Modeling Exercises

Queues as Continuous-time Markov chains

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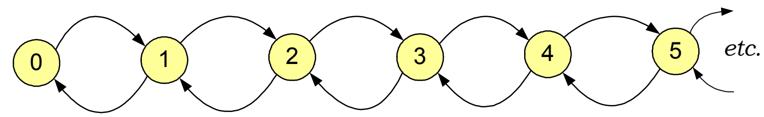
In each of the following cases, unless specified otherwise:

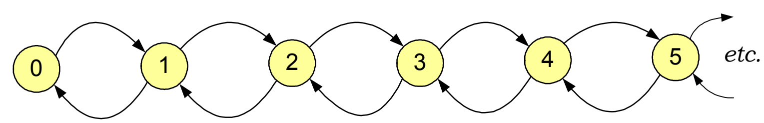
- customers arrive according to a Poisson process at the rate λ
- each of 2 servers works at the rate μ, with the service time having an exponential distribution.

Note: A birth-death model is not appropriate for all of these queues!

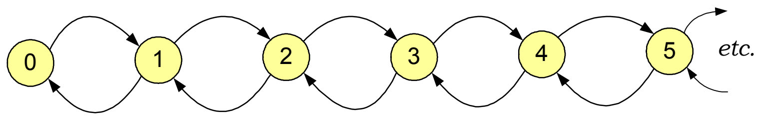
If a customer arrives and finds both servers busy, there is a 25% probability that he departs without entering the queue.



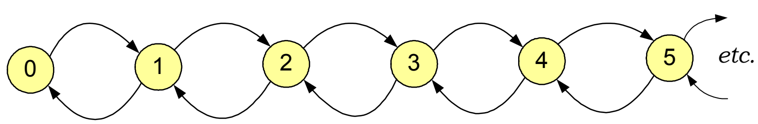
If a server finishes serving a customer and no customers are waiting, he helps out the other server if that server is busy, reducing the mean time for the job by 25%.



Arrivals are according to a Poisson process, but each arrival consists of either 1 or 2 customers, with probability 75% and 25%, respectively.



A waiting customer may get discouraged and leave the queue at any time--the length of time which he will wait having exponential distribution with mean $\frac{1}{3\lambda}$

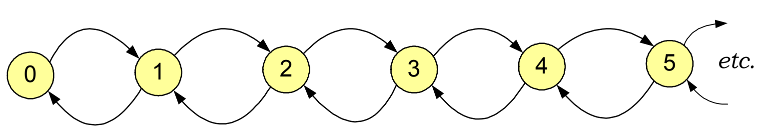


One-third of the customers require only a minor service, requiring only half the time of a regular service.

Server B works at half the rate of server A.
When both servers are idle, an arriving customer prefers server A, and if a customer is being served by B when A becomes free, he immediately switches to A.

Server B works at half the rate of server A.
When both servers are idle, an arriving customer prefers server A.
A customer may not switch servers once his service has begun.

There is a 10% probability that service of the customers is done improperly, in which case the customer re-enters the queue to be served again. (Mean service time in this case is the same as the original mean service time, $1/\mu$.)



Two types of customers arrive at a single-server queue, each according to a Poisson process:

VIPs with rate λ_1, and

NBs (nobodys) with rate λ_2.

Service rates are μ_1 and μ_2, respectively.
The VIPs have complete priority over NBs.
If a NB is being served when a VIP arrives, he is "dropped" immediately.
His service then resumes when no VIPs are in the system.

Each service operation for a customer consists of 2 separate tasks, each requiring a time having exponential distribution with mean $\frac{1}{2\mu}$.

There is a single server. When he becomes idle, he takes a break until 3 customers have arrived and wait for service.

There is a single server, who takes a break when he becomes idle. In this case, the length of the break is exponentially distributed with mean 15 minutes.

At any time, a "catastrophe" may occur, and all customers in the queue immediately depart. The time between such events is exponentially distributed with mean 5 hours.

At a taxi stand, taxis looking for customers and customers looking for taxis arrive according to Poisson processes with rates λ_t and λ_c, respectively.
A taxi will always wait if no customers are at the stand. However, an arriving customer waits only if there are 2 or fewer customers already waiting.

Four customers circulate between two single-server systems,
i.e., all customers leaving server A enter the queue of server B, and vice versa.
Server B works at half the rate of server A.

Customers arrive one at a time at a single-server queue, but the server processes the customers two at a time, unless only one customer remains in the queue when ready to begin the next service, in which case that single customer is served.
If a single customer is being served and a new customer arrives, the new customer must wait until service is completed.