end of period 4

i	1	2	3	4	5	6
S[i]	3	6	9	12	15	18
∆	3	3	3	3	3	3

Demand Distribution

d	P{d}
0	0.1
1	0.2
2	0.3
3	0.3
4	0.1

independent identically-distributed

Optimal Value Function

$f_n(S_n)$ = minimum expected cost for the next n periods, given current inventory position is S_n
($S_n>0$ ==> stock on hand,
$S_n<0$ ==> backorder position)

Recursive Definition

$$
f_n(S_n)= \begin{cases} \displaystyle\min_{(-S_n)^+ \le X_n \le 3} \sum_{d=0}^{4} p_d \left\{ h(S_n) + c(X_n) + s(d - S_n - X_n)^+ + f_{n-1}(S_n+X_n - d) \right\} & \text{if } n > 1 \\ - v(S_0) & \text{if } n = 0 \end{cases}
$$

backorders must be filled in next period $\Longrightarrow (-S_n)^+ \le X_n \le 3$

h(S) = storage cost
c(X) = production cost
s(S) = shortage cost
v(S) = salvage value

APL code

```
∇VALUE←F_prodn N;t;C;stock_cost
 ⍝
 ⍝           Optimal Value Function of DP model
 ⍝        of the Stochastic Production Planning Problem
 ⍝                (with Backordering)
 →LAST IF N=0

 ⍝              Compute Monthly Cost
 stock_cost←SHORT∆COST[1+0⌈-s]+G[1+0⌈s] ⍝ cost of stock
 C←((stock_cost∘.+H)+BIG×0>s∘.+x) ∘.+0×d

 ⍝              Compute Transition Matrix
 t←TRANSITION  (⌊/s)⌈(⌈/s)⌊ s ∘.+ x ∘.- d

 ⍝              Evaluate Optimal Value Function
 VALUE←P MIN∆E  C + (F_prodn N-1)[t]
 →0

 LAST: ⍝ Include salvage value as negative cost,
 ⍝       and include cost of production to fill
 ⍝       any remaining backorders
 VALUE←(SHORT∆COST[1+0⌈-s]+(-SALVAGE)[1+0⌈s]+H[1+0⌈(-s)]
```

Salvage value = $3 each
Setup cost = $25
unit prod'n cost = $5

d	0	1	2	3	4
p_d	.1	.2	.3	.3	.1

Stage 1
(final period)

s \ x:	0	1	2	3
¯3	999999.99	999999.99	999999.99	87.50
¯2	999999.99	999999.99	79.50	72.80
¯1	999999.99	71.50	64.80	55.80
0	38.50	56.80	47.80	40.30
1	27.80	43.80	36.30	35.30
2	14.80	32.30	31.30	33.30
3	3.30	27.30	29.30	31.30
4	¯1.70	25.30	27.30	29.60
5	¯3.70	23.30	25.60	28.50
6	¯5.70	21.60	24.50	28.30

Example: Let's compute the expected cost for S=1, X=2

Example computation: S=1, X=2

d	p{d}	Cost				Total	p{d} x Cost
		storage	prod'n	shortage	future		
0	0.1	1	35	0	-9	27	2.7
1	0.2	1	35	0	-6	30	6.0
2	0.3	1	35	0	-3	33	9.9
3	0.3	1	35	0	0	36	10.8
4	0.1	1	35	0	33	69	6.9
							36.3

salvage value

shortage + prod'n costs

Stage 1

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	87.50
-2	999999.99	999999.99	79.50	72.80
-1	999999.99	71.50	64.80	55.80
0	38.50	56.80	47.80	40.30
1	27.80	43.80	36.30	35.30
2	14.80	32.30	31.30	33.30
3	3.30	27.30	29.30	31.30
4	-1.70	25.30	27.30	29.60
5	-3.70	23.30	25.60	28.50
6	-5.70	21.60	24.50	28.30

$f_1(S_1)$

State	Optimal Values	Optimal Decisions
-3	87.50	3
-2	72.80	3
-1	55.80	3
0	38.50	0
1	27.80	0
2	14.80	0
3	3.30	0
4	-1.70	0
5	-3.70	0
6	-5.70	0

$f_1(S_1)$ is found by computing the minimum in each row

Stage 2

Using $f_1(S_1)$, compute $f_2(S_2)$

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	120.85
-2	999999.99	999999.99	112.85	$\boxed{103.81}$
-1	999999.99	104.85	95.81	85.61
0	71.85	87.81	77.61	68.76
1	58.81	73.61	64.76	58.12
2	44.61	60.76	54.12	49.50
3	31.76	50.12	45.50	43.65
4	21.12	41.50	39.65	41.00
5	12.50	35.65	37.00	40.30
6	6.65	33.00	36.30	40.50

S_1	$f_1(S_1)$
-3	87.50
-2	72.80
-1	55.80
0	38.50
1	27.80
2	14.80
3	3.30
4	-1.70
5	-3.70
6	-5.70

Example: Let's compute value for S=-2, X=3

Example computation: S=-2, X=3

d	p{d}	Cost				Total	p{d} x Cost
		storage	prod'n	shortage	future		
0	0.1	0	40	6	27.8	73.8	7.38
1	0.2	0	40	6	38.5	84.5	16.9
2	0.3	0	40	6	55.8	101.8	30.54
3	0.3	0	40	6	72.8	118.8	35.64
4	0.1	0	40	6	87.5	133.5	13.35
							103.81

Stage 2

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	120.85
-2	999999.99	999999.99	112.85	103.81
-1	999999.99	104.85	95.81	85.61
0	71.85	87.81	77.61	68.76
1	58.81	73.61	64.76	58.12
2	44.61	60.76	54.12	49.50
3	31.76	50.12	45.50	43.65
4	21.12	41.50	39.65	41.00
5	12.50	35.65	37.00	40.30
6	6.65	33.00	36.30	40.50

$f_2(S_2)$

State	Optimal Values	Optimal Decisions
-3	120.85	3
-2	103.81	3
-1	85.61	3
0	68.76	3
1	58.12	3
2	44.61	0
3	31.76	0
4	21.12	0
5	12.50	0
6	6.65	0

Compute $f_2(S_2)$ *by finding the minimum value in each row*

Stage 3

Use $f_2(S_2)$ *to compute* $f_3(S_3)$

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	152.48
-2	999999.99	999999.99	144.48	134.47
-1	999999.99	136.48	126.47	115.78
0	103.48	118.48	107.78	98.72
1	89.48	103.78	94.72	87.16
2	74.78	90.72	83.16	76.20
3	61.72	79.16	72.20	66.49
4	50.16	68.20	62.49	59.26
5	39.20	58.49	55.26	54.85
6	29.49	51.26	50.85	53.23

S_2	$f_2(S_2)$
-3	120.85
-2	103.81
-1	85.61
0	68.76
1	58.12
2	44.61
3	31.76
4	21.12
5	12.50
6	6.65

Stage 3

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	152.48
-2	999999.99	999999.99	144.48	134.47
-1	999999.99	136.48	126.47	115.78
0	103.48	118.48	107.78	98.72
1	89.48	103.78	94.72	87.16
2	74.78	90.72	83.16	76.20
3	61.72	79.16	72.20	66.49
4	50.16	68.20	62.49	59.26
5	39.20	58.49	55.26	54.85
6	29.49	51.26	50.85	53.23

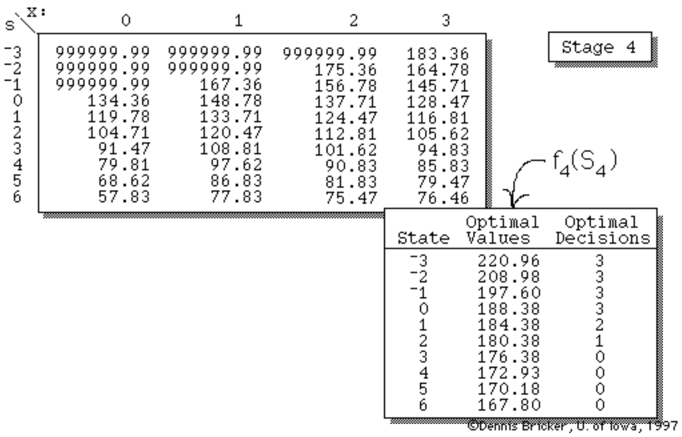
$f_3(S_3)$

State	Optimal Values	Optimal Decisions
-3	152.48	3
-2	134.47	3
-1	115.78	3
0	98.72	3
1	87.16	3
2	74.78	0
3	61.72	0
4	50.16	0
5	39.20	0
6	29.49	0

Stage 4

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	183.36
-2	999999.99	999999.99	175.36	164.78
-1	999999.99	167.36	156.78	145.71
0	134.36	148.78	137.71	128.47
1	119.78	133.71	124.47	116.81
2	104.71	120.47	112.81	105.62
3	91.47	108.81	101.62	94.83
4	79.81	97.62	90.83	85.83
5	68.62	86.83	81.83	79.47
6	57.83	77.83	75.47	76.46

S_3	$f_3(S_3)$
-3	152.48
-2	134.47
-1	115.78
0	98.72
1	87.16
2	74.78
3	61.72
4	50.16
5	39.20
6	29.49

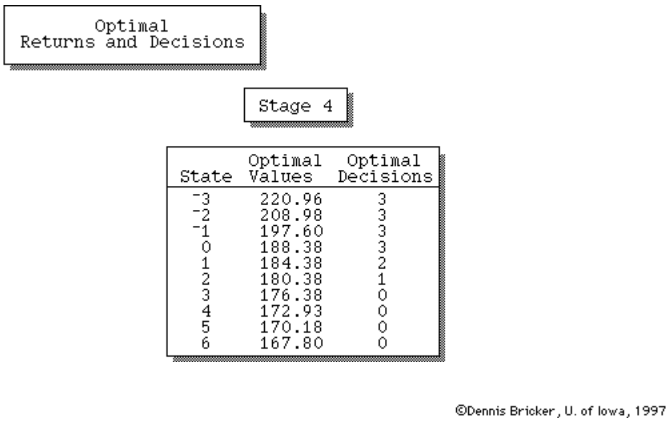


Stage 4

s\x:	0	1	2	3
¯3	999999.99	999999.99	999999.99	183.36
¯2	999999.99	999999.99	175.36	164.78
¯1	999999.99	167.36	156.78	145.71
0	134.36	148.78	137.71	128.47
1	119.78	133.71	124.47	116.81
2	104.71	120.47	112.81	105.62
3	91.47	108.81	101.62	94.83
4	79.81	97.62	90.83	85.83
5	68.62	86.83	81.83	79.47
6	57.83	77.83	75.47	76.46

$f_4(S_4)$

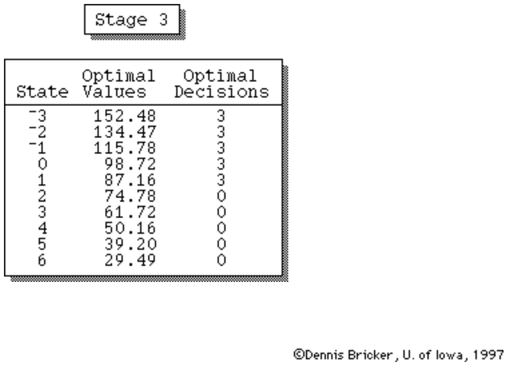
State	Optimal Values	Optimal Decisions
¯3	220.96	3
¯2	208.98	3
¯1	197.60	3
0	188.38	3
1	184.38	2
2	180.38	1
3	176.38	0
4	172.93	0
5	170.18	0
6	167.80	0



Optimal Returns and Decisions

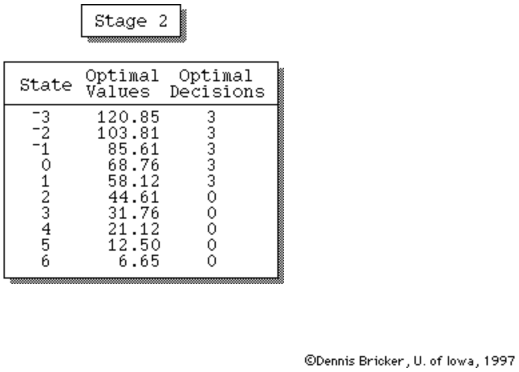
Stage 4

State	Optimal Values	Optimal Decisions
¯3	220.96	3
¯2	208.98	3
¯1	197.60	3
0	188.38	3
1	184.38	2
2	180.38	1
3	176.38	0
4	172.93	0
5	170.18	0
6	167.80	0



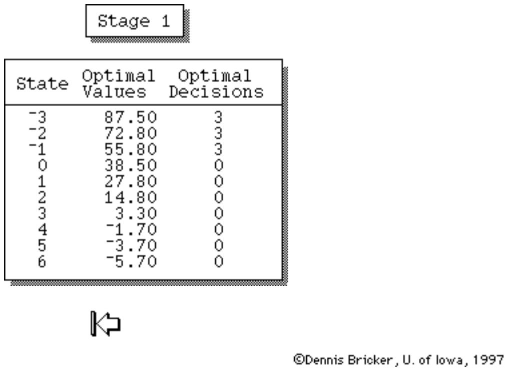
Stage 3

State	Optimal Values	Optimal Decisions
¯3	152.48	3
¯2	134.47	3
¯1	115.78	3
0	98.72	3
1	87.16	3
2	74.78	0
3	61.72	0
4	50.16	0
5	39.20	0
6	29.49	0



Stage 2

State	Optimal Values	Optimal Decisions
¯3	120.85	3
¯2	103.81	3
¯1	85.61	3
0	68.76	3
1	58.12	3
2	44.61	0
3	31.76	0
4	21.12	0
5	12.50	0
6	6.65	0



Stage 1

State	Optimal Values	Optimal Decisions
¯3	87.50	3
¯2	72.80	3
¯1	55.80	3
0	38.50	0
1	27.80	0
2	14.80	0
3	3.30	0
4	¯1.70	0
5	¯3.70	0
6	¯5.70	0