

# Power Plant Capacity Planning

This Hypercard stack was prepared by:  
 Dennis L. Bricker,  
 Dept. of Industrial Engineering,  
 University of Iowa,  
 Iowa City, Iowa 52242  
 e-mail: dennis-bricker@uiowa.edu



A power company is doing long-range planning, and has projected the following needs for additional power plants:

year	1	2	3	4	5	6	
# plants req'd	1	2	4	6	7	8	<i>cumulative</i>
cost/plant (\$million)	5.4	5.6	5.8	5.7	5.5	5.2	

There is a fixed cost of \$1.5 million if any plants are added in a year  
 At most 3 plants can be added in one year

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When should each of the 8 needed plants be added?

*(\$ might be saved by adding some plants in advance of the year in which they're needed, thereby avoiding the \$1.5million fixed cost each year.)*

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## DP Model

- Stage:  $n$  = # of years remaining in the planning period
- State:  $S_n$  = total # of plants which have been added
- Decision:  $X_n$  = # of plants to be added in the current year

*(Note that the stages are being numbered in reverse chronological order! That is, stage 6 is the first year in the planning period, etc.)*

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## Immediate cost of stage n

$$g_n(S_n, X_n) = \begin{cases} +\infty & \text{if } S_n < R_{n+1} \\ 0 & \text{if } X_n = 0 \text{ \& } S_n \geq R_n \\ 1.5 + C_n X_n & \text{if } X_n = 1, 2, \text{ or } 3 \text{ \& } S_n \geq R_n \end{cases}$$

*assures that sufficient plants have been added in previous year.*

*fixed cost*      *cost per plant*

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## Transition Function

If the decision  $X_n$  is made, when the current state is  $S_n$ , the resulting state will be

$$T(S_n, X_n) = S_n + X_n$$

*cumulative number of plants at beginning of next year*      *current number of plants*      *number of plants added*

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## Objective

- Minimizing Total Cost
- Minimizing Present Value of Total Cost

## Definition of Optimal Value

$f_n(S_n)$  = minimum total cost of the current & remaining stages ( $n, n-1, \dots, 0$ ) if stage  $n$  is entered in state  $S_n$

$$f_n(S_n) = \begin{cases} \text{minimum}_{(R_n - S_n) \leq X_n \leq 3} \{ g_n(S_n, X_n) + f_{n-1}(S_n + X_n) \} & \text{if } n \geq 1 \\ 0 & \text{if } n=0 \text{ \& } S_0 \geq R_1 \\ +\infty & \text{if } n=0 \text{ \& } S_0 < R_1 \end{cases}$$

$$z^+ = \max\{0, z\}$$

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# APL code

## Power Plant Capacity Planning

```

Z←F N;t;S;G
R
R      Optimal Value Function
R      Power Plant Capacity Planning problem
R
R→End IF N=0
R      Cost of adding plants in current year
G←((ρS)ρ0) ρ.+ (X>0)×1.5+COST[N]×X
R      Cumulative # of plants added at beginning of next yr.
S← Sρ.+X
R      Recursive definition of optimal value
Z← MIN G + (F N-1)[TRANSITION S+BIG×(S<RGMT[N])]
→0
End:Z←((ρS)ρ0),BIG
    
```

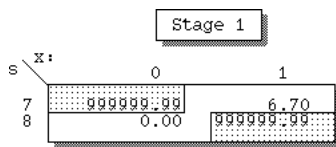
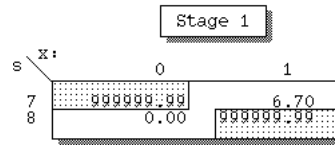
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Cost per plant =  $C_1 = 5.2$

Req'd # of plants =  $R_1 = 8$

$R_2 = 7$

999999.99 represents infinity (infeasible decision).



State	Optimal Values	Optimal Decisions	Resulting State
7	6.70	1	8
8	0.00	0	8

$f_1(S_1)$      $X_1^*$      $S_0^*$

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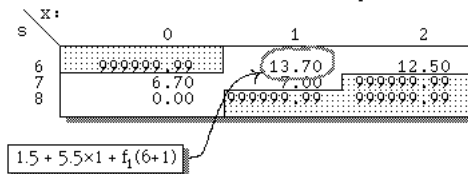
Stage 2

from computations just completed:

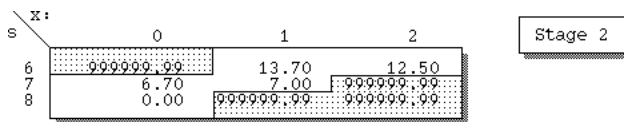
$$f_2(S_2) = \text{minimum}_{(R_2 - S_2) \leq X_2 \leq 3} g_2(S_2, X_2) + f_1(S_2 + X_2)$$

$S_2$	$f_1(S_1)$
7	6.70
8	0.00

Cost per plant =  $C_2 = 5.5$   
Req'd number of plants =  $R_2 = 7$



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State	Optimal Values	Optimal Decisions	Resulting State
6	12.50	2	8
7	6.70	0	7
8	0.00	0	8

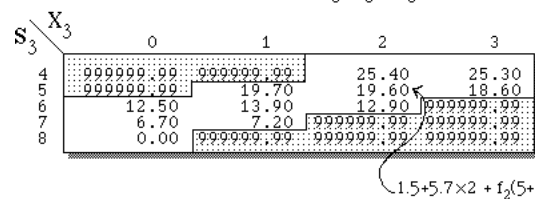
$f_2(S_2)$      $X_2^*$      $S_1^*$

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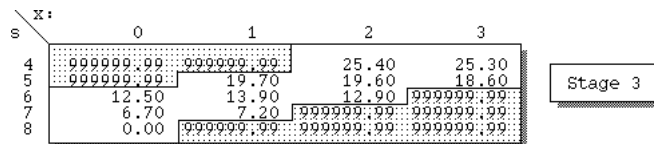
Stage 3

Cost per plant =  $C_3 = 5.7$

Req'd number of plants =  $R_3 = 6$

$$f_3(S_3) = \text{minimum}_{(R_3 - S_3) \leq X_3 \leq 3} g_3(S_3, X_3) + f_2(S_3 + X_3)$$


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State	Optimal Values	Optimal Decisions	Resulting State
4	25.30	3	7
5	18.60	3	8
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

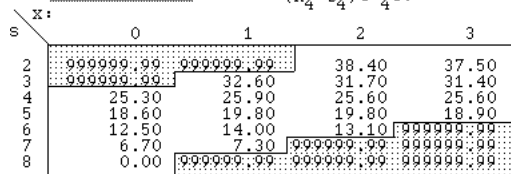
$f_3(S_3)$      $X_3^*$      $S_2^*$

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Stage 4

Cost per plant =  $C_4 = 5.8$

Req'd number of plants =  $R_4 = 4$

$$f_4(S_4) = \text{minimum}_{(R_4 - S_4) \leq X_4 \leq 3} g_4(S_4, X_4) + f_3(S_4 + X_4)$$


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		X:				
S		0	1	2	3	
2	999999.99	999999.99		38.40	37.50	Stage 4
3	999999.99	32.60	31.70	31.40		
4	25.30	25.90	25.60	25.60		
5	18.60	19.80	19.80	18.90		
6	12.50	14.00	13.10	999999.99		
7	6.70	7.30	999999.99	999999.99		
8	0.00	999999.99	999999.99	999999.99		

State	Optimal Values	Optimal Decisions	Resulting State
2	37.50	3	5
3	31.40	3	6
4	25.30	0	4
5	18.60	0	5
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

$f_4(S_4)$        $X_4^*$        $S_3^*$

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		X:				
S		0	1	2	3	
1	999999.99		44.60	44.10	43.60	Stage 5
2	37.50	38.50	38.00	36.90		
3	31.40	32.40	31.30	30.80		
4	25.30	25.70	25.20	25.00		
5	18.60	19.60	19.40	18.30		
6	12.50	13.80	12.70	999999.99		
7	6.70	7.10	999999.99	999999.99		
8	0.00	999999.99	999999.99	999999.99		

$S_4$	$f_4(S_4)$	Stage 5			
2	37.50	Cost per plant = $C_5 = 5.6$			
3	31.40	Req'd number of plants = $R_5 = 2$			
4	25.30	$f_5(S_5) = \text{minimum}_{(R_5 - S_5) \leq X_5 \leq 3} g_5(S_5, X_5) + f_4(S_5 + X_5)$			
5	18.60				
6	12.50				
7	6.70				
8	0.00				

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		X:				
S		0	1	2	3	
1	999999.99	44.60	44.10	43.60		Stage 5
2	37.50	38.50	38.00	36.90		
3	31.40	32.40	31.30	30.80		
4	25.30	25.70	25.20	25.00		
5	18.60	19.60	19.40	18.30		
6	12.50	13.80	12.70	999999.99		
7	6.70	7.10	999999.99	999999.99		
8	0.00	999999.99	999999.99	999999.99		

State	Optimal Values	Optimal Decisions	Resulting State
1	43.60	3	4
2	36.90	3	5
3	30.80	3	6
4	25.00	3	7
5	18.30	3	8
6	12.50	0	6
7	6.70	0	7
8	0.00	0	8

$f_5(S_5)$        $X_5^*$        $S_4^*$

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		X:				
S		0	1	2	3	
0	999999.99	50.50	49.20	48.50		Stage 6
1	43.60	43.80	43.10	42.70		
2	36.90	37.70	37.30	36.00		
3	30.80	31.90	30.60	30.20		
4	25.00	25.20	24.80	24.40		
5	18.30	19.40	19.00	17.70		
6	12.50	13.60	12.30	999999.99		
7	6.70	6.90	999999.99	999999.99		
8	0.00	999999.99	999999.99	999999.99		

$S_5$	$f_5(S_5)$	Stage 6			
1	43.60	Cost per plant = $C_6 = 5.4$			
2	36.90	Req'd number of plants = $R_6 = 1$			
3	30.80	$f_6(S_6) = \text{minimum}_{(R_6 - S_6) \leq X_6 \leq 3} g_6(S_6, X_6) + f_5(S_6 + X_6)$			
4	25.00				
5	18.30				
6	12.50				
7	6.70				
8	0.00				

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		X:				
S		0	1	2	3	
0	999999.99	50.50	49.20	48.50		Stage 6
1	43.60	43.80	43.10	42.70		
2	36.90	37.70	37.30	36.00		
3	30.80	31.90	30.60	30.20		
4	25.00	25.20	24.80	24.40		
5	18.30	19.40	19.00	17.70		
6	12.50	13.60	12.30	999999.99		
7	6.70	6.90	999999.99	999999.99		
8	0.00	999999.99	999999.99	999999.99		

State	Optimal Values	Optimal Decisions	Resulting State
0	48.50	3	3
1	42.70	3	4
2	36.00	3	5
3	30.20	3	6
4	24.40	3	7
5	17.70	3	8
6	12.30	2	8
7	6.70	0	7
8	0.00	0	8

$f_6(S_6)$        $X_6^*$        $S_5^*$

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Optimal Returns & Decisions

Stage 6:	Optimal State	Optimal Values	Optimal Decisions	Resulting State
	0	48.50	3	3
	1	42.70	3	4
	2	36.00	3	5
	3	30.20	3	6
	4	24.40	3	7
	5	17.70	3	8
	6	12.30	2	8
	7	6.70	0	7
	8	0.00	0	8

Stage 5:	Optimal State	Optimal Values	Optimal Decisions	Resulting State
	1	43.60	3	4
	2	36.90	3	5
	3	30.80	3	6
	4	25.00	3	7
	5	18.30	3	8
	6	12.50	0	6
	7	6.70	0	7
	8	0.00	0	8

Stage 4:	Optimal State	Optimal Values	Optimal Decisions	Resulting State
	2	37.50	3	5
	3	31.40	3	6
	4	25.30	0	4
	5	18.60	0	5
	6	12.50	0	6
	7	6.70	0	7
	8	0.00	0	8

Stage 3:	Optimal State	Optimal Values	Optimal Decisions	Resulting State
	4	25.30	3	7
	5	18.60	3	8
	6	12.50	0	6
	7	6.70	0	7
	8	0.00	0	8

Stage 2:	Optimal State	Optimal Values	Optimal Decisions	Resulting State
	6	12.50	2	8
	7	6.70	0	7
	8	0.00	0	8

Stage 1:	Optimal State	Optimal Values	Optimal Decisions	Resulting State
	7	6.70	1	8
	8	0.00	0	8

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**Optimal Solution**

The initial state (entering stage 6) is 0

**Power Plant Capacity Planning**

\*\*\* Optimal value is 48.5 \*\*\*

STAGE	STATE	DECISION
6	0	3
5	3	3
4	6	0
3	6	0
2	6	2
1	8	0
0	8	



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**Discounting Future Costs**

Let  $r$  = rate of interest

Then

\$1 invested now has the same value as  $\$1(1+r)^n$  after  $n$  periods,

and

\$1 paid  $n$  periods hence is equivalent to

$$\$1 \frac{1}{(1+r)^n} = \$1 \beta^n \text{ where } \beta = \frac{1}{1+r} \text{ (discount factor)}$$

paid now.



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Minimizing total discounted future costs, i.e., present value of future costs, in a DP model:

In the power plant capacity planning model:

$$f_n(S_n) = \begin{cases} \text{minimum}_{(R_n - S_n) \leq X_n \leq 3} \{ g_n(S_n, X_n) + \beta f_{n-1}(S_n + X_n) \} & \text{if } n \geq 1 \\ 0 & \text{if } n=0 \text{ \& } S_0 \geq R_1 \\ +\infty & \text{if } n=0 \text{ \& } S_0 < R_1 \end{cases}$$

**discount factor**

= minimum present value of costs of last  $n$  stages ("present"  $\leftrightarrow$  stage  $n$ )

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**Power Plant Capacity Planning**

**State Vector**

i	1	2	3	4	5	6	7	8	9
sf(i)	0	1	2	3	4	5	6	7	8

**Decision Vector**

i	1	2	3	4
xf(i)	0	1	2	3

Suppose that  $r=20\%$ , so that  $\beta = \frac{1}{1.2} = 0.83333$

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**Power Plant Capacity Planning**

```
z=F N;t;S;G
n
n      Optimal Value Function
n      Power Plant Capacity Planning problem
n      (minimizing present value of future costs, with
n      discount factor beta)
n
->End IF N=0
n
n      Cost of adding plants in current year
G←((ρS)ρ0)+.(x>0)×1.5+COST(N)×x
n
n      Cumulative # of plants added at beginning of next yr
S←S+.×x
n      Recursive definition of optimal value
z←MIN G + beta×(F N-1) [TRANSITION S+BIG×(S<RQMT(N))]
->0
End:z←((ρS)ρ0),BIG
Note: recursion is backward
```

**APL Code**

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**Power Plant Capacity Planning**

Recursion type: backward

S \ x:	0	1	2	3	—Stage 1—
5	9999.99	9999.99	9999.99	17.10	
6	9999.99	9999.99	11.90	9999.99	
7	9999.99	6.70	9999.99	9999.99	
8	0.00	9999.99	9999.99	9999.99	

State	Optimal Values	Optimal Decisions	Resulting State
5	17.10	3	8
6	11.90	2	8
7	6.70	1	8
8	0.00	0	8

$f_1(S_1)$

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S \ x:	0	1	2	3
4	9999.99	9999.99	9999.99	23.58
5	9999.99	9999.99	18.08	18.00
6	9999.99	12.58	12.50	9999.99
7	5.58	7.00	9999.99	9999.99
8	0.00	9999.99	9999.99	9999.99

S <sub>1</sub>	f <sub>1</sub> (S <sub>1</sub> )
5	17.10
6	11.90
7	6.70
8	0.00

S \ x:	0	1	2	3
3	9999.99	9999.99	9999.99	29.02
4	9999.99	9999.99	23.32	23.25
5	9999.99	17.62	17.55	18.60
6	10.42	11.85	12.90	9999.99
7	4.65	7.20	9999.99	9999.99
8	0.00	9999.99	9999.99	9999.99

S <sub>2</sub>	f <sub>2</sub> (S <sub>2</sub> )
4	23.58
5	18.00
6	12.50
7	5.58
8	0.00

---Stage 2---

State	Optimal Values	Optimal Decisions	Resulting State
4	23.58	3	7
5	18.00	3	8
6	12.50	2	8
7	5.58	0	7
8	0.00	0	8

$f_2(S_2)$

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---Stage 3---

State	Optimal Values	Optimal Decisions	Resulting State
3	29.02	3	6
4	23.25	3	7
5	17.55	2	7
6	10.42	0	6
7	4.65	0	7
8	0.00	0	8

$f_3(S_3)$

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s \ x:	0	1	2	3	S <sub>3</sub>	f <sub>3</sub> (S <sub>3</sub> )
1	9999.99	9999.99	9999.99	38.28	3	29.02
2	9999.99	9999.99	32.48	33.53	4	23.25
3	9999.99	26.68	27.73	27.58	5	17.55
4	19.38	21.93	21.78	22.78	6	10.42
5	14.63	15.98	16.98	18.90	7	4.65
6	8.68	11.18	13.10	9999.99	8	0.00
7	3.88	7.30	9999.99	9999.99		
8	0.00	9999.99	9999.99	9999.99		

State	Optimal Values	Optimal Decisions	Resulting State
1	38.28	3	4
2	32.48	2	4
3	26.68	1	4
4	19.38	0	4
5	14.63	0	5
6	8.68	0	6
7	3.88	0	7
8	0.00	0	8

---Stage 4---

f<sub>4</sub>(S<sub>4</sub>)

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s \ x:	0	1	2	3	S <sub>4</sub>	f <sub>4</sub> (S <sub>4</sub> )
0	9999.99	9999.99	39.76	40.53	1	38.28
1	9999.99	34.16	34.93	34.45	2	32.48
2	27.06	29.33	28.85	30.49	3	26.68
3	22.23	23.25	24.89	25.53	4	19.38
4	16.15	19.29	19.93	21.53	5	14.63
5	12.19	14.33	15.93	18.30	6	8.68
6	7.23	10.33	12.70	9999.99	7	3.88
7	3.23	7.10	9999.99	9999.99	8	0.00
8	0.00	9999.99	99			

State	Optimal Values	Optimal Decisions	Resulting State
0	39.76	2	2
1	34.16	1	2
2	27.06	0	2
3	22.23	0	3
4	16.15	0	4
5	12.19	0	5
6	7.23	0	6
7	3.23	0	7
8	0.00	0	8

---Stage 5---

f<sub>5</sub>(S<sub>5</sub>)

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s \ x:	0	1	2	3	S <sub>5</sub>	f <sub>5</sub> (S <sub>5</sub> )
0	999999.99	35.37	34.85	36.23	0	39.76
1	28.47	29.45	30.83	31.16	1	34.16
2	22.55	25.43	25.76	27.86	2	27.06
3	18.53	20.36	22.46	23.73	3	22.23
4	13.46	17.06	18.33	20.39	4	16.15
5	10.16	12.93	14.99	17.70	5	12.19
6	6.03	9.59	12.30	9999.99	6	7.23
7	2.69	6.90	9999.99	9999.99	7	3.23
8	0.00	9999.99	9999.99	9999.99	8	0.00

State	Optimal Values	Optimal Decisions	Resulting State
0	34.85	2	2
1	28.47	0	1
2	22.55	0	2
3	18.53	0	3
4	13.46	0	4
5	10.16	0	5
6	6.03	0	6
7	2.69	0	7
8	0.00	0	8

--Stage 6---

f<sub>6</sub>(S<sub>6</sub>)

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Power Plant Capacity Planning

\*\*\* Optimal value is 34.85369084 \*\*\*

STAGE	STATE	DECISION
6	0	2
5	2	0
4	2	2
3	4	3
2	7	0
1	7	1
0	8	



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